Chapter Three

Relativistic Four Vectors

3.1 Space and Time

In special theory of relativity now we can consider the position vector is four-dimensional vector which can be given by 3 spatial (x, y, z) coordinate fourth -time coordinate (ict).

From S frame the distance is identified as $s^2 = x^2 + y^2 + z^2 + (ict)^2 = x^2 + y^2 + z^2 - c^2t^2$

Then it can be shown that $x^2 + y^2 + z^2 - c^2t^2$ is invariant under Lorentz transformation

$$x^2 + y^2 + z^2 - c^2t^2 = \gamma^2 (x' + vt')^2 + (y')^2 + (z')^2 - c^2 \left(t' + v\frac{x'}{c^2} \right)^2 \gamma^2$$

$$\gamma^2 \left(x'^2 + v^2t'^2 + 2x'vt' - c^2t'^2 - \frac{v^2x'^2}{c^2} - 2x'vt' \right) + y'^2 + z'^2$$

$$\frac{1}{\left(1 - \frac{v^2}{c^2}\right)} \left(x'^2 \left(1 - \frac{v^2}{c^2}\right) - c^2t'^2 \left(1 - \frac{v^2}{c^2}\right) \right) + y'^2 + z'^2 = x'^2 + y'^2 + z'^2 - c^2t'^2$$

$$x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t'^2 \text{ because } y = y', z = z'$$

$\Rightarrow x^2 - c^2t^2 = x'^2 - c^2t'^2$ which will analogically rotation of coordinate system.

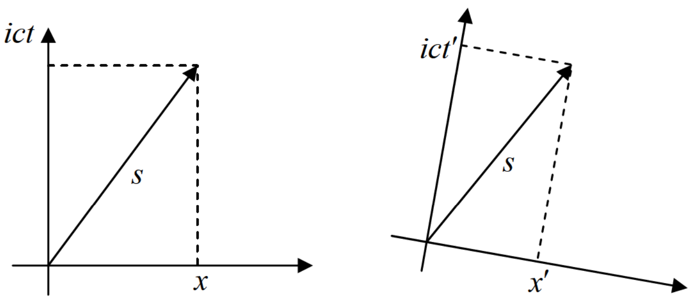

Figure 3.1

Space time interval $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 = c^2(\Delta t')^2 - (\Delta x')^2 = c^2\tau_0^2$, where τ_0 is proper time interval.

(i) Space-like Intervals: Time separation between the two event is less than the time taken by light in covering the distance between them.

$$\frac{\Delta x}{c} > \Delta t$$

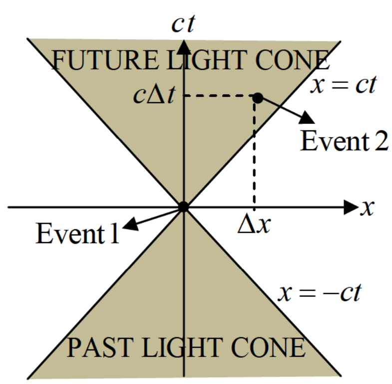

Figure 3.2

(ii) Time-like intervals: Time separation between two event is more than the time taken by light in covering the distance between them

$$\frac{\Delta x}{c} < \Delta t$$

(iii) Light-like intervals: Time separation between two event is equal to time taken by light in covering the distance between them

$$\frac{\Delta x}{c} = \Delta t$$