Chapter One

LORENTZ TRANSFORMATION

1.1 Inertial Frame of Reference

Any frame can be denoted by a coordinate system with origin O and unit vector $\hat{i}$, $\hat{j}$ and $\hat{k}$ along direction x, y and z respectively as shown in figure.

If we are defining two different frame which means two different coordinate system.

The frame which is moving with constant velocity identified as inertial frame of reference. From any inertial frame of reference law of nature must be invariant. An inertial frame is also defined in which Newton's law of motion is defined.

Galilean Transformations:

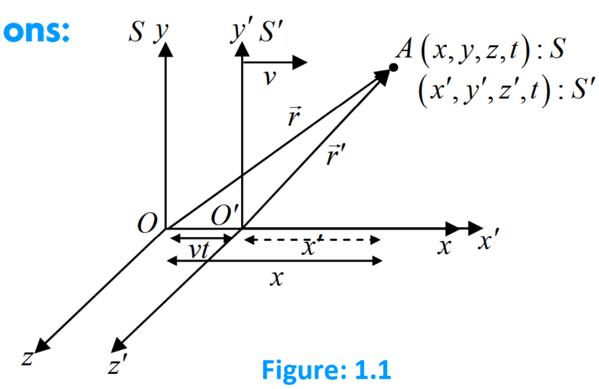

Figure: 1.1

There are two inertial frames denoted by S with origin O and S' with origin O' as shown in figure. At time $t=0$ origin of both frames coincides. A frame S' which is moving with constant velocity v relative to an inertial frame S. Frame S' moving along $\hat{i}$ direction with respect to S. After time t the distance between O and O' is vt.

There is point A in space. After time t the position vector of A from origin O and O' is measured as $\vec{r}$ and $\vec{r}'$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{r}' = x'\hat{i} + y'\hat{j} + z'\hat{k}$ respectively.

From the geometry $x = x' + vt$, $y = y'$ and $z = z'$.

We can write $x' = x - vt,\ y' = y$ and $z = z'$ which is linear transformation of co-ordinates from one inertial frame to another are referred as Galilean transformations.

Inverse Galilean transformation can be achieved just by changing the sign of velocity v to $-v$ given by

$$x = x' + vt, \qquad y = y', z = z',$$

In term of vector we can represent Galilean transformation and Inverse Galilean transformation as $\vec{r}' = \vec{r} - \vec{v}t$ and $\vec{r} = \vec{r}' + \vec{v}t$

Relative Velocity in Galilean Transformation

Let us assume point A is moving with respect to S' is u'_x, u'_y and u'_z in $\hat{i}, \hat{j}$ and $\hat{k}$ direction where $u'_x = \frac{dx'}{dt}$ $u'_y = \frac{dy'}{dt}$ and $u'_z = \frac{dz'}{dt}$

The same point A is moving with respect to S with respect to u_x, u_y and u_z in $\hat{i}, \hat{j}$ and $\hat{k}$ direction where $u_x = \frac{dx}{dt}$ $u_y = \frac{dy}{dt}$ and $u_z = \frac{dz}{dt}$

The relation ship between speed measured by S and S' by differentiating with respect to time t to equation $x = x' + vt, \qquad y = y',\ z = z'$,

$\frac{dx}{dt} = \frac{dx'}{dt} + v \Rightarrow u_x = u'_x + v$, $\frac{dy}{dt} = \frac{dy'}{dt} \Rightarrow u_y = u'_y$ and $\frac{dz}{dt} = \frac{dz'}{dt} \Rightarrow u_z = u'_z$

Now we can have formula

$$u_x = u'_x + v \text{ or } u'_x = u_x - v$$

$$u_y = u'_y,\ u_z = u'_z$$

In the vector form relative velocity or velocity transformation is given as

$$\frac{d\vec{r}}{dt} = \vec{v} + \frac{d\vec{r}'}{dt}, \vec{u} = \vec{v} + \vec{u}'$$

The Acceleration Transformation

Assume point A is moving with acceleration $\vec{a}' = \frac{d^2\vec{r}'}{dt^2}$ from frame S'. If observer will be attached to frame S then he will measure acceleration of $\vec{a} = \frac{d^2\vec{r}}{dt}$.

Now let us find relation between $\vec{a}$ and $\vec{a}'$

Differentiate with respect to time t $\vec{r} = \vec{r}' + \vec{v}t$ we get $\frac{d\vec{r}}{dt} = \frac{d\vec{r}'}{dt} + \vec{v}$

Again differentiate $\frac{d\vec{r}}{dt} = \frac{d\vec{r}'}{dt} + \vec{v}$ with respect to time t we get $\frac{d^2\vec{r}}{dt^2} = \frac{d^2\vec{r}'}{dt^2} + \frac{d\vec{v}}{dt}$

$\vec{v}$ is constant so $\frac{d\vec{v}}{dt} = 0$, therefore, $\frac{d^2\vec{r}}{dt^2} = \frac{d^2\vec{r}'}{dt^2} \Rightarrow a = a'$

It is found that acceleration measured on both frames is same.

Inertial frame is also known as frame where Newton's law of motion is applicable.

Discrepancy in Galilean Transformation

Let us assume the light pulse is emerged from origin of frame S' and move along $\hat{i}$ direction. The observer attached with S' frame will measure speed of light as $c = 3\times10^8\, m/\text{sec}$ in $\hat{i}$ direction.

If the observer will measure the speed of same light from S frame and if he will use Galilean transformation then he use formula $u_x = u_x' + v$, $u_y = u_y'$, $u_z = u_z'$, where $u_x' = c$, $u_y' = 0$ $u_z' = 0$. So measurement of velocity components of light from S is $u_x = c + v$, $u_y = 0$ and $u_z = 0$.

So we have seen that speed of light from S as $u' = c$, and same light will move from S' is with speed $u = c + v$ where $c = 3\times10^8\, m/\text{sec}$ is speed of light.