PYQ Solution [IIT-JAM]

(Chapter 3 Radioactivity)

Ans. 1: 87.5

Solution: $N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}} = N_0\left(\frac{1}{2}\right)^{15/5} = \frac{N_0}{8}$

In 15 days the probability of decay $= \frac{N_0 - N}{N_0} \times 100 = \frac{7}{8} \times 100 = 87.5\%$

Ans. 2: (d)

Solution: $R = \lambda N$, where $N = \frac{10}{60} \times \left(6 \times 10^{23}\right) = 10^{23}$

$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{2\ln 2 \times 10^8} = \frac{0.693}{2 \times 2.303 \times 0.3010 \times 10^8} = \frac{0.693}{1.386 \times 10^8} = \frac{0.693}{1.386 \times 10^8} = 5 \times 10^{-9}\, s^{-1}$ Thus,

$R = 5 \times 10^{-9} \times \frac{10}{60} \times \left(6 \times 10^{23}\right) = 5 \times 10^{14}$.

Ans. 3: (d)

Solution: $\lambda = \frac{1}{t}\ln\left(\frac{R_0}{R}\right) = \frac{1}{30}\ln\left(\frac{R_0}{3/4R_0}\right) = \frac{1}{30}\ln\left(\frac{4}{3}\right) = \frac{1}{30}(1.4 - 1.1) = \frac{1}{100}$

$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{1/100} = 69.3$ day.

Ans. 4: 6.0

Solution: $\left|\frac{dN}{dt}\right| = \lambda N = \frac{0.693}{3.942 \times 10^6\,(s)} \times \frac{\left(70 \times 10^3\right)}{40} \times \frac{3.24 \times 10^{-5}}{100} \times 6.022 \times 10^{23}$

$= 6.0 \times 10^{13}$ Disintegrations/s $= 6.0 \times 10^{13}$ Bq $= 6.0 \times 10^{10}$ kBq

Ans. 5: (d)

Ans. 6: (b) and (c)

Solution: From given statement only (b) and (c) are correct.

Ans. 7: 3.57

Solution: $\frac{K^{40}}{A^{40}} = \frac{N_0 e^{-\lambda t}}{N_0 - N_0 e^{-\lambda t}} = 50 \Rightarrow e^{\lambda t} = \frac{51}{50} = 1.02 \Rightarrow \lambda t = \ln(1.02)$

$\Rightarrow t = \frac{1.25 \times 10^9}{0.693}\ln(1.02) \Rightarrow t = 3.57 \times 10^7$ years.

PYQ Solution [GATE]

Ans. 1: 25.995

Solution: The height of coulomb barrier for α particle from

$${}_{90}Th^{230} \rightarrow {}_{88}X^{226} + 2He^{4} \quad (\alpha\text{ - particle})$$

$$V_C = \frac{1}{4\pi \epsilon_0}\left(\frac{2ze^2}{R}\right)$$

Here, $R_0 = 1.3\ fm,\ \frac{e^2}{4\pi \epsilon_0} = 1.44\ MeV\ fm$

And $R = R_0 A^{1/3}$

Here, we consider pure Columbic interaction

$A_{Th}^{1/3} = A_X^{1/3} + A_\alpha^{1/3} = (226)^{1/3} + (4)^{1/3} = (6.09 + 1.58) = 7.67$, $\quad R = R_0 A_{Th}^{1/3} = 1.3(7.67)$

Hence, $V_C = \left(\frac{e^2}{4\pi \epsilon_0}\right)\frac{2\times 90}{1.3(7.67)} = \frac{180\times 1.44}{1.3\times 7.67}\frac{MeV}{fm}$

$V_C = 25.995\ MeV$

Ans. 2: (b)

Ans. 3: (a)

Solution: $^{13}C_6 + \nu_e \rightarrow {}^{13}N_7 + X \Rightarrow {}^{13}C_6 \rightarrow {}^{13}N_7 + X + \overline{\nu}_e$

$L_e = 0 \qquad 0 \ \ +1 \ \ -1$

To conserve the Lepton number L_e, X should be e^-

Ans. 4: 40

Solution: Branching ratio is the fraction of particles (here β) which decays by an individual decay mode with respect to the total number of particles which decays

$$BR = \frac{\left(\frac{dN}{dt}\right)_x}{\left(\frac{dt}{dt}\right)_\beta} = \frac{(T_{1/2})_x}{(T_{1/2})_\beta} \Rightarrow (T_{1/2})_\beta = \frac{(T_{1/2})_x}{BR} = \frac{30}{0.75} = 40 \text{ hours}$$

PYQ Solution [NET-JRF]

Ans. 1: (a)

Solution:

$$X \xrightarrow{\lambda_1} Y \xrightarrow{\lambda_2} Z$$

	X	Y	Z
$t=0$	N_0	0	0
t	N_1	N_2	N_3

Rate equations $N_1 = N_0 e^{-\lambda_1 t}, \dfrac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2, \dfrac{dN_3}{dt} = \lambda_2 N_2$

$$N_3 = N_0\left[1 + \frac{\lambda_1 e^{-\lambda_2 t}}{(\lambda_2-\lambda_1)} - \frac{\lambda_2 e^{-\lambda_1 t}}{(\lambda_2-\lambda_1)}\right]$$

$$= N_0\left[1 + \frac{\lambda_1}{(\lambda_2-\lambda_1)}\left(1-\lambda_2 t + \frac{\lambda_2^2 t^2}{2}\right) - \frac{\lambda_2}{(\lambda_2-\lambda_1)}\left(1-\lambda_1 t + \frac{\lambda_1^2 t^2}{2}\right)\right]$$

$$= N_0\left[1 + \frac{\lambda_1}{(\lambda_2-\lambda_1)} - \frac{\lambda_1\lambda_2 t}{(\lambda_2-\lambda_1)} + \frac{\lambda_1}{(\lambda_2-\lambda_1)}\frac{\lambda_2^2 t^2}{2} - \frac{\lambda_2}{(\lambda_2-\lambda_1)} + \frac{\lambda_2\lambda_1 t}{(\lambda_2-\lambda_1)} - \frac{\lambda_2}{(\lambda_2-\lambda_1)}\frac{\lambda_1^2 t^2}{2}\right]$$

$$= N_0\left[\frac{\lambda_1}{(\lambda_2-\lambda_1)} \times \frac{\lambda_2^2 t^2}{2} - \frac{\lambda_2}{(\lambda_2-\lambda_1)} \times \frac{\lambda_1^2 t^2}{2}\right] = \frac{\lambda_1\lambda_2 t^2}{2} N_0\left[\frac{\lambda_2}{\lambda_2-\lambda_1} - \frac{\lambda_1}{\lambda_2-\lambda_1}\right] = \frac{1}{2}\lambda_1\lambda_2 N_0 t^2$$