PYQ Solution [IIT-JAM]

(Newton's Law One Dimensional)

Ans. 1: 3.225

Solution: $f_r = \mu N = 0.1 \times 2 \times 10 = 2N$

$\because m = 2\,kg$

Applied force is more than friction

$$ma = F - \mu N = 3 - 2 = 1 \Rightarrow a = \frac{1}{m} = \frac{1}{2} = 0.5\ m/s^2$$

$\because v = u^2 + 2as \ \Rightarrow v = \sqrt{2as} = \sqrt{2 \times 0.5 \times 10} = \sqrt{10} = 3.225\ m/s$

$\because u = 0,\ s = 10m$

Ans. 2: 2.51

Solution: $\because y = u_y t - \frac{1}{2} g t^2$, Time of flight $T = \frac{2u_y}{g}$

$\because x = u_x t + \frac{1}{2}\frac{qE}{m} t^2$, Range $R = u_x T + \frac{1}{2}\frac{qE}{m} T^2$

$$R = u_x \frac{2u_y}{g} + \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{2u_y}{g}\right)^2$$

where $u_x = \frac{5}{\sqrt{2}}, u_y = \frac{5}{\sqrt{2}}, g = 10,\ \frac{qE}{m} = \frac{10^{-3} \times 10}{0.2} = \frac{1}{20}$

$$R = \frac{5}{\sqrt{2}}\frac{2 \times 5}{\sqrt{2}}\frac{1}{10} + \frac{1}{2}\left(\frac{1}{20}\right)\left(\frac{2 \times 5}{\sqrt{2} \times 10}\right)^2 = \frac{25}{10} + \frac{1}{80} = 2.5 + 0.0125 = 2.512$$

Newton's Law Two Dimensional

Ans. 1: (d)

Solution: Maximum $h_{\max} = \frac{v^2 \sin^2\theta}{2g}$ and range $R = \frac{v^2 \sin 2\theta}{g}$ where $\theta = \frac{\pi}{4}$

$$\tan\alpha = \frac{h_{\max}}{\frac{R}{2}} = \frac{1}{2}$$

From the figure $a_r = -g\cos(90 - \alpha) = -g\sin\alpha = \frac{-g}{\sqrt{5}}$

$$a_\theta = -g\cos\alpha = \frac{-2g}{\sqrt{5}}$$

Ans. 2: (d)

Ans. 3: (a), (c) and (d)

Solution: (a) $\vec{r}(t) = A\cos\omega t\,\hat{i} + B\sin\omega t\,\hat{j} \Rightarrow x = A\cos\omega t,\ y = B\sin\omega t$

$$\Rightarrow \frac{x}{A} = \cos\omega t,\ \frac{y}{B} = \sin\omega t \Rightarrow \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1 \text{ (Ellipse)}$$

(b) $\frac{d\vec{r}}{dt} = -A\omega\sin\omega t\,\hat{i} + B\omega\cos\omega t\,\hat{j}$

Speed $= \left|\frac{d\vec{r}}{dt}\right| = \sqrt{A^2\omega^2\sin^2\omega t + B^2\omega^2\cos^2\omega t}$. Speed is function of time, so not constant.

(c) $\frac{d\vec{r}^2}{dt^2} = -A\omega^2\cos\omega t\,\hat{i} - B\omega^2\sin\omega t\,\hat{j} = -\omega^2\vec{r}$. Force act towards origin.

(d) $L = (\vec{r}\times\vec{p}) = m\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ A\cos\omega t & B\sin\omega t & 0 \\ -A\omega\sin\omega t & B\omega\cos\omega t & 0 \end{pmatrix} \Rightarrow L = m\omega AB\hat{k}$

Ans. 4: (b)

Solution: $v_r = 12\,m/s\quad v_\theta = \omega r \Rightarrow 2\times 8 = 16m/\sec$

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{144+256} = \sqrt{400} = 20m/\sec$$

