

STR Tutorial-1 (Solution)

Solution 1: $x_+ = x + ct \Rightarrow x'_+ = x' + ct' = \left(\frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \right) + c \left(\frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}} \right)$

$$\Rightarrow x'_+ = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(x \left(1 - \frac{v}{c} \right) + ct \left(1 - \frac{v}{c} \right) \right) = \frac{1 - \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} (x + ct) = \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} x_+ = \sqrt{\frac{1 - \beta}{1 + \beta}} x_+$$

$x'_- = x' - ct' \Rightarrow \left(\frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \right) - c \left(\frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}} \right)$

$$x'_- = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(x \left(1 + \frac{v}{c} \right) - ct \left(1 + \frac{v}{c} \right) \right) = \frac{1 + \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} (x - ct) = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} x_- = \sqrt{\frac{1 + \beta}{1 - \beta}} x_-$$

Solution 2: $x'_2 - x'_1 + 9 \times 10^9 m$ and $t'_2 - t'_1 = 0$. Then

$$t_2 - t_1 = \left(\frac{t'_2 + \frac{v}{c^2} x'_2}{\sqrt{1 - \frac{v^2}{c^2}}} \right) - \left(\frac{t'_1 + \frac{v}{c^2} x'_1}{\sqrt{1 - \frac{v^2}{c^2}}} \right) \Rightarrow t_2 - t_1 = \frac{t'_2 - t'_1}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{v}{c^2} \frac{(x'_2 - x'_1)}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{v}{c^2} \frac{(x'_2 - x'_1)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Put $v = 0.8c \Rightarrow t_2 - t_1 \approx 40 \text{ sec}$

Solution 3: $\Delta t = 0$, $t'_2 - t'_1 = 5 \mu s$, $x'_2 - x'_1 = 5 \text{ km}$ $v = ?$

$$t_2 - t_1 = \frac{t'_2 + \left(\frac{-v}{c^2} \right) x'_2}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{t'_1 + \left(\frac{-v}{c^2} \right) x'_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow \frac{\left[(t'_2 - t'_1) - \frac{v}{c^2} (x'_2 - x'_1) \right]}{\sqrt{1 - \frac{v^2}{c^2}}} = 0 \Rightarrow 5 \times 10^{-6} - \frac{v}{c^2} \times 5 \times 10^3 = 0$$

$$\Rightarrow \frac{v}{c^2} = \frac{5 \times 10^{-6}}{5 \times 10^3} = 10^{-9} \Rightarrow v = 3 \times 10^8 \times c \times 10^{-9} = 0.3 c$$

Solution 4: $u'_z = v \cos \theta$ $u'_x = v \sin \theta$ $u'_y = 0$

$$v = u$$

$$u_z = \frac{u'_z + v}{1 + \frac{u'_z v}{c^2}} = \frac{v \cos \theta + u}{1 + \frac{uv \cos \theta}{c^2}}$$

$$u_x = \frac{u u'_x \sqrt{1 - v^2/c^2}}{1 + \frac{u'_z v}{c^2}} = \frac{v \sin \theta \sqrt{1 - v^2/c^2}}{1 + \frac{uv \cos \theta}{c^2}}; u_y = 0$$

$$\therefore u = \sqrt{u_x^2 + u_y^2 + u_z^2} = \sqrt{\frac{u^2 + u^2 + 2uv \cos \theta - \frac{u^2 v^2}{c^2} \sin^2 \theta}{\left(1 + \frac{uv}{c^2} \cos \theta\right)}}$$

$$u = \sqrt{\frac{u^2 + 2uv \cos \theta + u^2 \left(1 - \frac{u^2}{c^2} \sin^2 \theta\right)}{1 + \frac{uv}{c^2} \cos \theta}}$$

$$\text{for } v \rightarrow c \quad u = \frac{\sqrt{c^2 + 2cu \cos \theta + u^2(1 - \sin^2 \theta)}}{1 + \frac{u}{c} \cos \theta} = \frac{(c + u \cos \theta)}{1 + \frac{u}{c} \cos \theta} = \frac{c \left(1 + \frac{u}{c} \cos \theta\right)}{\left(1 + \frac{u}{c} \cos \theta\right)} = c$$

Solution 5: **Case 1:-** speed of B with respect to A choose speed of A along x -axis

$$V = -u \quad u'_x = v \cos \theta, u'_y = v \sin \theta$$

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}} = \frac{v \cos \theta - u}{1 - \frac{uv \cos \theta}{c^2}} \text{ and } u_y = \frac{u'_y \sqrt{1 - \frac{u^2}{c^2}}}{1 + \frac{u'_x v}{c^2}} = \frac{v \sin \theta \sqrt{1 - \frac{u^2}{c^2}}}{1 - \frac{uv \cos \theta}{c^2}}$$

Case 2- speed of A with respect to B choose speed of B along x -axis .

$$V = -v \quad u'_x = u \cos \theta, u'_y = u \sin \theta$$

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}} = \frac{u \cos \theta - v}{1 - \frac{uv \cos \theta}{c^2}}$$

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}} = \frac{v \cos \theta - u}{1 - \frac{uv \cos \theta}{c^2}} \text{ and } u_y = \frac{u'_y \sqrt{1 - \frac{u^2}{c^2}}}{1 + \frac{u'_x v}{c^2}} = \frac{u \sin \theta \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{uv \cos \theta}{c^2}}$$

