Solution (Class Test)

STR-Lorentz Transformation

Ans. 1: (c)

Solution: $v=\frac{c}{2}\hat{i} \quad u_x'=0,\, u_y'=\frac{c}{2},\, u_z'=0$

$$u_x=\frac{u_x'+v}{1+\frac{u_x'v}{c^2}}=\frac{c}{2} \quad , \quad u_x=\frac{u_y'\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{u_x'v}{c^2}}=\frac{c}{2}\sqrt{1-\frac{1}{4}}=\sqrt{3}\frac{c}{4},\; u_z=\frac{u_z'\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{u_x'v}{c^2}}=0$$

$$u=\sqrt{\frac{c^2}{4}+\frac{3c^2}{16}}=\frac{\sqrt{7}c}{4}=0.6c$$

Ans. 2: (b)

Solution: $t_2-t_1=0$ and $x_2-x_1=10^8$

$$t_2'-t_1'=\frac{(t_2-t_1)}{\sqrt{1-v^2/c_2}}-\left(\frac{x_2-x_1}{\sqrt{1-v^2/c_2}}\right)\frac{v}{c_2}$$

$$=-\frac{(x_2-x_1)\frac{v}{c^2}}{\sqrt{1-v^2/c^2}}=-\frac{(x_2-x_1)\frac{v}{c^2}}{\sqrt{1-v^2/c^2}}=-\frac{10^8\times\frac{0.8c}{c^2}}{\sqrt{1-0.64}}=\frac{\cdot 8\times 10^8}{\cdot 6\times 3\times 10^8}=\frac{8}{18}=0.44\,\text{sec}.$$

Ans. 3: (c)

Solution: $\Delta t=\frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}\Rightarrow \Delta t=\frac{2.2\times 10^{-6}}{\sqrt{1-(0.998)^2}}=34.8\times 10^{-6}\,\text{sec}$

Now distance will be $=\Delta t\times v=34.8\times 10^{-6}\times 0.998\times 3\times 10^8=10.4192\,km$

Apparent thickness $\Delta X=\Delta t\times v=2.2\times 10^{-6}\times 0.998\times 3\times 10^8=0.658\,km$

Ans. 4: (a)

Solution: Velocity of S' with respect to S is $v=0.6c$

$$t_A'=\frac{t_A-\frac{v}{c^2}y}{\sqrt{1-\frac{v^2}{c^2}}}$$

For event A, $t_A=0, y=0$. So $ct_A'=0$

$$t_B' = \frac{t_B - \frac{v}{c^2} y}{\sqrt{1 - \frac{v^2}{c^2}}}$$

For event B, $t_B = 0, y = 2$. So $ct_B' = -\frac{3}{2}$

Ans. 5: (d)

Solution: $v = -\frac{c}{2}, u_x' = \frac{2c}{3}$

$$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}} = \frac{c}{5}$$

$$l = l_0 \sqrt{1 - \frac{u_x^2}{c^2}} = 0.97\ l_0$$

Ans. 6: (b)

Solution: $$u_{A,B} = \frac{\frac{4}{5}c - \frac{3}{5}c}{1 - \frac{4}{5}c . \frac{3}{5}c . \frac{1}{c^2}} = \frac{\frac{c}{5}}{\frac{13}{25}} = \frac{5}{13}c$$

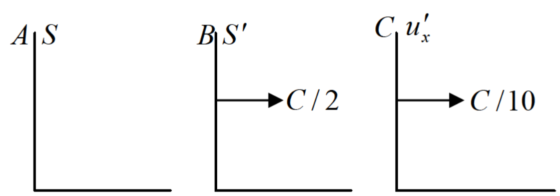
Kinematic equation is given by

$$\frac{5}{13} c \times t = L\sqrt{1 - \frac{25}{169}} + L \Rightarrow t = \frac{5L}{c} \Rightarrow \alpha = 5$$

Ans. 7: (b)

Solution: $v = \frac{c}{2}, \quad u_x' = \frac{c}{10}$

$$u_x = \frac{u_x' + v}{1 + \frac{u' v_x}{c^2}} = \frac{4c}{7} = 0.57c$$



Ans. 8: (d)

Solution: $$u' = \frac{c}{n} \text{ and } u = \frac{u' + v}{1 + \frac{u' v}{c^2}} \Rightarrow u = \frac{c}{n}\left(\frac{1 + \frac{nv}{c}}{1 + \frac{v}{nc}}\right) = \frac{c}{1.5}\left(\frac{1 + (1.5 \times 0.8)}{1 + \left(\frac{0.8}{1.5}\right)}\right) = \frac{c}{1.5}\left(\frac{2.2}{1.53}\right) = 0.95c$$

Ans. 9: (a)

Solution $u_x' = c\cos\theta_0 \ \ u_y' = c\sin\theta \quad v = u$

$u_y = \dfrac{c\sin\theta\sqrt{1-\dfrac{u^2}{c^2}}}{1+c\cos\theta u/c^2} = \qquad \sin\theta = \dfrac{u_y}{c} \Rightarrow \sin\theta = \dfrac{\sin\theta_0\sqrt{1-\dfrac{u^2}{c^2}}}{1+\dfrac{u}{c}\cos\theta_0}$

Ans. 10: (a)

Solution: Area of disc from S - frame is 1 i.e., $\pi a^2 = 1$ or $\pi a \cdot a = 1$

Area of disc from S' frame is $\pi a \cdot b = \pi a \cdot a\sqrt{1-\dfrac{u^2}{c^2}} = 1 \cdot \sqrt{1-\dfrac{u^2}{c^2}} = \sqrt{1-\dfrac{u^2}{c^2}}$

Where, $b = a\sqrt{1-\dfrac{u^2}{c^2}} = \sqrt{1-(0.8)^2} = 0.6$.