Chapter One

LORENTZ TRANSFORMATION

1.4 Consequences of Lorentz Transformation

(b) Time Dilation:

Moving clock ticks slower

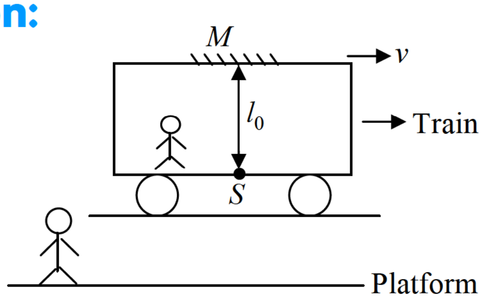


Figure: 1.6

Assume a train is moving with respect to platform. The train is moving with velocity v along positive x direction .The light source and detector is attached at the floor of train.

A mirror is attached roof of train which is just above the light source and detector. The height between roof and floor is l_0. The experiment is defined as light pulse will emerge from source hit the mirror and again hit detector. There two observers one observer A is attached to train and another observer is B attached to platform. We need to find time interval such that light pulse will emerge from source and again detected by the detector.

From observer A point of view the time interval is $\Delta t_A = \frac{2l_0}{c} = \Delta t_0$.

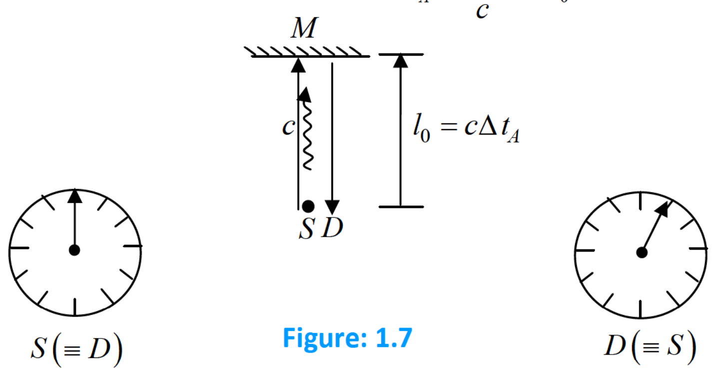


Figure: 1.7

From observer B point of view the time interval is ΔT_B. From the geometry and using Pythagoras law $l_0 = \sqrt{c^2\left(\frac{\Delta t_B}{2}\right)^2 - v^2\left(\frac{\Delta t_B}{2}\right)^2} \Rightarrow \Delta t_B = \frac{\frac{2l_0}{c}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\Delta t_A}{\sqrt{1-\frac{v^2}{c^2}}}$

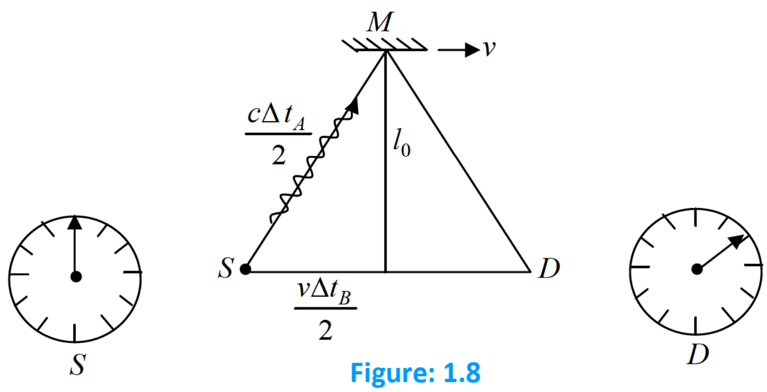


Figure: 1.8

We can see $\Delta t_B > \Delta t_A$

Proper Time: If clock is rest with respect to observer then observer will observe proper time

Time Dilation: When two observers are in relative uniform motion and uninfluenced by any gravitational mass, the point of view of each will be that the other's (moving) clock is ticking at a *slower* rate than the local clock which will measure proper time. The faster the relative velocity, the greater the magnitude of time dilation. This case is sometimes called special relativistic time dilation.

