Chapter Two

MASS ENERGY EQUIVALENCE

2.1 Derivation of Relativistic Mass

There are two identical particles A and B, where the rest mass of particle A is $m_{A,R} = m_0$ and rest mass of particle B is $m_{B,R} = m_0$.

There are two inertial frame S and S' where S' is moving with velocity with respect to S is $v\hat{i}$.

$-v_A\hat{j}$ is velocity of A from S frame. and $v'_B\hat{i}$ is velocity of B from S' frame. They are separated by distance Y at $t = 0$. If T_0 is time in which A and B travel distance Y from S and S' frame respectively. So $v_A = v'_B = \frac{Y}{T_0}$

From symmetry we can conclude that initial momentum of system is zero..

From S' frame, $v_{B,S'} = v_B\hat{j}$ $m_{B,S'} = m'_B$

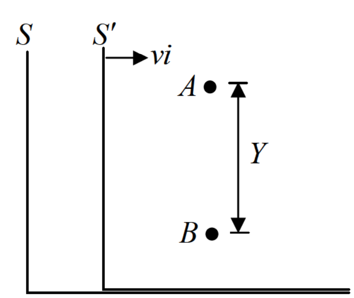

Figure 2.1

From S frame, $v_{S',S} = v\hat{i}$, $v_{A,S} = -v_A\hat{j}$ $m_{A,S} = m_A$

$$\vec{v}_{B,S} = v\hat{i} + v'_B\sqrt{1-\frac{v^2}{c^2}}\hat{j} = v\hat{i} + v_B\hat{j} \quad \text{where } v_B = v'_B\sqrt{1-\frac{v^2}{c^2}}$$

$$v_A = \frac{Y}{T_0} \quad v_B = \frac{Y}{T} = \frac{Y}{T_0}\sqrt{1-\frac{v^2}{c^2}}$$

From conservation of momentum

$$m_A v_A = m_B v_B = m_A\frac{Y}{T_0} = m_B\frac{Y}{T_0}\sqrt{1-\frac{v^2}{c^2}} \Rightarrow m_B = \frac{m_A}{\sqrt{1-\frac{v^2}{c^2}}}$$

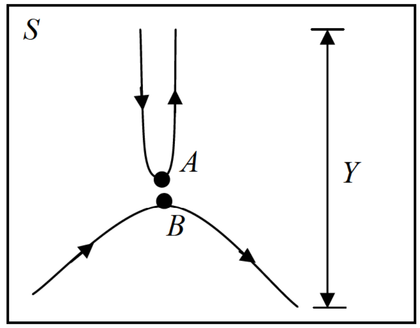

Figure 2.2

Suppose A is rest with respect to A $m_A = m_{A,R} = m_0 = m_{B,R}$

$$m_B = \frac{m_A}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_{B,R}}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow m_B > m_{B,R} \qquad \text{so } m(v) = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Relativistic Mass

The mass of a body moving at a speed v relative to an observer is larger than its mass when it is at rest relative to the observer.

$m = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$, where m_0 is rest mass of body and m is observed mass.

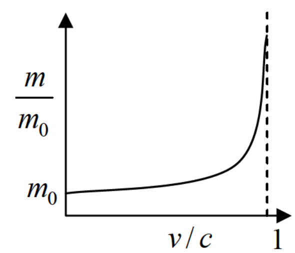

Figure 2.3

Relativistic Momentum

$$\vec{p} = m\vec{v} = \frac{m_0\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Relativistic force in one dimension and Newton's law of motion

The force is defined as $F = \dfrac{dp}{dt}$ where p is relativistic momentum.

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = \frac{d}{dt}\left(\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\frac{dv}{dt} + m_0 v\frac{d}{dt}\left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

$$\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\frac{dv}{dt} + \frac{m_0 v}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\frac{v}{c^2}\frac{dv}{dt} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\frac{dv}{dt}\left(1+\frac{\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}}\right) = \frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\frac{dv}{dt}$$

Relativistic Kinetic Energy

Using work energy theorem $T=\int F.ds=\int_0^s \frac{dp}{dt}dx=\int_0^v dmu.\frac{dx}{dt}=\int_0^v dmu.u$

$$T=\int_0^v udm.u+mu.du$$

$$m=\frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}=m^2\left(1-\frac{u^2}{c^2}\right)=m_0^2\Rightarrow m^2c^2-m^2u^2=m_0^2c^2\Rightarrow 2mdmc^2-2mdmu^2-m^2 2udu=0$$

$$dmc^2=dmu^2+mudu=T=\int_0^v udm.u+mu.du=\int_{m=m_0}^{m(v)} c^2dm=m(v)c^2-m_0c^2\Rightarrow T=\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}-m_0c^2$$

Case when $\frac{v}{c}\to 0$,

$$T=m_0c^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right)=m_0c^2\left(\left(1-\frac{v^2}{c^2}\right)^{-1/2}-1\right)=m_0c^2\left(1+\frac{1}{2}\frac{v^2}{c^2}...-1\right)=\frac{1}{2}m_0v^2$$

2.2 Rest Mass Energy and Relativistic Energy

Einstein suggested that, if particle is rest with respect to observe then he will measure rest mass m_0 and equivalent energy is measured as rest mass energy as m_0c^2.

If particle of rest mass m_0 moving with observer then with speed v then equivalent energy is identified as relativistic energy as $E=mc^2\Rightarrow\frac{m_0c^2}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}$

Relationship between Total Energy and Momentum for Free Particle

Total energy and momentum are given by $E=\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$ $p=\frac{m_0v}{\sqrt{1-\frac{v^2}{c^2}}}$

$$\Rightarrow p^2\left(1-\left(\frac{v^2}{c^2}\right)\right)=m_0^2v^2\Rightarrow m_0^2v^2+\frac{p^2v^2}{c^2}=p^2\Rightarrow v^2=\frac{p^2c^2}{m_0^2c^2+p^2}\Rightarrow v=\left(\frac{p^2c^2}{m_0^2c^2+p^2}\right)^{1/2}$$

Putting the value of $v^2=\frac{p^2c^2}{m_0^2c^2+p^2}$ in equation $E=\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$

We get the equation as $E^2=m_0^2c^4+p^2c^2\Rightarrow E=\left(m_0^2c^4+p^2c^2\right)^{1/2}$