Ans. 5: 9

Solution: $a_r = \ddot{r} - \dot{\theta}^2 r \Rightarrow e^t - 8\times e^t$ at $t = 0$,$1 - 8\times 1 = -7m/\sec^2$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \Rightarrow e^t\times 0 + 2e^t\sqrt{8} = 2\sqrt{8}\quad |a| = \sqrt{(-7)^2 + 4\times 8} = \sqrt{49+32} = \sqrt{81} = 9m/\sec^2$$

Ans. 6: (b)

Solution: $\vec{F} = -k\vec{r} = -k(r\hat{r} + z\hat{z})$

$F_r = -kr$, $F_\theta = 0$

$F_z = -kz \Rightarrow m\ddot{z} = -kz$ the motion along z is simple harmonic motion.

Ans. 7: 5.28

Solution: From S_2 Frame

$$\vec{r}' = (x - vt)\hat{i} + y\hat{i} = (R\cos\omega t - vt)\hat{i} + R\sin\omega t\hat{i}$$

$$= \vec{v}' = (\dot{x} - v)\hat{i} + \dot{y}\hat{j} = (-R\omega\sin t - v)\hat{i} + R\omega\cos\omega t\hat{j}$$

$$\vec{L} = \vec{r}'\times\vec{P} = m\left[(R\cos\omega t - vt)\hat{i} + R\sin\omega t\hat{i}\right]\times\left[(-R\omega\sin t - v)\hat{i} + R\omega\cos\omega t\hat{j}\right]$$

$$= m\left(\omega R^2\cos^2\omega t - vtR\cos\omega t + R^2\omega\sin^2\omega t + vR\sin\omega t\right)\hat{k}$$

$$= m\left(R^2\omega - vtR\omega\cos\omega t + vR\sin\omega t\right)$$

at $t = \frac{2\pi}{\omega}$, $\vec{L} = m\omega R^2(1-2\pi) = -5.28m\omega R^2 \Rightarrow |\vec{L}| = 5.28\ m\omega R^2$

Ans. 8: (c)

Solution: $x=5\cos(8\pi t), y=5\sin(8\pi t)$ and $z=5t$, $\Rightarrow x^2+y^2=5^2, z=5t$ motion is Helical

Ans. 9: (b)

Solution: $\vec{v}=\dot{r}\hat{r}+r\dot{\theta}\hat{\theta}\Rightarrow\frac{dv}{dt}=\vec{a}=\left(\ddot{r}-r\dot{\theta}^2\right)\hat{r}+\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)\hat{\theta}$

$$\Rightarrow a_r=\frac{d^2r}{dt^2}-r\left(\frac{d\theta}{dt}\right)^2$$

PYQ Solution [GATE]

Ans. 1: 4.94

Solution: $m\frac{dv}{dt} = mg - kv \Rightarrow \frac{dv}{dt} = g - \frac{k}{m}v \Rightarrow \frac{dv}{g - \frac{k}{m}v} = dt \Rightarrow \int_{10}^{u} \frac{dv}{g - \frac{k}{m}v} = \int_{0}^{0.2} dt$

$$\Rightarrow -\frac{m}{k}\left[\ln\left[g - \frac{k}{m}v\right]\right]_{10}^{u} = [t]_0^{0.2} \Rightarrow -\frac{m}{k}\left\{\left[\ln\left(g - \frac{k}{m}u\right)\right] - \ln\left(g - \frac{10k}{m}\right)\right\} = 0.2$$

$$\Rightarrow -\frac{m}{k}\left\{\ln\left(10 - \frac{0.05}{0.01}u\right) - \ln\left(10 - 10\times\frac{.05}{.01}\right)\right\} = 0.2$$

$$\Rightarrow -\frac{m}{k}\{\ln(10 - 5u) - \ln(-40)\} = 0.2$$

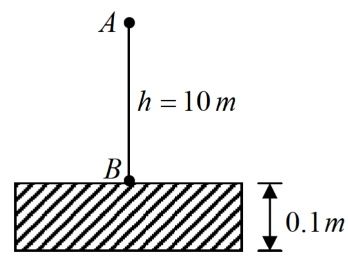
$$\ln\left(\frac{8}{u-2}\right) = \frac{0.2k}{m} \Rightarrow \ln\left(\frac{8}{u-2}\right) = \frac{0.2k}{m} \Rightarrow \frac{8}{u-2} = e \Rightarrow u = \frac{8}{e} + 2 = 4.94\ m/s$$

