## WORKSHEET QUESTIONS

Q1. In a cathode ray oscilloscope, the deflection of electrons by the superposition of two mutually perpendicular electric fields is given by:

$$\begin{aligned} x &= 3\cos(\omega t) \\ y &= 3\cos(\omega t + \pi/3) \end{aligned}$$

Find and draw the resultant trajectory of the electrons.

Q2. Consider two light waves: $E_x = A_x\cos(\omega_x t + \phi_x)$ and $E_y = A_y\cos(\omega_y t + \phi_y)$. Their superposition will result in a plane polarized light, if:

(a) $A_x = A_y$, $\phi_x = \pi/2$, $\phi_y = \pi$ (b) $A_x \neq A_y$, $\phi_x = \phi_y = 0$, $\omega_x = \omega_y$
(c) $A_x = A_y, \phi_x \neq \phi_y, \omega_x = \omega_y$ (d) $\omega_x = \omega_y, \phi_x = \pi/2, \phi_y = \pi$

Q3. Find the state and direction of polarization of the following EM waves

(i)

$$\begin{aligned} E_x &= 2E_0\cos(\omega t - kz), and \\ E_y &= E_0\sin(\omega t - kz) \end{aligned}$$

(ii)

$$\begin{aligned} E_x &= E_0\cos(\omega t + \pi/2), and \\ E_y &= E_0\cos(\omega t) \end{aligned}$$

(iii)

$$\begin{aligned} E_x &= 2E_0\sin(\omega t), and \\ E_y &= E_0\sin(\omega t + \pi/2) \end{aligned}$$

(iv)

$$\begin{aligned} E_x &= 2E_0\sin(\omega t - \pi/2), and \\ E_y &= E_0\sin(\omega t + \pi/2) \end{aligned}$$

(v)

$$\begin{aligned} E_x &= 2E_0\sin(\omega t + \pi/2), and \\ E_y &= E_0\sin(\omega t + 4\pi/2) \end{aligned}$$

(vi)

$$\begin{aligned} E_x &= E_0\cos(\omega t + 3\pi/2), and \\ E_y &= E_0\sin(\omega t + \pi) \end{aligned}$$

## Solutions

**Ans 1.** Using eq. 23 above,

$$\frac{x^2}{E_1^2} + \frac{y^2}{E_2^2} - 2\frac{xy}{E_1E_2}\cos\theta = \sin^2\theta \tag{1.24}$$

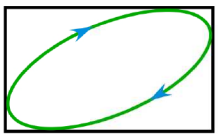
$E_1 = E_2 = 3$; $\delta\theta = \pi/3$;
$\therefore$, the above equation becomes

$$\frac{x^2}{9} + \frac{y^2}{9} - 2\frac{xy}{9}\cos -\pi/3 = \sin^2 -\pi/3 \tag{1.25}$$

$$\frac{x^2}{9} + \frac{y^2}{9} - \frac{xy}{9} = \frac{3}{4} \tag{1.26}$$

$$x^2 + y^2 - xy = \frac{27}{4} \tag{1.27}$$

This is an equation for ellipse and since $\delta\theta = -\pi/3$, therefore, the rotation will be clockwise direction. Hence, the superposition of the above equations will result in the formation Left Elliptically Polarised trajectory inclined at an angle of $\pi/3$ to the coordinate axes.




**Ans 2.** A plane polarized light will be produced on superposition of two waves if the following conditions are met:

(i) their frequencies are equal, i.e, $\omega_x = \omega_y$, and (ii) the phase difference between the waves is zero, i.e, $\delta\theta = 0$.

$\delta\theta = \phi_x + \pi/2 - \phi_y = 0$

$\implies \phi_y = \phi_x + \pi/2 = \pi$ and $\phi_x = \pi/2$

**Hence, the correct ans is option (d).**

**Ans 3(i)** $\delta\theta = 0 \implies$ Linearly polarized light.

**Ans 3(ii)** $\delta\theta = \pi/2 \implies$ anticlockwise direction;
$A_x = A_y$ and $\omega_x = \omega_y \implies$ circularly polarized light moving in anticlockwise direction.

