Chapter One
Electrostatics

1.1 Coulomb's Law

Electrostatics deals with the study of forces, fields and potentials arising from static charges.

Charge

Charge: It is a physical quantity, and it is like mass, a fundamental property of matter.

- charge on electron to be negative and charge on proton to be positive is a convention.
- Charge of an isolated system is always conserved
- Charge is always quantized. i.e. Q = ne, where $n = 0, \pm 1, \pm 2 -$
  e = charge of an electron $= 1.6 \times 10^{-19}$C.
- For Quarks this quantization rule is not valid.

Properties of Electric Charge

(a) Additivity of charges

(b) Charge is conserved

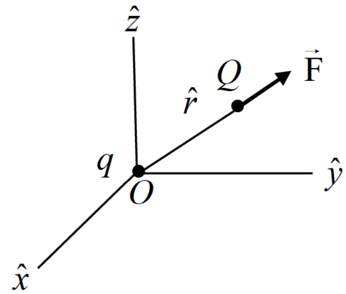
(c) Quantization of charge

Unit: coulomb

Smallest charge = e = 1.602192 × 10–19 C Charge on electron and proton

Coulomb's Law for Discrete Charges

It gives the force on a test charge Q due to a single point charge q, that is at rest a distance r away.



Note: $\vec{F} \Leftrightarrow \mathbf{F}$

$$\boxed{\mathbf{F}=\frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}\hat{r}}$$

The constant ϵ_0 is called the permittivity of free space. In SI units, where force is in newtons (N), distance in meters (m), and charge in coulombs (C),

$$\epsilon_0 = 8.85\times 10^{-1}\ \frac{\mathrm{C}^2}{\mathrm{N}\cdot\mathrm{m}^2}$$

The Electric Field

Consider several point charges $q_1, q_2, \ldots, q_n$, at distances $r_1, r_2, \ldots, r_n$ from Q, the total force on Q is given by

$$\mathbf{F}=\mathbf{F}_1+\mathbf{F}_2+\cdots=\frac{1}{4\pi\epsilon_0}\left(\frac{q_1Q}{r_1^2}\hat{r}_1+\frac{q_2Q}{r_2^2}\hat{r}_2+\cdots\right)$$

$$=\frac{Q}{4\pi\epsilon_0}\left(\frac{q_1}{r_1^2}\hat{r}_1+\frac{q_2}{r_2^2}\hat{r}_2+\frac{q_3}{r_3^2}\hat{r}_3+\cdots\right)$$

Or $$\mathbf{F}=Q\mathbf{E}$$

Where,

$$\boxed{\mathbf{E}(\mathbf{r})\equiv\frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i}$$

For a small charge dq, the Electric Field is given by,

$$\mathbf{dE}(\mathbf{r})=\frac{1}{4\pi\epsilon_0}\frac{\mathrm{dq}}{\mathrm{r}}\hat{r}$$

For Electric field due to total charge $E=\int dE$

For continuous system a small charge $dq\to\lambda dl\sim\sigma da\sim\rho d\tau$

Where, λ is line charge distribution, σ is surface charge distribution and ρ is volume charge distribution.

Note: Coulomb's law and the principle of superposition constitute the physical input for electrostatics-the rest, except for some special properties of matter, is mathematical elaboration of these fundamental rules.

Example: The electron revolving around the nucleus have a charge $+e$ in a primitive atom, can be represented by the spherical charge distribution.

$$\rho(r) = \frac{-e}{\pi r_0^3} e^{-2r/r_0}$$

where r_0 is called the Bohr radius. Calculate the total charge of the cloud.

Solution: Small charge is given by charge is given by $dq = \rho d\tau$. Here we must take volume element in spherical polar coordinate.

$$Q = \int \rho d\tau = \iiint \rho\, r^2 dr \sin\,\theta d\theta d\phi = \int_0^\infty \rho(r) r^2 dr \int_0^\pi \sin\,\theta d\theta \int_0^{2\pi} d\phi$$

$$Q = \frac{-e}{\pi r_0^3} \cdot 4\pi \int_0^\infty r^2 e^{-2r/r_0} dr = -e$$ Use gamma integral $\int_0^\infty r^n\, e^{-\alpha r}\, dr = \frac{\Gamma(n+1)}{\alpha^{n+1}}$

Example:

(a) Twelve equal charges, *q*, are situated at the comers of a regular 12-sided polygon (for instance, one on each numeral of a clock face). What is the net force on a test charge *Q* at the centre?

(b) Suppose *one* of the 12 *q* 'sis removed (the one at "6 o'clock"). What is the force on *Q*? Explain your reasoning carefully.

