Chapter 3 (Wave Nature of Particle)
Solution of PYQ [IIT-JAM]

$A_2 \to$ De Bogile Hypthesis

Ans. 1: (b)

Solution: $E^2 = p^2c^2 + m_0^2c^4$

$$2E\frac{dE}{dp} = 2pc^2 \Rightarrow E.v_g = pc^2 \Rightarrow \frac{E}{p} = \frac{c^2}{v_g} = v_p \Rightarrow v_p = \frac{c^2}{\beta c} \Rightarrow \frac{c}{\beta}$$

Ans. 2: (c)

Solution: For photon $p_{ph} = \frac{E}{c}$, $\lambda_{ph} = \frac{h}{p} = \frac{hc}{E}$

For electron $p_e = \frac{\sqrt{E^2 - m^2c^4}}{c}$, $\lambda_e = \frac{h}{p} = \frac{hc}{\sqrt{E^2 - m^2c^4}}$

Ans. 3: 1.43

Solution: $E^2 = p^2c^2 + \left(m_0c^2\right)^2 \Rightarrow p = \sqrt{\frac{E^2 - \left(m_0c^2\right)^2}{c^2}} = \frac{\sqrt{1-0.25}}{c} = \frac{\sqrt{0.75}MeV}{c}$

As, $\lambda = \frac{h}{p} = \frac{6.6\times10^{-34}\times3\times10^{8}}{\sqrt{.75}\times1.6\times10^{-13}} = \frac{19.8\times10^{-13}}{1.38} = 14.34\times10^{-13}m = 1.43\times10^{-12}m$

Ans. 4: (c)

Solution: Group velocity, $V_g = \frac{d\omega}{dk}$ and phase velocity is $V_p = \frac{\omega}{k}$

$\omega^2 = \alpha k + \beta k^3$ ....(A)

Differentiating both sides we get $2\omega.\frac{d\omega}{dk} = \alpha + 3\beta k^2$

Now dividing both sides by k we will get

$$2\frac{\omega}{k}.\frac{d\omega}{dk} = \frac{\alpha}{k} + 3\beta k \quad \Rightarrow 2V_p.V_g = \frac{\alpha}{k} + 3\beta k$$

For $k = k_0$ and $V_p = V_g$

$$2V_p^2 = \frac{\alpha}{k_0} + 3\beta k_0 \quad \Rightarrow V_p = \left(\frac{\alpha}{2k_0} + \frac{3\beta k_0}{2}\right)^{1/2}$$

From equation (A) $V_p = \frac{\omega}{k} = \left(\frac{\alpha}{k_0} + \beta k_0\right)^{1/2}$

Thus, $\left(\dfrac{\alpha}{2k_0}+\dfrac{3\beta k_0}{2}\right)^{1/2}=\left(\dfrac{\alpha}{k_0}+\beta k_0\right)^{1/2}\Rightarrow\dfrac{\alpha}{2k_0}-\dfrac{\beta k_0}{2}=0\Rightarrow k_0=\sqrt{\dfrac{\alpha}{\beta}}$

Ans. 5: (d)

Solution: For $\omega=\omega_0=100$ and $k=k_0=10$ the phase velocity is $v_p=\dfrac{\omega_0}{k_0}$ =10

The group velocity is $v_g=\dfrac{\Delta\omega}{\Delta k}=\dfrac{\omega_2-\omega_1}{k_2-k_1}=\dfrac{110.25-90.25}{10.5-9.5}=20$

$$\frac{v_g}{v_p}=\frac{20}{10}=2$$

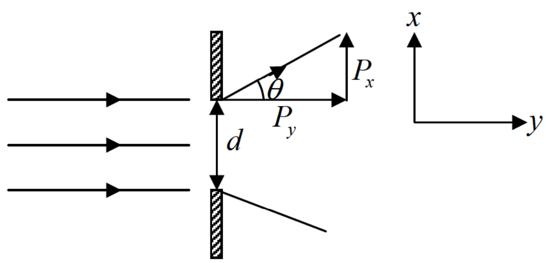
Ans. 6: (d)

Solution: $\omega^2=\left(\omega_0^2+c^2k^2\right)$

$$2\omega\frac{d\omega}{dk}=2c^2k\Rightarrow\frac{\omega}{k}\cdot\frac{d\omega}{dk}=c^2\Rightarrow v_p.v_g=c^2$$

Ans. 7: (b) and (d)

Solution: The electrons beam before slit is collimated in y - direction as shown in figure. Thus, before slit



$P_y=P$ and $P_x=0$

also $\Delta x\to\infty$ as $\Delta P_x=0$

Thus options (a) and (c) are not correct.

Now, after the slit $\Delta P_x=d$ as a result $\Delta P_x=\dfrac{\hbar}{\Delta x}=\dfrac{\hbar}{d}$

i.e., $P_x\cong\dfrac{\hbar}{d}$

Thus, option (b) is correct.

Whereas presence of slit does not affect the uncertainty in momentum in y - direction. Thus option (d) is also correct.