PYQ Solutions [IIT-JAM]

[Chapter 1 Bohr Sommerfeld theory and Hydrogen atom]

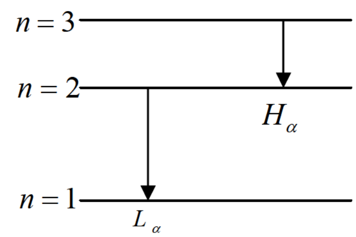
Ans. 1: (c)

Solution: $B.E.=\left(M_p+M_e-M_H\right)c^2 \Rightarrow M_H=M_p+M_e-\dfrac{B.E.}{c^2}$, where $B.E.=-13.6eV$

$$\Rightarrow M_H=M_p+M_e+\frac{13.6eV}{c^2}$$

Ans. 2: 0.185

Solution: According to Bohr Theory : $\dfrac{1}{\lambda}=R\left(\dfrac{1}{n_f^2}-\dfrac{1}{n_i^2}\right)$,



R is Rydberg constant, $R=1.097\times10^7 m^{-1}$

The longest wavelength in the Lyman series:

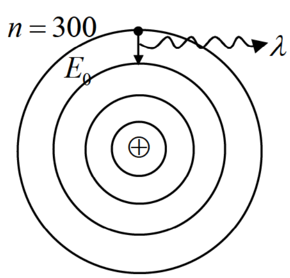
$$\Rightarrow\frac{1}{\lambda_L}=R\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=R\left(\frac{3}{4}\right)\quad\Rightarrow\lambda_L=\frac{4}{3R}$$

The longest wavelength in the Balmer series:

$$\Rightarrow\frac{1}{\lambda_B}=R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=R\left(\frac{5}{36}\right)\quad\Rightarrow\lambda_B=\frac{36}{5R}\Rightarrow\frac{\lambda_L}{\lambda_B}=\frac{4}{3R}\times\frac{5R}{36}=\frac{5}{27}=0.1852$$

Ans. 3: (d)

Solution: $\dfrac{1}{\lambda}=R\left(\dfrac{1}{n_f^2}-\dfrac{1}{n_i^2}\right)$, where $R=1.097\times10^7 m^{-1}$



$$n_f=299 \text{ and } n_i=300\Rightarrow\frac{1}{\lambda}=R\left[\frac{1}{299^2}-\frac{1}{300^2}\right]$$

$$\Rightarrow\frac{1}{\lambda}=R\left[\frac{300^2-299^2}{300^2\times299^2}\right]=R\left[\frac{599}{804.6\times10^7}\right]$$

$$\frac{1}{\lambda}=1.097\times10^7\left[\frac{599}{804.6\times10^7}\right]=0.8167\qquad\lambda=\frac{1}{0.8167}\Rightarrow\lambda=1.2244m$$

This wavelength corresponds to $RF\left(\lambda\sim m\right)$, so correct option will be (d)

Ans. 4: (d)

Solution: From Bohr model the kinetic energy and Total energy, $\langle T\rangle=-\langle E\rangle$

where $E_g=\dfrac{E_0}{1}$: For ground state, $n=1$ & $E_e=\dfrac{E_0}{16}$: For third excited state, n=4

$$\Rightarrow\frac{T_g}{T_e}=\frac{-E_g}{-E_e}=\frac{16}{1}=16:1=16$$

PYQ Solutions [GATE]

Ans. 1: (c)

Solution: Orbital with sub shell quantum number $l=0$ are called s orbital. All orbital are spherical in shape and have spherical symmetry. This means that the wave function will depend only on the distance from the nucleus and not on the direction.