(c) Now 13 equal charges, *q*, are placed at the comers of a regular 13-sided polygon. What is the force on a test charge *Q* at the centre?

(d) If one of the 13 *q's* is removed, what is the force on *Q*? Explain your reasoning.

Solution:

(a)

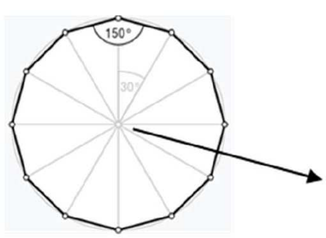


Fig (a)

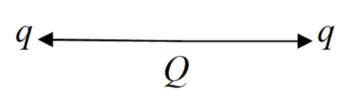
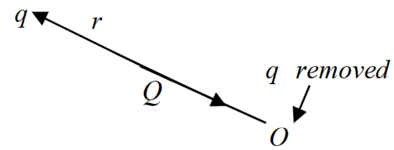


Fig (b)

The system of 12 charges is shown in the fig(a) sitting at the corners. We can see that two charges are sitting opposite to each other as shown in the fig(b). Hence the force on charge Q will be cancelled. This will happen with all 12 charges. So, $F_{net} = 0$. Force will be cancelled.

(b) If one charge is removed the force that will appear is given by,

$$F=\frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$$ **(coulomb's law)**

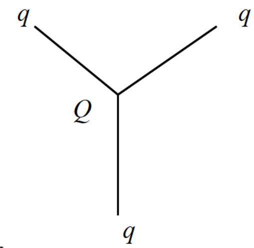


That is the amount of force vanished after removal of q will be the force acting on Q.

Since other charges will cancel each other like they were doing in previous case (a).

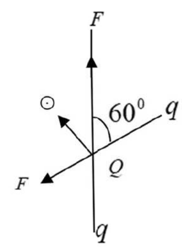
(c) Instead of 13 let us see what happens with three (odd number as 13)

$F_x=0$, $F_y=0$. The force on Q is $F=0$.



(d) The force on Q after the charge was removed will be F.

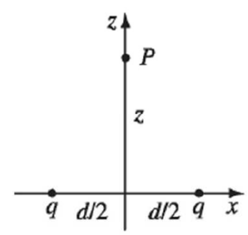
$$F=\frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$$



That is the amount of force vanished after removal of q will be the force acting on Q.

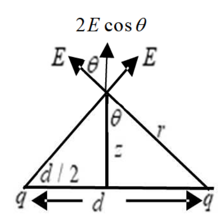
Example: (a) Find the electric field (magnitude and direction) a distance z above the midpoint between equal and opposite charges, q, a distance d apart.

(b) Repeat part (a), only this time make left-hand charge -q instead of q.



Solution:

(a) The Electric field at P is calculated using coulombs law. Cosine components will add up in z direction and sine components will cancel
The net field is given by, $\vec{E}_{net}=2E\cos\theta\hat{z}$,



where, $E=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$, and $r^2=\frac{d^2}{4}+z^2$

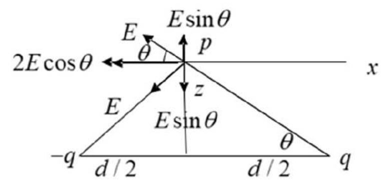
$$\vec{E}_{net}=2\times\frac{1}{4\pi\varepsilon_0}\frac{q}{z^2+\frac{d^2}{4}}\cdot\frac{z}{\sqrt{z^2+\frac{d^2}{4}}}\hat{z}=2\times\frac{1}{4\pi\varepsilon_0}\frac{qz}{\left(z^2+\frac{d^2}{4}\right)^{3/2}}\hat{z}$$

For $z >> d$, $r^2 = z^2$, $\vec{E}_{net} = 2 \times \frac{1}{4\pi\varepsilon_0} \frac{q}{z^2} \hat{z}$

Note: When you go very far it appears that 2q charge is at mid-point.

(b) Sine components will cancel each other, and cosine components will

Add up as shown in fig.



$$\vec{E}_{net} = 2E\cos\theta(-\hat{x})$$

$$= \cancel{2} \times \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \cdot \frac{d/\cancel{2}}{r}(-\hat{x}) = \frac{1}{4\pi\varepsilon_0} \frac{qd}{\left(z^2 + \frac{d^2}{4}\right)^{3/2}}(-\hat{x}) \qquad \cos\theta = \frac{d/2}{r}$$

Where $r^2 = \frac{d^2}{a} + z^2$

In the situation when $d \to 0, \quad E \to 0$ Since $\quad q_{net} = 0$