PYQ Solution [NET-JRF]

Newton's Law One Dimensional

Ans. 1: (c)

Solution: From conservation at energy



$$mgh=\frac{1}{2}mv^2 \Rightarrow v=\sqrt{20g}$$

The equation at motion when partition the cushion

$$mv\frac{dv}{dx}=mg-k$$

$$\int_{\sqrt{20g}}^{0} vdv=\int_{0.1}^{0}\left(g-\frac{k}{m}\right)dx \quad \left.\frac{v^2}{2}\right|_{\sqrt{20g}}^{0}=\left(g-\frac{k}{m}\right)\times 0.1$$

$$-\frac{20g\times 0.1}{2}=\left(g-\frac{k}{m}\right)=a \ -100\ g=a$$

option (c) is correct

Ans. 2: (b)

Solution: $m\frac{d^2x}{dt^2}=mg-\gamma\frac{dx}{dt}$

Putting the values of m,γ and g and simplifying we obtain

$$\frac{d^2x}{dt^2}+\frac{dx}{dt}=10$$

The general solution of this equation is $x(t)=c_1+c_2t+10e^{-t}$

Using the initial conditions $x(0)=0, x'(0)=0$

We obtain, $c_1=-10$ and $c_2=10$

Hence the required solution is $x(t)=10\left(t-1+e^{-t}\right)$

Ans. 3: (b)

Solution: The acceleration of cyclist is

$$a(t)=\frac{d}{dt}\left[\frac{300}{t+30}\right]=\frac{-300}{(t+30)^2}$$

Hence net force acting on the cyclist

$$F(t)=\frac{-300\times 80}{(t+30)^2}$$

Since frictional force is the only force acting on the cyclist, this net force is equal to functional force. We can write

$$F(t)=\frac{-80}{300}\left(\frac{300}{t+30}\right)^2=\frac{-4}{15}\left[v(t)\right]^2$$

When the cyclist moves at a constant speed, the frictional force is $F(t)=\frac{-4}{15}(8)^2$.

The displacement during this interval is 8×60. Thus the work done by frictional force $=\frac{-4}{15}\times 64\times 8\times 60$ is $-8.192\ KJ$. Hence in order to maintain constant speed the cyclist must supply an energy equal to $8.192\ KJ$.

Ans. 4: (c)

Solution: From Newton's second law, $m\frac{dv}{dt}=-\gamma v^2\Rightarrow\frac{dv}{v^2}=-\frac{\gamma}{m}dt$

Integrating both sides gives $-\frac{1}{v}=-\frac{\gamma}{m}t+c$

where c is a constant of integration

Since $v=v_0$ at $t=0$, we obtain

$$-\frac{1}{v_0}=-\frac{\gamma}{m}.0+c\Rightarrow c=-\frac{1}{v_0}$$

Hence, $-\frac{1}{v}=-\frac{\gamma}{m}t-\frac{1}{v_0}$

$$\Rightarrow\frac{1}{v}=\frac{\gamma t}{m}+\frac{1}{v_0}=\frac{\gamma v_0 t+m}{mv_0}\Rightarrow v=\frac{mv_0}{\gamma v_0 t+m}=\frac{mv_0}{m+\gamma v_0 t}$$

Ans. 5: (d)

Solution: Since table is frictionless then there is not any tangential force, so ball will have zero speed.

