# Class Test Solution

## (Relativistic Electrodynamics and Relativistic Quantum Mechanics)

Ans. 1: (a)

Ans. 2: (c)

Solution: $\because E^2 - B^2 = E'^2 - B'^2 = \text{constant} \Rightarrow (9+16) - 9 = 25 - B'^2 \Rightarrow |B'| = 3$

Ans. 3: (d)

Solution: $E_x = E'_x,\ E_y = \gamma\left(E'_y + vB'_z\right)$ and $E_z = \gamma\left(E'_z - vB'_y\right)$

$$B_x = B'_x, B_y = \gamma\left(B'_y - \frac{v}{c^2}E'_z\right) \text{ and } B_z = \gamma\left(B'_z + \frac{v}{c^2}E'_y\right)$$

$$\vec{E}' = -\vec{\nabla}V' - \frac{\partial \vec{A}'}{\partial t} = 0,\ \vec{B}' = \vec{\nabla} \times \vec{A}' = a\,\hat{j}$$

$$E_x = 0, E_y = \gamma(0 - v \times 0) = 0,\ E_z = \gamma(0 + va) = \gamma va$$

(replace $v$ by $-v$) $\Rightarrow$ $\vec{E} = v\gamma a\hat{z}$

$$B_x = 0,\ B_y = \gamma\left(a + \frac{v}{c^2} \times 0\right) = \gamma a,\ B_z = \gamma\left(0 - \frac{v}{c^2} \times 0\right) = 0$$

$$\Rightarrow \vec{B} = \gamma a\,\hat{j}$$

Ans. 4: (b)

Solution: $v = \frac{c}{4}$ $\qquad u'_x = \frac{c}{2}$

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}} = \frac{\frac{c}{2} + \frac{c}{4}}{1 + \frac{1}{c^2}\frac{c}{2}\cdot\frac{c}{4}} = \frac{2c}{3}$$

Therefore, $\lambda = \dfrac{\lambda_0}{\sqrt{1 - \frac{u_x^2}{c^2}}} = \dfrac{3\lambda_0}{\sqrt{5}}$

Ans. 5: (d)

Solution: where $\rho' = \lambda_0, J_x'$ $J_x = \dfrac{J'_x + v\rho'}{\sqrt{1 - \frac{v^2}{c^2}}} = \dfrac{v\lambda_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

Ans. 6: (c)

Ans. 7: (c)

Solution: The Klein Gordon equation is given by $-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = -c^2\hbar^2\nabla^2\psi + m_0^2 c^4 \psi$

Multiply the equation with $\psi^*$ we get $-\hbar^2\psi^* \frac{\partial^2 \psi}{\partial t^2} = -c^2\hbar^2\psi^*\nabla^2\psi + m_0^2 c^4 \psi^*\psi$..........$A$

The Klein Gordon equation for $\psi^*$ $-\hbar^2 \frac{\partial^2 \psi^*}{\partial t^2} = -c^2\hbar^2\nabla^2\psi^* + m_0^2 c^4 \psi^*$

Multiply equation with $\psi$ we get $-\hbar^2\psi \frac{\partial^2 \psi^*}{\partial t^2} = -c^2\hbar^2\psi\nabla^2\psi^* + m_0^2 c^4 \psi\psi^*$......$B$

Subtract equation A to B $-\hbar^2\left(\psi^* \frac{\partial^2 \psi}{\partial t^2} - \psi \frac{\partial^2 \psi^*}{\partial t^2}\right) = -\hbar^2 c^2\left(\psi^*\nabla^2\psi - \psi\nabla^2\psi^*\right)$

$$\left(\psi^* \frac{\partial^2 \psi}{\partial t^2} - \psi \frac{\partial^2 \psi^*}{\partial t^2}\right) = c^2\left(\psi^*\nabla^2\psi - \psi\nabla^2\psi^*\right) \Rightarrow \frac{\partial}{\partial t}\left(\psi^* \frac{\partial \psi}{\partial t^2} - \psi \frac{\partial \psi^*}{\partial t^2}\right) = c^2\vec{\nabla}.\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right)$$

$$\frac{\partial}{\partial t}\frac{\hbar}{2imc^2}\left(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t}\right) = \vec{\nabla}.\left(\frac{\hbar}{2im}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right)\right)$$

$\frac{\partial}{\partial t}\frac{i\hbar}{2m_0c^2}\left(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t}\right) + \vec{\nabla}.\left(\frac{\hbar}{2im_0}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right)\right) = 0 \Rightarrow \frac{\partial \rho}{\partial t} + \vec{\nabla}.\vec{J} = 0$ equivalent to continuity equation.

Where $\rho = \frac{i\hbar}{2m_0c^2}\left(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t}\right)$

Ans. 8: (b)

Ans. 9: (b)

Solution: $i\hbar\frac{\partial \psi}{\partial t} = \left[c\vec{\alpha}.\vec{\pi} + \beta mc^2\right]\psi$

Where $\psi(t) = |\psi\rangle \exp\left(-\frac{iEt}{\hbar}\right)$ where $|\psi\rangle = \begin{bmatrix}\chi \\ \phi\end{bmatrix}$, $\chi$ and $\phi$ are two component spinor .

$\begin{bmatrix}0 & \vec{\sigma} \\ \vec{\sigma} & 0\end{bmatrix}$ and $\beta = \begin{bmatrix}1 & 0 \\ 0 & -1\end{bmatrix}$

The matrix form of Dirac equation is given by

$$\begin{bmatrix}E - mc^2 & -c\vec{\alpha}.\vec{\pi} \\ -c\vec{\alpha}.\vec{\pi} & E + mc^2\end{bmatrix}\begin{bmatrix}\chi \\ \phi\end{bmatrix} = 0$$

$\left(E - mc^2\right)\chi - c\vec{\sigma}.\vec{\pi}\phi = 0$ .......(1) $\left(E + mc^2\right)\phi - c\vec{\sigma}.\vec{\pi}\chi = 0$ .........(2)

Ans. 10: (d)

Solution: Let us compute $\left(\vec{\pi}\times\vec{\pi}\right)=\left(\vec{p}-\frac{q\vec{A}}{c}\right)\times\left(\vec{p}-\frac{q\vec{A}}{c}\right)=\vec{p}\times\vec{p}+\frac{q^2}{c^2}\left(\vec{A}\times\vec{A}\right)-\frac{q}{c}\left(\vec{p}\times\vec{A}\right)-\frac{q}{c}\left(\vec{A}\times\vec{p}\right)$

We know $\vec{p}\times\vec{p}=0, \vec{A}\times\vec{A}=0$ now let us calculate

$-\frac{q}{c}\left(\vec{p}\times\vec{A}\right)-\frac{q}{c}\left(\vec{A}\times\vec{p}\right)\left|\psi\right\rangle=-\frac{q}{c}\left(-i\hbar\vec{\nabla}\times\vec{A}\right)\left|\psi\right\rangle-\left(\vec{A}\times-i\hbar\vec{\nabla}\right)\left|\psi\right\rangle=-i\hbar\frac{q}{c}\vec{B}-0$ we know

$\left(\vec{\pi}\times\vec{\pi}\right)=\frac{i\hbar q}{c}\vec{B}$