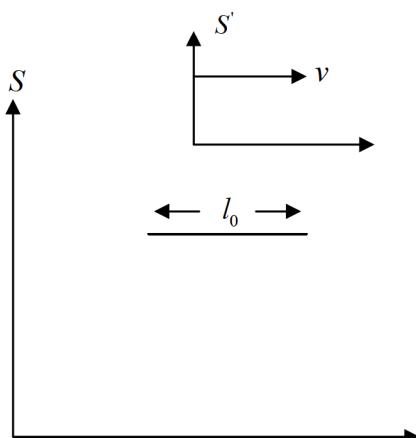
Solution 6: $u_x = u$ $V = v$

$$\therefore u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$u'_x = \frac{(u-v)c^2}{c^2 - uv} \Rightarrow \frac{u'^2}{c^2} = \frac{(u-v)^2 c^2}{(c^2 - uv)^2}$$

\therefore length of rod with respect to S' ,

$$\begin{aligned} l &= l_0 \sqrt{1 - \frac{u'^2}{c^2}} = l_0 \sqrt{1 - \frac{(u-v)^2 c^2}{(c^2 - uv)^2}} \\ &= \frac{l_0}{(c^2 - uv)} \sqrt{(c^2 - uv)^2 - (u-v)^2 c^2} \\ &= \frac{l_0}{(c^2 - uv)} \sqrt{c^4 + u^2 v^2 - 2c^2 uv - c^2 u^2 - c^2 v^2 + 2c^2 uv} \\ &= \frac{l_0}{(c^2 - uv)} \sqrt{c^2 (c^2 - u^2) - v^2 (c^2 - u^2)} = l_0 \frac{\sqrt{(c^2 - u^2)(c^2 - v^2)}}{(c^2 - uv)} \end{aligned}$$



Solution 7: (a) The speed of bullet with respect to target is, $u_t = \frac{v-u}{1 - \frac{vu}{c^2}}$ and the bullet have to travel the

distance from target reference, $L_1 = L \sqrt{1 - \frac{v^2}{c^2}} = L\gamma_1$

So, time taken by the bullet to hit the target from reference of target is,

$$\frac{L_1}{u_t} = \gamma_1 L \left(1 - \frac{vu}{c^2}\right) / (u - v)$$

(b) The speed of target with respect to bullet is, $u_b = \frac{u-v}{1 - \frac{vu}{c^2}}$ and the bullet have to travel the

distance from bullet reference, $L_2 = L \sqrt{1 - \frac{u^2}{c^2}} = L\gamma_2$

So, time taken by the bullet to hit the target from reference of bullet is,

$$\frac{L_2}{u_t} = \gamma_2 L \left(1 - \frac{vu}{c^2}\right) / (u - v)$$

Solution 8: (a) $\Delta t = \frac{1.77 \times 10^{-8} \text{ sec}}{\sqrt{1 - (0.99)^2}} = 1.3 \times 10^{-7} \text{ sec}$

$$d = 0.99c \times \Delta t = 2.97 \times 10^8 \text{ m/sec} \times 1.3 \times 10^{-7} \text{ sec} = 39 \text{ m}$$

(b) $d' = d \sqrt{1 - \frac{v^2}{c^2}} = \Delta \tau = \frac{d \sqrt{1 - \frac{v^2}{c^2}}}{0.99c} = \frac{39 \sqrt{1 - (0.99)^2}}{0.99c} = 1.77 \times 10^{-8} \text{ sec}$

Solution 9: The speed of Space ship A with respect to observer attached to earth is $0.6c$. If t_0 is proper

time $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$. It is given that $t = 1 \text{ year}$, $v = 0.6c \Rightarrow t_0 = 0.8 \text{ year}$

The speed of space ship A with respect to space ship B

$$\text{i.e. } v_1 = \frac{0.6c + 0.6c}{1 + \frac{0.6c \times 0.6c}{c^2}} = \frac{1.2c}{1.36} = .88c$$

$$t_B = \frac{t_0}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{0.8}{\sqrt{1 - (.88)^2}} = \frac{0.8}{\sqrt{1 - 0.77}} = \frac{0.8}{\sqrt{.225}} = \frac{0.8}{.47} = \frac{80}{47} = 1.7 \text{ years}$$

Solution 10: (a) Speed of train A with respect to C is $v_1 = \frac{4c}{5}$

Speed of train B with respect to C is $-\frac{3}{5}c$

$$v_1 t_A = l_0 \sqrt{1 - \frac{v_1^2}{c^2}} + l_0 \sqrt{1 - \frac{v_2^2}{c^2}} - v_2 t_A$$

$$\frac{4}{5} c t_A = l_0 \sqrt{1 - \frac{16}{25}} + l_0 \sqrt{1 - \frac{9}{25}} - \frac{3}{5} c t_A$$

$$\frac{7}{5} c t_A = l_0 \sqrt{1 - \frac{16}{25}} + l_0 \sqrt{1 - \frac{9}{25}} = l_0 \times \frac{3}{5} + l_0 \times \frac{4}{5} = \frac{7}{5} l_0 \Rightarrow t_A = \frac{l_0}{c}$$

(b) when observer is on B the speed of A with respect to B

$$v = -\left(-\frac{3}{5}c\right) = \frac{3}{5}c, u'_x = \frac{4}{5}c$$

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}} = \frac{\frac{3c}{5} + \frac{4c}{5}}{1 + \frac{\frac{3c}{5} \times \frac{4c}{5}}{c^2}} = \frac{\frac{7c}{5}}{\frac{37}{25}} = \frac{35c}{37}$$

$$u_x t_A = l_0 \sqrt{1 - \frac{u_x^2}{c^2}} + l_0$$

$$\frac{35c}{37} t_A = l_0 \sqrt{1 - \left(\frac{35}{37}\right)^2} + l_0 = l_0 \sqrt{1 - 0.89} + l_0 = 1.1l_0$$

$$t_A = \frac{37 \times 1.1}{35} \frac{l_0}{c} \approx \frac{41l_0}{35c}$$