Newton's Law Two Dimensional

Ans. 1: (d)

Solution: The acceleration in Polar coordinate

$$\vec{a}=a_r\hat{r}+a_\theta\hat{\theta}=\left(\ddot{r}-r\dot{\theta}^2\right)\hat{r}+\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)\hat{\theta}$$

$\dot{r}=v_0\quad \ddot{r}=0$ and $\dot{\theta}=\omega_0\quad \ddot{\theta}=0$

$$\vec{a}=\left(-r\omega_0^2\right)\hat{r}+\left(2v_0\omega_0\right)\hat{\theta}$$

option (d) is correct

PYQ Solution [JEST]

Ans. 1: (c)

Solution: Equation of motion $\frac{mdv}{dt} = mg + kv^2 \Rightarrow \frac{dv}{dt} = g + \frac{k}{m}v^2 \Rightarrow \frac{dv}{g + \frac{k}{m}v^2} = dt$

$$\Rightarrow \int \frac{dv}{g + \frac{k}{m}v^2} = \int dt \Rightarrow \int \frac{dv}{\frac{k}{m}\left(\frac{gm}{k} + v^2\right)} = \int dt \Rightarrow \frac{m}{k} \times \frac{1}{\sqrt{\frac{gm}{k}}} \tan^{-1} \frac{v}{\sqrt{\frac{gm}{k}}} = t$$

$$\Rightarrow v = \sqrt{\frac{mg}{k}} \tan\left(\sqrt{\frac{kg}{m}} \cdot t\right)$$

Ans. 2: (b)

Solution: $T\sin\theta = ma$, $T\cos\theta = mg$, $\tan\theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\frac{a}{g} = \tan^{-1}\left(\frac{1.2}{9.8}\right) = 6.98^0 \approx 7^0$

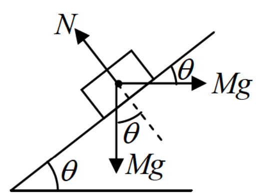
Ans. 3: (c)

Solution: The free body diagram of the block is shown below:

The normal force on the block can be calculated using Newton's second law in the direction perpendicular to the incline.

$$N - Mg\cos\theta - Mg\sin\theta = 0$$

$$\Rightarrow N = Mg(\sin\theta + \cos\theta)$$



Maximum value of static frictional force

$$f_s = \mu Mg(\sin\theta + \cos\theta)$$

The coefficient μ tells us that $|F_f| \le \mu N$. Using Eq this inequality becomes

$$Mg|\sin\theta - \cos\theta| \le \mu Mg(\cos\theta + \sin\theta) \qquad ......(1).$$

The absolute value here signifies that we must consider two cases:

- If $\tan\theta \ge 1$, then Eq.(1) becomes

$$\sin\theta - \cos\theta \le \mu(\cos\theta + \sin\theta) \quad \Rightarrow \quad \tan\theta \le \frac{1+\mu}{1-\mu}.$$

We divided by $1-\mu$, so this inequality is valid if $\mu < 1$, we see from the first inequality here that any value of θ (subject to our assumption, $\tan\theta \le 1$) works.

- If $\tan\theta \le 1$, then Eq. (1) becomes

$$-\sin\theta + \cos\theta \le \mu(\cos\theta + \sin\theta) \quad \Rightarrow \quad \tan\theta \ge \frac{1-\mu}{1+\mu}.$$

Putting these two ranges for θ together, we have

$$\frac{1-\mu}{1+\mu} \le \tan\theta \le \frac{1+\mu}{1-\mu}.$$

Ans. 4: (c)

Solution: $m\dfrac{dv}{dt} = -\eta v^{\alpha} \Rightarrow \int_0^t dt = -\dfrac{m}{\eta}\int_{v_0}^{0}\dfrac{dv}{v^{\alpha}} = \dfrac{m}{\eta}\dfrac{v_0^{1-\alpha}}{1-\alpha}$

Newton's law two dimensional

Ans. 1: (d)

Solution: $v_{relative} = v_A - v_B \Rightarrow T\left(v_A - v_B\right) = 2\pi R$

$$TR\left(\omega_A - \omega_B\right) = 2\pi R \Rightarrow TR\left(\frac{2\pi}{T_A} - \frac{2\pi}{T_B}\right) = 2\pi R \Rightarrow T = \left(\frac{1}{T_A} - \frac{1}{T_B}\right)^{-1}$$