**Ans 3(iii)** $\delta\theta = -\pi/2 \implies$ clockwise direction;
$A_x \neq A_y$ and $\omega_x = \omega_y \implies$ elliptically polarized light moving in anticlockwise direction.

**Ans 3(iv)** $\delta\theta = -\pi/2 - \pi/2 = -\pi \implies$ Linearly polarized light;

**Ans 3(v)** $\delta\theta = \pi/2 - 4\pi/2 = -3\pi/2 \implies$ anticlockwise direction;
$A_x \neq A_y$ and $\omega_x = \omega_y \implies$ elliptically polarized light moving in anticlockwise direction.

**Ans 3(vi)** $\delta\theta = 3\pi/2 - \pi = \pi/2 \implies$ anticlockwise direction;
$A_x = A_y$ and $\omega_x = \omega_y \implies$ circularly polarized light moving in anticlockwise direction.

## Superposition Principle

**Q1.** At a given point in space the total light wave is composed of three phasors $P_1 = a$; $P_2 = \frac{a}{2}e^{i\theta}$; $P_3 = \frac{a}{2}e^{-i\theta}$. The intensity of light at this point will be given by:

(a) $4a^2\cos^2\frac{\theta}{2}$ (b) $4a^2\cos^4\frac{\theta}{2}$

(c) $a^2\cos^2\theta$ (d) $4a^2\cos^2 2\theta$

**Sol.** The net phasor at the given point is given by:

$$\begin{aligned} P &= P_1 + P_2 + P_3 \\ &= a + \frac{a}{2}e^{i\theta} + \frac{a}{2}e^{-i\theta} \\ &= \frac{a}{2}(2 + \cos\theta + i\sin\theta + \cos\theta - i\sin\theta) \\ &= a(1+\cos\theta) \\ &= 2a\cos^2\frac{\theta}{2} \end{aligned}$$

$$I = P^2 = 4a^2\cos^4\left(\frac{\theta}{2}\right)$$

**The correct option is (b).**

**Q2.** Calculate the net intensity of the wave for $\phi = \pi/3$ comprising the following electric fields:
$E_1 = ae^{i(\phi+\pi)}$; $E_2 = ae^{-i(\phi+\pi)}$; $E_3 = \frac{a}{2}e^{-i(\phi)}$; $E_4 = \frac{a}{2}e^{i\phi}$.

(a) $4a^2$ (b) $\frac{a^2}{2}$

(c) $\frac{4a^2}{16}$ (d) $\frac{a^2}{6}$

**Sol.** The net electric field is given by:

$$\begin{aligned} E &= E_1 + E_2 + E_3 + E_4 \\ &= ae^{i(\phi+\pi)} + ae^{i(\phi+\pi)} + \frac{a}{2}e^{-i\phi} + \frac{a}{2}e^{i\phi} \\ &= \frac{a}{2}\left[2\left(\cos(\phi+\pi) + i\sin(\phi+\pi)\right) + 2\left(\cos(\phi+\pi) - i\sin(\phi+\pi)\right) + \cos\phi + i\sin\phi + \cos\phi - i\sin\phi\right] \end{aligned}$$

$$= \frac{a}{2}(4\cos(\phi+\pi)+2\cos\phi$$

$$= \frac{a}{2}(-4\cos(\phi)+2\cos\phi$$

$$= -\frac{a}{2}(2\cos\phi$$

$$I = E^2 = a^2\cos^2\phi$$

$$I|_{\phi=\pi/3} = a^2\cos^2\frac{\pi}{3}$$
$$= \frac{a^2}{4}$$

**The correct option is (c).**

**Q3.** Find the resultant motion of the following orthogonal fields:

$$x = a_1\cos(2\omega_o t+\phi_1); \quad \text{and}$$

$$y = a_2\cos\omega_o t.$$

for $\phi = 0,\ \pi/2,\ \pi$

**Sol.** Case I: For $\phi = 0$:

$$x = a_1\cos 2\omega_o t = a_1\left(2\cos^2\omega_o t - 1\right)$$

and

$$y = a_2\cos\omega_o t$$

This gives,

$$\frac{x}{a_1} = \frac{2y^2}{a_2^2} - 1$$

Rearranging the terms, we get:

$$y^2 = \frac{a_2^2}{2a_1}(x+a_1)$$

This is an equation for **Parabola**.

