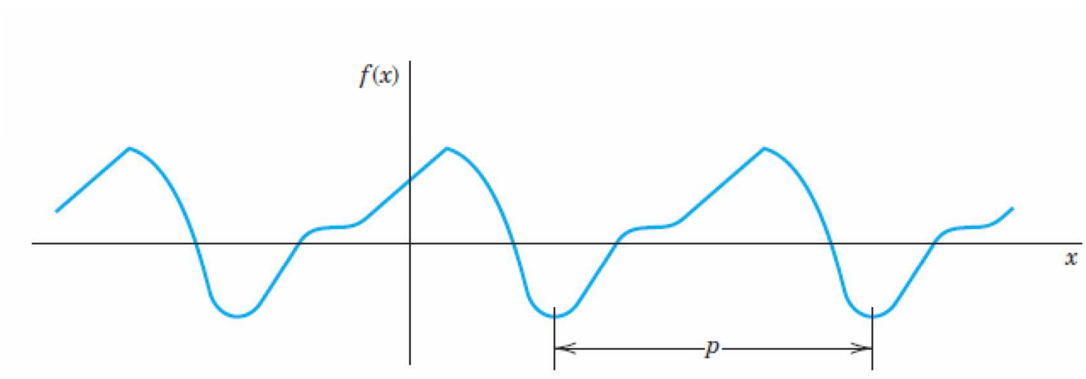
Chapter One

FOURIER SERIES

Fourier series (FS) are *infinite* series that represent *periodic* functions in terms of cosines and sines. Fourier series are of greatest importance to the physicist, engineer, and applied mathematician. To define Fourier series, we first need some background material.

Periodic Function: A function $f(x)$ is called a periodic function if $f(x)$ is defined for all real x, except possibly at some points, and if there is some positive number p, called a **period** of $f(x)$, such that

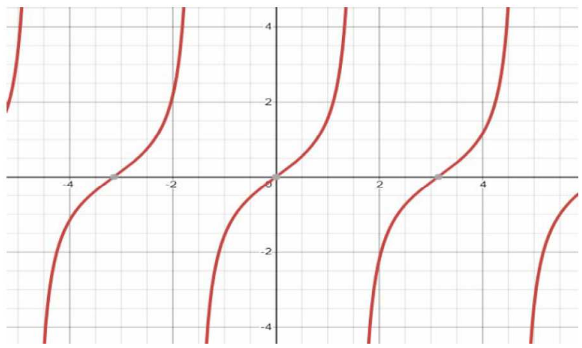
$$f(x+np)=f(x)$$



Example: $f(x) = \tan x$

Solution: $f(x) = \tan x$ is a periodic function that is not defined for all real x but undefined for some points (more precisely, countably many points), that is $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, ...$

The graph of a periodic function has the characteristic that it can be obtained by periodic repetition of its graph in any interval of length p.



Fundamental Period: The smallest positive period is called the *fundamental period*(for all x).

Examples of periodic functions: cosine, sine, tangent, and cotangent.

Examples of Non-periodic functions: x, x^2, x^3, e^x, $\cosh x$, and $\ln x$

If $f(x)$ and $g(x)$ have period p, then $\left(af(x)+bg(x)\right)$ with any constants a and b also has the period p.

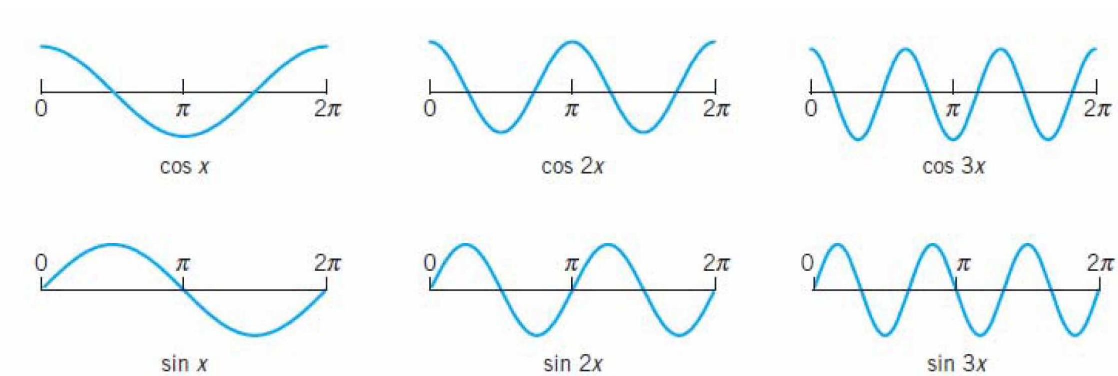


Figure: Cosine and sine functions having the period 2π

The infinite series to be written here will be trigonometric series,

$$a_0 + a_1\cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + ...$$

$$= a_0 + \sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right)$$