

Chapter 4 (Relativistic Electrodynamics)

Solution of PYQ [GATE]

Ans. 1: (b)

Ans. 2: 6

Solution: In Cartesian co-ordinate, three Independent coordinate for electric field, (E_x, E_y, E_z) and three Independent co-ordinate for magnetic field (B_x, B_y, B_z) .

Ans. 3: (b)

Solution: Under parity operation $r \rightarrow -r$

$$E = -\frac{\partial V}{\partial r} ; \quad E : P \rightarrow -E$$

$$B = \vec{I} \times \vec{r} ; \quad B : P \rightarrow +B$$

$$L = \vec{r} \times \vec{p} ; \quad L : P \rightarrow +L$$

$$E = -\frac{\partial A}{\partial t} ; \quad A : P \rightarrow -A$$

Solution of PYQ [NET-JRF]

Ans. 1: (a)

Solution: Assume the wavelength at radiation if choose is rest with respect to observe = $\lambda_0 = L$

From Doppler effect if relation move towards observe

$$\lambda = L \sqrt{\frac{1-\beta}{1+\beta}} \sqrt{\frac{1+\beta}{1+\beta}}$$

$$\lambda = L \sqrt{\frac{(1-\beta)(1+\beta)}{(1+\beta)(1+\beta)}}$$

$$\lambda = \frac{L}{1+\beta} \sqrt{1-\beta^2} \quad \lambda = \frac{1}{\beta} \sqrt{1-\beta^2} \quad 1+\beta \approx \beta$$

Ans. 2: (b)

Ans. 3: (c)

Solution: $\lambda = \frac{\lambda_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{Q}{L \sqrt{1-\frac{v^2}{c^2}}}$

Ans. 4: (d)

Solution: $E_x = E'_x$, $E_y = \gamma(E'_y + vB'_z)$ and $E_z = \gamma(E'_z - vB'_y)$

$$B_x = B'_x, B_y = \gamma\left(B'_y - \frac{v}{c^2}E'_z\right) \text{ and } B_z = \gamma\left(B'_z + \frac{v}{c^2}E'_y\right)$$

$$\vec{E}' = -\vec{\nabla}V' - \frac{\partial \vec{A}'}{\partial t} = 0, \vec{B}' = \vec{\nabla} \times \vec{A}' = a\hat{j}$$

$$E_x = 0, E_y = \gamma(0 - v \times 0) = 0, E_z = \gamma(0 + va) = \gamma va$$

$$(\text{replace } v \text{ by } -v) \Rightarrow \vec{E} = v\gamma a\hat{z}$$

$$B_x = 0, B_y = \gamma\left(a + \frac{v}{c^2} \times 0\right) = \gamma a, B_z = \gamma\left(0 - \frac{v}{c^2} \times 0\right) = 0$$

$$\Rightarrow \vec{B} = \gamma a \hat{j}$$

Ans. 5: (b)

Solution: Charge of q_2 seen by rest frame of $q_1 = \gamma q_2$; $F = \frac{1}{4\pi\epsilon_0} \frac{\gamma q_1 q_2}{l^2}$

Ans. 6: (c)

Solution: $\because E^2 - B^2 = E'^2 - B'^2 = \text{constant} \Rightarrow (9+16) - 9 = 16 - B'^2 \Rightarrow B' = 0$

Ans. 7: (a)

Solution: $|\vec{E}|^2 - |\vec{B}|^2 = 29$

In another Frame $|\vec{E}'|^2 - |\vec{B}'|^2 = 29$

$$\vec{B}' = 2\sqrt{5}\hat{k} \Rightarrow |\vec{B}'|^2 = 4 \times 5 = 20 \Rightarrow |\vec{E}'|^2 = 49$$

$$\text{It is given } \vec{E} \perp \vec{B} \text{ so } \vec{E}' = \frac{7}{\sqrt{2}}(\hat{i} + \hat{j})$$

Ans. 8: (c)

Solution: From S

$E \cdot B$ is invariant and $E^2 - B^2$ is invariant

$$E'_x{}^2 - B'_y{}^2 = -3\alpha^2$$

$$E'_x + B'_y = \alpha \text{ (Solving these two equation)}$$

$$E'_x = -\alpha, \quad B'_y = 2\alpha$$

Option 4 is correct.

Ans. 9: (d)

Solution: $E_x = E'_x, E_y = \gamma(E'_y + vB'_z)$ and $E_z = \gamma(E'_z - vB'_y)$

$$B_x = B'_x, B_y = \gamma\left(B'_y - \frac{v}{c^2}E'_z\right) \text{ and } B_z = \gamma\left(B'_z + \frac{v}{c^2}E'_y\right)$$

$$\vec{E}' = -\vec{\nabla}V' - \frac{\partial \vec{A}'}{\partial t} = 0, \quad \vec{B}' = \vec{\nabla} \times \vec{A}' = a\hat{j}$$

$$E_x = 0, E_y = \gamma(0 - v \times 0) = 0, E_z = \gamma(0 + va) = \gamma va$$

$$(\text{replace } v \text{ by } -v) \Rightarrow \vec{E} = v\gamma a\hat{z}$$

$$B_x = 0, B_y = \gamma\left(a + \frac{v}{c^2} \times 0\right) = \gamma a, B_z = \gamma\left(0 - \frac{v}{c^2} \times 0\right) = 0$$

$$\Rightarrow \vec{B} = \gamma a \hat{j}$$