Chapter One

LORENTZ TRANSFORMATION

1.4 Consequences of Lorentz Transformation

(c) Loss of Simultaneity

The loss of simultaneity in Special relativity - is that real or created due to the fact that light takes time to travel. So even though two events are simultaneous but since light takes time to travel, they may not be simultaneous for two different observers.

The loss of simultaneity can be understood with following experiment.

A train of proper length l_0 moving with speed v with respect to a platform in positive $\hat{x}$ direction. There are two detectors A and B are attached at two opposite ends of the train. Two observers X and Y attached to train and platform respectively.

Suddenly two light pulse immerged from center of train O and moves in straight line towards detectors A and B. We

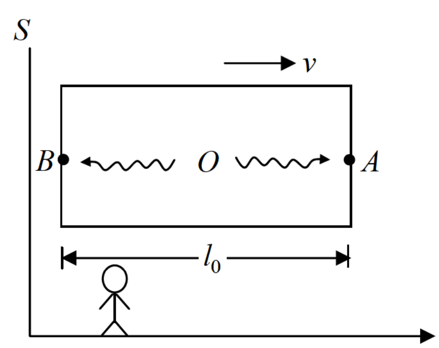


Figure: 1.6

need to calculate time interval such that light pulse is detected by A and B with respect to observer X and Y.

With respect to observer X:

Event-1: Let us assume that $\Delta t_{A,X}$ is time interval measured by observer X such that light pulse emerge from O and detected by detector A so $\Delta t_{A,X} = \frac{l_0}{2c}$

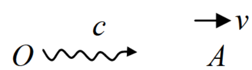
Event-2: Let us assume that $\Delta t_{B,X}$ is time interval measured by observer X such that light pulse emerge from O and detected by detector B so $\Delta t_{B,X} = \frac{l_0}{2c}$

We can see $\Delta t_{A,X} = \Delta t_{B,X}$ which means two events are simultaneity with respect to observer X

With respect to observer Y:

Event-1: Let us assume that $\Delta t_{A,Y}$ is time interval measured by observer Y such that light pulse emerge from O and detected by detector A.

Using kinematics relation

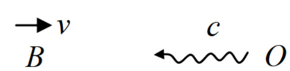


$$c\Delta t_{A,Y} = \frac{l_0}{2}\sqrt{1-\frac{v^2}{c^2}} + v\Delta t_{A,Y} \Rightarrow (c-v)\Delta t_{A,Y} = \frac{l_0}{2}\sqrt{1-\frac{v^2}{c^2}}$$

$$\Delta t_{A,Y} = \frac{l_0\sqrt{1-\frac{v^2}{c^2}}}{2(c-v)} = \frac{l_0}{2c}\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

Event-2: Let us assume that $\Delta t_{B,Y}$ is time interval measured by observer Y such that light pulse emerge from O and detected by detector B so

Using kinematics relation



$$c\Delta t_{B,Y} = \frac{l_0}{2}\sqrt{1-\frac{v^2}{c^2}} - v\Delta t_{A,Y} \Rightarrow (c+v)\Delta t_{B,Y} = \frac{l_0}{2}\sqrt{1-\frac{v^2}{c^2}}$$

$$\Delta t_{B,Y} = \frac{l_0\sqrt{1-\frac{v^2}{c^2}}}{2(c+v)} = \frac{l_0}{2c}\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

One can see $\Delta t_{A,Y} > \Delta t_{B,Y}$

We can conclude the two events is simultaneous with respect to observer A but if these two event are not simultaneous with respect to observer B

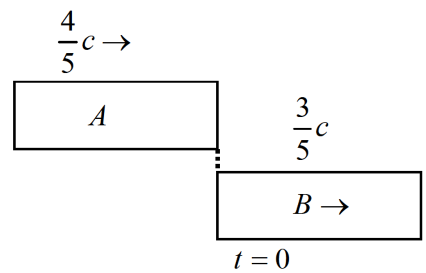
Example: Two trains A and B having same proper length l_0 are moving with respect to platform in parallel track with speed $\frac{4}{5}c$ and $\frac{3}{5}c$ respectively. At $t=0$ Engine of train A is just behind the last buggy of train B.

(a) If observer is attached to platform what will be time measured such that train A will completely crossed the train B

(b) If observer is attached to train B what will be time measured such that train A will completely crossed the train B.

(Train A will completely cross to train B means engine of train B will just behind the last buggy of train A)

Solution: (a) Speed of train A and train B with respect to observer is $\frac{4}{5}c$ and $\frac{3}{5}c$. Let's assume in time t train A will cross train B. Length of both trains are contracted with respect to observer.



Using kinematics relation $\frac{4}{5}c.t = l_0\sqrt{1-\frac{16}{25}} + l_0\sqrt{1-\frac{9}{25}} + \frac{3}{5}ct$

$$\frac{c}{5}t = l_0 \times \frac{3}{5} + l_0 \times \frac{4}{5} \Rightarrow t = \frac{7l_0}{c}$$

(b) Velocity of train A with respect to platform $v_{A,P} = \frac{4}{5}c$

Velocity of train B with respect to platform $v_{B,P} = \frac{3}{5}c$

We need to find speed of A with respect to B $v_{A,B} = ?$

So we assume B is equivalent to S frame so platform will equivalent to S' frame $v_{S',S} = v = -\frac{3}{5}c\hat{i}$, $v_{A,P} = v_{A,S'} = u'_x = \frac{4}{5}c$ and $v_{A,B} = v_{A,S} = u_x = ?$

Using formula $u_x = \dfrac{u'_x + v}{1+\dfrac{u'_x . v}{c^2}} = \dfrac{\dfrac{4c}{5} - \dfrac{3c}{5}}{1 - \dfrac{1}{c^2} \times \dfrac{4c}{5} \times \dfrac{3c}{5}} = \dfrac{\dfrac{c}{5}}{\dfrac{13}{25}} = \dfrac{5}{13}c$

With respect to observer the length of A will contracted but length of B will not change.

Let us assume total time t such that train A will pass train B with respect to observer using kinematics relation

$$\frac{5}{13}c \times t = l_0\sqrt{1-\frac{25}{169}} + l_0 \Rightarrow \frac{5}{13}ct = \frac{12}{13}l_0 + l_0 \Rightarrow \frac{5}{13}ct = \frac{25}{13}l_0 \Rightarrow t = \frac{5l_0}{c}$$