Solution (Class Test)

(STR-Mass Energy Equivalence & Four Vectors)

Ans. 1: (c)

Solution: $E=\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow 1-\frac{v^2}{c^2}=\left(\frac{mc^2}{E}\right)^2 \Rightarrow \frac{v^2}{c^2}=1-\frac{m^2c^4}{E^2} \Rightarrow v=c\sqrt{1-\left(\frac{mc^2}{E}\right)^2}=c\sqrt{1-\left(\frac{1}{2}\right)^2}=\frac{\sqrt{3}}{2}c$

Ans. 2: (c)

Solution: Since $E=210MeV$ and $m_0=105MeV/c^2$.

$$E=mc^2 \Rightarrow E=\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}210=\frac{105}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \sqrt{1-\frac{v^2}{c^2}}=\frac{105}{210}=\frac{1}{2}\text{ Now, } t=\frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{2}{0.5}=4\mu s$$

Ans. 3: (b)

Solution: From conservation of momentum, massless particle and particle of mass m have same momentum p and from conservation of energy, $Mc^2=\sqrt{p^2c^2+m^2c^4}+pc$

$$p=\frac{c}{2M}\left(M^2-m^2\right)\ p=\frac{c}{2M}\sqrt{M^2-\frac{M^2}{4}}=\frac{\sqrt{3}}{4}Mc$$

Ans. 4: (d)

Solution: Energy before collision is $E=2m_0c^2$ so momentum before and after collision is

$$p=\frac{\sqrt{E^2-m_0^2c^4}}{c}=\sqrt{3}m_0c^2$$

From conservation of energy $2m_0c^2+m_0c^2=\sqrt{p^2c^2+M^2c^4} \Rightarrow 3m_0c^2=\sqrt{3m_0^2c^4+M^2c^4}$

$$M=\sqrt{6}m_0$$

Ans. 5: (c)

Ans. 6: (a)

Solution: $\frac{dp}{dt}=e\varepsilon$

$p=e\varepsilon t+c$

At $t=0$, $p=0$, $c=0$

$$\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}=e\varepsilon t$$

$$t=\frac{m}{e\varepsilon}\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}$$ Put $v=\frac{c}{2}$, $t=\frac{m}{e\varepsilon}\frac{c/2}{\sqrt{1-\frac{1}{4}}}=\frac{mc}{\sqrt{3}e\varepsilon}\Rightarrow t=\frac{mc}{\sqrt{3}eE}$

Ans. 7: (c)

Solution: $v'=v_0$, $v=\frac{\sqrt{3}}{2}c,\theta=30^0\Rightarrow v=\frac{v'\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c}\cos\theta}=\frac{v_0\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c}\cos 30}=\frac{v_0\sqrt{1-\frac{3}{4}}}{1-\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}}=\frac{v_0\frac{1}{2}}{1-\frac{3}{4}}=2v_0$

Ans. 8: (b)

Solution: On the paper, the point $(x',ct')=(1,0)$, which we just found to be the point $(x,ct)=(\gamma,\gamma v/c)$, is a distance $\gamma\sqrt{1+v^2/c^2}$ from the origin. So, as in the ct' - axis case,

$$\frac{\text{one } x' \text{ unit}}{\text{one } x \text{ unit}}=\sqrt{\frac{1+\beta^2}{1-\beta^2}}$$

Ans. 9: (b)

Ans. 10: (b)