Chapter Four

Relativistic Electrodynamics

4.1 Four Current Vectors

Relativistic Electromagnetism is a physical phenomenon explained in electromagnetic field theory due to coulomb's law, Maxwell four equation and Lorentz transformation.

The requirement that the equations remain consistent when viewed from various moving observers led to special relativity, a geometric theory of 4-space where intermediation is by light and radiation.

Analogy

We have relation in space $(\vec{r})$ time (t) as $c^2t^2 - r^2 = c^2\tau_0^2$ where τ_0 is proper time

Similar relation we have for relativistic mass (m) and momentum (p) as $c^2m^2 - p^2 = c^2m_0^2$, where m_0 is rest mass.

On the basis of these analogy we have relation for charge density (ρ) and charge density $\vec{J}$ as $c^2\rho^2 - j^2 = c^2\rho_0^2$ where ρ_0 is identify as proper charge density which is measured from rest frame of charge .

So $\rho = \dfrac{\rho_0 m}{m_0}$ and $\vec{j} = \dfrac{\rho_0}{m_0}\vec{p}$

So, we have equivalent Lorentz transformation as well as inverse Lorentz transformation for current density $\vec{j}$ and charge density ρ.

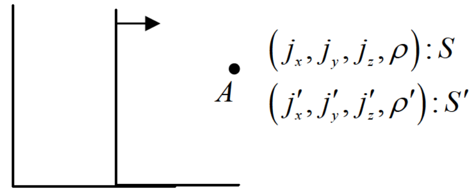


Figure 4.1

$$j'_x = \frac{j_x - \rho v}{\sqrt{1 - v^2/c^2}}, \qquad j'_y = j_y, \qquad j'_z = j_z$$

and $\rho = \dfrac{\rho' - v j_x / c^2}{\sqrt{1 - v^2/c^2}}$

Similarly, we have inverse Lorentz transformation

$$j_x = \frac{j'_x + \rho' v}{\sqrt{1 - v^2/c^2}}, \qquad j_y = j'_y, \qquad j_z = j'_z \text{ and } \rho = \frac{\rho' + v j'_x / c^2}{\sqrt{1 - v^2/c^2}}$$

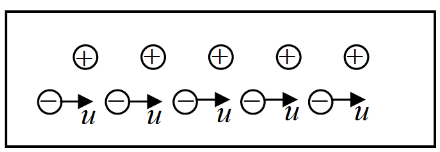
Example: Consider a long straight wire rest in S frame. The free electrons moves with drift velocity u towards right as shown in figure. If number of free electron per unit volume is n.

(a) Find the net charge density from S frame

(b) Find net current density from S

(c) If another inertial frame S' moving with respect to S with velocity v as same direction of Drift velocity u Find total charge density with respect to S'

(d) If another inertial frame S' moving with respect to S with velocity v as same direction of Drift velocity u Find total charge density with respect to S'



Solution: From S frame negative charge density id due to is free electron is $\rho^- =$ negative there is similar number of positive charge $\rho^+ = +ne$ the positive charge is rest with respect to S frame. The distance between negative electron and positive charge is same.

(a) $\rho = \rho^- + \rho^+ = (-ne) + (ne) = 0$

(b) $j_x^- = \rho^- u = -neu$, $j_x^+ = 0$

(c) $\rho^{-\prime} = \dfrac{\rho^- - v j_x^- / c^2}{\sqrt{1 - v^2/c^2}} = \dfrac{\rho^- - v\rho^- u / c^2}{\sqrt{1 - v^2/c^2}} \Rightarrow \rho^{-\prime} = \rho^- \dfrac{(1 - vu/c^2)}{\sqrt{1 - v^2/c^2}}$ where $j_x^- = \rho^- u$

$\rho^{+\prime} = \dfrac{\rho^+ - v j_x^+ / c^2}{\sqrt{1 - v^2/c^2}}$ $\rho^{+\prime} = \dfrac{\rho^+}{\sqrt{1 - v^2/c^2}}$ where

$$\rho' = \rho'^{+} + \rho'^{-} \Rightarrow \rho' = \frac{ne}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{ne(1 - vu/c^2)}{\sqrt{1 - v^2/c^2}} = \frac{nevu/c^2}{\sqrt{1 - v^2/c^2}}$$

(d) $j_x'^- = \frac{j_x^- - \rho^- v}{\sqrt{1-v^2/c^2}}, = \frac{\rho^- u - \rho^- v}{\sqrt{1-v^2/c^2}} = \rho^- \frac{u-v}{\sqrt{1-v^2/c^2}} \Rightarrow j_x'^+ = -ne\frac{u-v}{\sqrt{1-v^2/c^2}}$

$$j_x'^+ = \frac{j_x^+ - \rho^+ v}{\sqrt{1-v^2/c^2}}, = \frac{0-\rho^+ v}{\sqrt{1-v^2/c^2}} = \frac{-\rho^+ v}{\sqrt{1-v^2/c^2}} = \frac{-nev}{\sqrt{1-v^2/c^2}}$$

$$j_x' = j_x'^+ + j_x'^- = \frac{-nev}{\sqrt{1-v^2/c^2}} + -ne\frac{u-v}{\sqrt{1-v^2/c^2}} = \frac{-nue}{\sqrt{1-v^2/c^2}}$$