Derivation of Time Dilation from Lorentz Transformation

A clock being at rest in the S' frame measures the time t_2' and t_1' of two events occurring at a fixed position x', then time interval Δt measures from S frame appears slow (Δt_0) from S′ frame i.e. to the observer the moving clock will appear to go slow.

$$\Delta t' = t_2' - t_1' = \Delta t_0 \Rightarrow \Delta t = t_2 - t_1 = \gamma\left(t'_2 + \frac{vx'}{c^2}\right) - \gamma\left(t_1 + \frac{vx'}{c^2}\right) \Rightarrow \Delta t = \frac{t'_2 - t'_1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where Δt_0 proper time which is measured from rest frame of clock, and Δt is dilated time which is measured from moving frame of clock where $\Delta t > \Delta t_0$.

Example: A muon is having life time $2.2\mu\sec$ moving towards earth. The speed of muon respect to earth is $0.998c$. Ignore the mass of muon find the distance travelled by muon with respect to observer at earth before it decays.

Solution: The proper life time is given as $t_0 = 2.2\times10^{-6}\sec$

The speed of muon with respect to observer is $v = 0.998c$

The clock on the muon is moving with respect to observer on earth so with respect to observer the life time is dilated and measured as

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2.2\times10^{-6}\sec}{\sqrt{1-\left(\frac{0.998c}{c}\right)^2}} = 34.6\times10^{-6}\sec.$$

Let the distance traveled by muon with respect to earth is d

$d = vt = 0.998c\times34\times10^{8}\,m/\sec = 0.998\times3\times10^{8}\times34\times10^{8}\,m/\sec = 10.4km$.

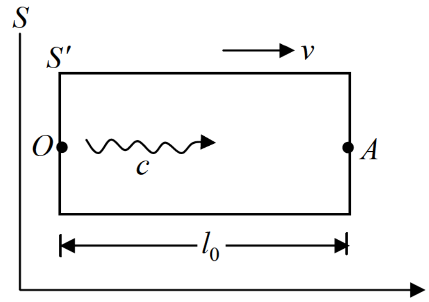
Example: A space craft is moving with respect to earth. As an observer on earth finds that between $1pm$ to $2pm$ according to her clock 3601 sec elapsed the on spacecraft clock .what is speed of space craft with respect to earth.

Solution: The proper time interval during $1pm$ to $2pm$ is $\Delta t_0 = 3600\sec$

The clock attached to space craft is moving with respect to earth so it will be measured dilated time i.e. $\Delta t = 3601\sec$

So $\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \sqrt{1-\frac{v^2}{c^2}} = \frac{\Delta t_0}{\Delta t} \Rightarrow v = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = c\sqrt{1-\left(\frac{3600}{3601}\right)^2} = 7.1\times10^{6}\,m/\sec$

Example: The distance between O to A is proper length l_0 in rest frame of S' as shown in figure, what is time measured by observer attached to rest frame of S such that light emerge from O to reach to A. It is given that velocity of S' with respect to S is v.



Solution:

Method 1: From frame S' $x_2' - x_1' = l_0, t_2' - t_1' = \frac{l_0}{c}$

From frame S $t_2 - t_1 = \left(\frac{t_2' + v\frac{x_2'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\right) - \left(\frac{t_1' + v\frac{x_1'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \left(\frac{t_2' - t_1'}{\sqrt{1-\frac{v^2}{c^2}}}\right) + \frac{v}{c^2\sqrt{1-\frac{v^2}{c^2}}}\left(x_2' - x_1'\right)$

$$t_2 - t_1 = \left(\frac{\frac{l_0}{c}}{\sqrt{1-\frac{v^2}{c^2}}}\right) + \frac{vl_0}{c^2\sqrt{1-\frac{v^2}{c^2}}} = \frac{l_0}{c}\frac{1+\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{l_0}{c}\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

Method 2: From frame S

$$c\Delta t = l_0\sqrt{1-\frac{v^2}{c^2}} + v\Delta t \Rightarrow \Delta t = (c-v)\Delta t = l_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\Delta t = \frac{l_0\sqrt{1-\frac{v^2}{c^2}}}{c-v} = \frac{l_0}{c}\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$