Ans. 2: (b)

Solution: For dipole moment, Energy = $-eEr\cos\theta$

$$E_1^1 = \langle -eEr\cos\theta \rangle = -eE\langle r\rangle\langle\cos\theta\rangle = 0 \qquad [\because \langle\cos\theta\rangle = 0]$$

Ans. 3: (b)

Solution: $3p$ radial wave function is, $R_{3p} \propto r\left(1-\frac{r}{6a_0}\right)e^{-\frac{r}{3a_0}}$

Ans. 4: 1.06

Solution: $r_n = a_0\frac{n^2}{z}\left(\frac{m_e}{\mu}\right)$

Where $\mu = \frac{m_{e^-}m_{e^+}}{m_{e^-}+m_{e^+}} = \frac{m_e^2}{2m_e} = \frac{m_e}{2}$, $z=1$ & $n=1$ $\Rightarrow r_1 = a_0\left(\frac{2m_e}{m_e}\right) = 2\times 0.53 = 1.06A^0$

Ans. 5: 14

Solution: $\frac{N_1}{N_0} = e^{-\Delta E/kT}$

$$\Delta E = \left(\frac{13\cdot 6}{1^2} - \frac{13\cdot 6}{2^2}\right)ev = (13\cdot 6 - 3\cdot 4) = 10\cdot 2eV$$

$$\therefore \frac{\Delta E}{kT} = \frac{10.2eV}{8.617\times 10^{-5}ev/k\times 5800k} = 20.41$$

Thus $\frac{N_1}{N_0} = e^{-20\cdot 41} \Rightarrow N_1 = 6\cdot 023\times 10^{23}\times 1\cdot 37\times 10^{-9} = 8.25151\times 10^{14}$

$$N_1 \approx 8\times 10^{14}, \quad so\ n = 14$$

PYQ Solutions [NET-JRF]

Ans. 1: (b)

Solution: For positronium, its reduced mass only differs from the electron by a factor of 2 so the energy expression for positronium atom will be $E_n = -\frac{13.6}{2n^2}(eV)$

For n = 1, $E_1 = \frac{-13.6}{2}(eV) = -6.8eV$, $E_1 = -6.8eV$

Ans. 2: (b)

Solution: In case of muonic atom, the reduce mass is $m' = \frac{m_\mu m_p}{m_\mu + m_p} = 180m_e \left(m_\mu = 200m_e\right)$

$E_n' = \left(\frac{m'}{m_e}\right)\frac{E_1}{n^2} = 180\frac{E_1}{n^2}$ where, $E_1 = -13.6eV$

For ground state of muonic atom *n*=1, $E_1' = 180E_1$

For first excited state of muonic atom *n*=2, $E_2' = 45E_1$

The longest wavelength of the photon corresponds to the transition between first excited state and ground state of muonic atom.

The energy difference between first excited state and ground state is

$\Delta E = E_1' - E_2' = 135E_1 = 1836eV = 2938\times10^{-19}J$

In term of wavelength

$\Delta E = hv = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{6.6\times10^{-34}\times3\times10^8}{2938\times10^{-19}} = 6.74\times10^{-10}m = 6.74\,\mathring{A}$

Ans. 3: (c)

Solution: For Normal Li-atom:

$\mu = \frac{7m_p\times m_e}{7m_p + m_e} \Rightarrow \frac{\mu}{m_e} = \frac{7\times1836}{(7\times1836)+1} \cong 1$

Ionization energy: $I = \frac{13.6z^2}{n^2}\times\frac{\mu}{m_e} = 13.6\times9 = 122.4eV$

For Muonic Li-atom:

$\mu = \frac{7m_p\times m_{\mu^-}}{7m_p\times m_{\mu^-}} \cong \frac{7\times1836\times200m_e^2}{((7\times1836)+200)m_e} = 197\,m_e \Rightarrow \frac{\mu}{m_e} = 197$

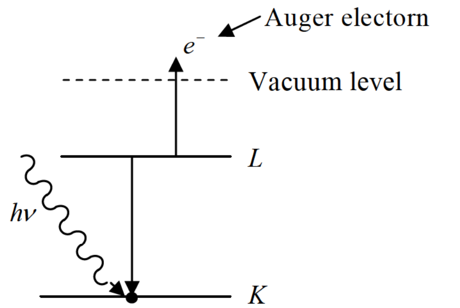
Ionization energy: $I' = \frac{13.6z^2}{n^2}\times\frac{\mu}{m_e} = 13.6\times9\times197 = 24112.8eV$

$[I' = 200I]$

Thus correct option is (c)

Note: Answer does not match

Ans. 4: (d)