**Case II:** For $\phi = \pi/2$

$$x = -a_1 \sin 2\omega_o t, \quad \text{or} \quad -\frac{x}{a_1} = 2\sin\omega_o t \cos\omega_o t$$

$$\text{and} \quad y = a_2 \cos\omega_0 t \quad \text{or} \quad \cos\omega_o t = y/a_2$$

$$\sin\omega_o t = \sqrt{1 - y^2/a_2^2}$$

Equating the values of $\cos\omega_0 t$ and $\sin\omega_0 t$ in the equation for x, we get:

$$-\frac{x}{a_1} = 2\frac{y}{a_2}\sqrt{1 - y^2/a_2^2}$$

Squaring both sides, we get,

$$4\frac{y^2}{a_2^2}\left(\frac{y^2}{a_2^2} - 1\right) + \frac{x^2}{a_1^2} = 0$$

The above equation is order two in x and order four in y. Thus, the equation will have 2 roots in x and 4 roots in y intersecting at 2 & 4 points on the lines parallel to x-axis and y-axis respectively. Thus, it will form a figure of '8' on the screen.

**Q4.** Consider the superposition of two harmonic waves given by the equations:

$$x = a\sin(\omega_o t)$$
$$y = a\sin(3\omega_o t + \phi_1), \quad \text{when } \phi_1 = \pi/2$$

Draw the Lissajous figure representing the resultant motion.

**Soln.**

$$\frac{y}{a} = \sin(3\omega_0 t + \pi/2) = \cos(3\omega_0 t) = \left(3\cos\omega_0 t - 4\cos^3\omega_0 t\right)$$

$$\frac{y}{a} = \sqrt{1 - \sin^2\omega_0 t}\left(3 - 4\left(1 - \sin^2\omega_0 t\right)\right)$$

$$= \sqrt{1 - x^2/a^2}\left(4x^2/a^2 - 1\right)$$

Squaring both sides give,

$$\frac{y^2}{a^2} = \left(1 - \frac{x^2}{a^2}\right)\left(4\frac{x^2}{a^2} - 1\right)^2$$

**Q5.** Calculate the net field amplitude $E_0$ and the phase difference $\theta$ of the wave for $\phi = \pi/2$ comprising the following electric fields:
$E_1 = 4\cos\omega t$; $E_2 = 2\cos\omega t + \phi$; $E_3 = 4\cos(\omega t + \phi/2)$; $E_4 = \cos(\omega t + \phi/2 + \pi)$.

(a) $E_0 = 4\sqrt{2}$; $\theta = \pi/2$

(b) $E_0 = 8\sqrt{2}$; $\theta = \pi/4$

(c) $E_0 = 4/\sqrt{2}$; $\theta = \pi/2$

(d) $E_0 = 8/\sqrt{2}$; $\theta = \pi/4$

**Sol.**

$$\begin{aligned} E &= E_1 + E_2 + E_3 + E_4 \\ &= E_0\cos(\omega t + \theta) \end{aligned}$$

$$\begin{aligned} E_0\cos(\theta) &= 4\cos(0) + 4\cos(\phi) + 4\cos(\phi/2) + 4\cos(\phi/2 + \pi) \\ &= 4\left[1 + \cos(\phi) + \cos(\phi/2) - \cos(\phi/2)\right] \\ &= 4(1 + \cos\phi) \end{aligned}$$

$$\begin{aligned} E_0\sin(\theta) &= 4\sin(0) + 4\sin(\phi) + 4\sin(\phi/2) + 4\sin(\phi/2 + \pi) \\ &= 4\sin\phi + 4\sin(\phi/2) - 4\sin(\phi/2) \\ &= 4\sin\phi \end{aligned}$$

$$\begin{aligned} E_0^2 &= 16(1 + \cos^2\phi) + 16\sin^2\phi \\ &= 16 + 16\cos^2\phi + 32\cos\phi + 16\sin^2\phi \quad = 32(1 + \cos\phi) \\ &= 32 \cdot 2 \cdot \cos^2\phi/2 \\ &= 64\cos^2\phi/2 \end{aligned}$$

$$\therefore \quad E_0 = 8\cos\phi/2.$$

$$E_0|_{\phi=\pi/2} = 8 \cdot \cos\pi/4 = 8/\sqrt{2}$$

$$\tan\theta = \frac{4\sin(\phi)}{4(1 + \cos\phi)} = \frac{\sin(\phi)}{1 + \cos\phi}$$

$$\tan\theta = \frac{\sin\pi/2}{1+\cos\pi/2} = 1$$

$$\theta = \tan^{-1} = \pi/4$$

**The correct option is (d).**

**Q6.** Calculate the net field amplitude $E_0$ and the phase difference $\theta$ of the wave for $\phi = \pi/2$ comprising the following electric fields:
$E_1 = 2\cos\omega t$; $E_2 = 2\sin(\omega t - \phi)$; $E_3 = 4\cos(\omega t + \phi + 3\pi/2)$.

(a) $E_0 = 8$; $\theta = \pi/2$ (b) $E_0 = 64$; $\theta = 0$

(c) $E_0 = 8/$; $\theta = 0$ (d) $E_0 = 64$; $\theta = \pi/2$

**Sol.**

$$E_1 = 2\cos(\omega t)$$
$$E_2 = 4\sin(\omega t - \phi) = 4\cos(\omega t + \pi/2 - \phi)$$
$$E_3 = 2\cos(\omega t + \phi + 3\pi/2)$$

$$E = E_1 + E_2 + E_3$$
$$= E_0\cos(\omega t + \theta)$$

$$E_0\cos\theta = 2 + 4\cos(\pi/2 - \phi) + 2\cos(\phi + 3\pi/2)\Big)$$
$$E_0\sin\theta = 4\sin(\pi/2 - \phi) + 2\sin(\phi + 3\pi/2)$$

$$E_0^2 = 4 + 16\cos^2(\pi/2 - \phi) + 4\cos^2(\phi + 3\pi/2)$$
$$+ 16\cos(\pi/2 - \phi) + 8\cos(\phi + 3\pi/2)$$
$$+ (16\cos(\pi/2 - \phi)\cos(\phi + 3\pi/2))$$
$$+ 16\sin^2(\pi/2 - \phi) + 4\sin^2(\phi + 3\pi/2)$$
$$+ 16\sin(\pi/2 - \phi)\sin(\phi + 3\pi/2)$$
$$= 4 + 16 + 4 + 16\cos\left(-\frac{\pi}{2} + \phi + \phi + 3\pi/2\right)$$
$$+ 16\cos(\pi/2 - \phi) + 8\cos(\phi + 3\pi/2)$$

$$E_0^2 = 24 - 16\cos(2\phi) + 24\sin\phi$$

$$\begin{aligned} I_{\phi=\pi/2} &= 24 - 16\cos(\pi) + 24\sin(\pi/2) \\ &= 24 + 16 + 24 \\ &= 64 \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{4\sin(\pi/2-\phi) + 2\sin(\phi+3\pi/2)}{2+4\cos(\pi/2-\phi)+2\cos(\phi+3\pi/2)} \\ &= \frac{2\cos\phi}{2+6\sin\phi} \end{aligned}$$

$$\begin{aligned} \theta|_{\phi=\pi/2} &= \tan^{-1}\left[\frac{2\cos\pi/2}{2+6\sin\pi/2}\right] \\ &= \tan^{-1}(0) \\ &= 0. \end{aligned}$$

**The correct option is (b).**