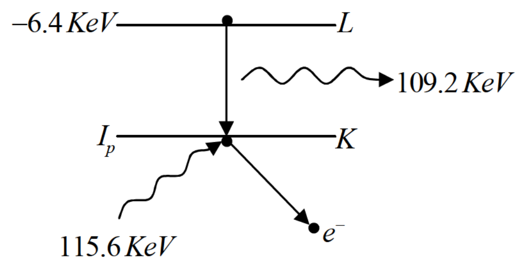
Solution: $K.E.$ of Auger electron $=(E_K - E_L) - E_L$

$$= E_K - 2E_L$$

$$= (25.4 - 2\times 3.34)\, keV$$

$$= 18.7\, keV$$

Ans. 5: (c)



Solution: Binding energy of K-shell electron

$$I_p = 6.4\ KeV + 109.2\, KeV = 115.6\ KeV$$

Thus, K.E. of ionized electron is

$$= 115.62\ KeV - 115.60\, KeV = 0.02\, KeV = 20\, eV$$

Ans. 6: (c)

Solution: Energy required to dissociate KCl is $\left(KCl \to K^+ + Cl^-\right)$

$$V = \frac{1}{4\pi \in_0}\frac{q_1 q_2}{r_{12}} = \left(9\times 10^9\ \frac{Nm^2}{c^2}\right)\frac{\left(1.6\times 10^{-19} c\right)^2}{0.3\times 10^{-9}\ m} = 7.7\times 10^{-19}\ J = 4.81\ eV$$

The band dissociation energy is the energy required to dissociate a molecule into its component atom $KCl \to K + Cl$

To find the energy required to dissociate KCl into K and Cl, we must add an electron to the K^+ ion, which releases the potassium ionization energy. Remove one electron from Cl^- ion which requires the chlorine electron affinity energy.

Given ionization energy of $K = E_{ie} = 4.34\ eV$

Electron affinity of $Cl = 3.82\ eV$

Thus the energy gained in the transformation from ion to atom is

$$= E_{ie} - E_{ai} = 4.34 - 3.82 = 0.52\ eV$$

Thus dissociation energy $= 4.79 - 0.52 = 4.27\ eV$

Ans. 7: (d)

Solution: The conservation law of energy and momentum gives:

$$Mc^2 + h\nu = \left[(M+\Delta)^2 c^4 + p^2c^2\right]^{1/2} \text{ and } \frac{h\nu}{c} = p$$

$$M^2c^4 + h^2\nu^2 + 2Mc^2h\nu = M^2c^4 + \Delta^2c^4 + 2M\Delta c^4 + p^2c^2$$

$$M^2c^4 + h^2\nu^2 + 2Mc^2h\nu = M^2c^4 + \Delta^2c^4 + 2M\Delta c^4 + h^2\nu^2 \quad \Rightarrow 2Mc^2h\nu = \Delta^2c^4 + 2M\Delta c^4$$

$$\Rightarrow 2Mc^2h\nu = 2M\Delta c^4\left(1+\frac{\Delta}{2M}\right) \quad \Rightarrow \nu = \frac{\Delta c^2}{h}\left(1+\frac{\Delta}{2M}\right) \quad \Rightarrow \nu = \frac{\Delta c^2}{2Mh}(\Delta + 2M).$$

Ans. 8: (a)

Solution: $\mu = \frac{m_p m_e}{m_e + m_p} = \frac{1836 m_e^2}{m_e + 1836 m_e} = \frac{1836}{1837} m_e$: For hydrogen atom

$\mu' = \frac{6 m_p m_e}{m_e + 6 m_p} = \frac{6\times 1836 m_e^2}{m_e + (6\times 1836) m_e} = \frac{11016}{11017} m_e$: For z=6

First Balmer line

$$\frac{hc}{\lambda} = -\frac{13.6\mu}{m_e}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) eV \quad \text{and} \quad \frac{hc}{\lambda'} = -\frac{13.6\mu'}{m_e}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) Z^2 eV, \qquad Z = 6$$

$$\frac{\mu}{\lambda'} = \frac{\mu' z^2}{\lambda} \qquad \Rightarrow \lambda' = 18.2 nm$$