Chapter 4 (Relativistic Electrodynamics)

Worksheet

Q1. Infinite charge sheet with density σ is kept in x, y plane. If the sheet is moving with speed v with respect to observer in positive x direction. Which of the following is correct expression of electric field and magnetic field at distance a in $\hat{z}$ direction. $\left(\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$?

(a) $E_z = \frac{\gamma\sigma}{\varepsilon}, B_z = -\frac{\gamma v\sigma}{\varepsilon}$

(b) $E_z = \frac{\gamma\sigma}{\varepsilon}, B_y = -\frac{\gamma v\sigma}{\varepsilon}$

(c) $E_y = \frac{\gamma\sigma}{\varepsilon}, B_z = -\frac{\gamma v\sigma}{\varepsilon}$

(d) $E_z = \frac{\gamma\sigma}{\varepsilon}, B_y = \frac{\gamma v\sigma}{\varepsilon}$

Q2. A point charge q is at rest at the origin in system S_0. Another system S_A is moving to right at velocity v relative to S_0. The Y component of electric field measure from S_A at distance (x, y, z, ict) which is measured from the origin of S_A is equal to

(where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$)

(a) $\frac{qy}{4\pi\varepsilon_0\gamma\left(x^2+y^2+z^2\right)^{3/2}}$

(b) $\frac{\gamma qy}{4\pi\varepsilon_0\gamma\left(x^2+y^2+z^2\right)^{3/2}}$

(c) $\frac{qy}{4\pi\varepsilon_0\gamma\left(\gamma^2\left(x+vt\right)^2+y^2z^2\right)^{3/2}}$

(d) $\frac{\gamma qy}{4\pi\varepsilon_0\left(\gamma^2\left(x+vt\right)^2+y^2+z^2\right)^{3/2}}$

Q3. A rod has charge density λ_0 (seen by observer which is at rest with respect to the observer) and is moving with speed $\frac{c}{2}$ with respect to frame A. Frames A is moving with respect to frame B with speed $\frac{c}{2}$. The Frames A and B are moving in the same direction along the length of the rod. If observer is attached to frame B then the charge density measured by observer is

(a) λ_0

(b) $\frac{5\lambda_0}{3}$

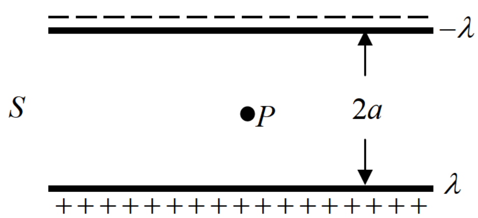
(d) $\frac{\sqrt{21}\lambda_0}{5}$

(c) $\frac{2\lambda_0}{\sqrt{3}}$

Q4. A rod have charge density λ_0 (seen by observer which is at rest with respect to the observer) and is moving with speed $\frac{c}{2}$ with respect to frame A. Frames A is moving with respect to frame B with speed $\frac{c}{4}$. The Frames A and B are moving in the same direction along the length of the rod. If observer is attached to frame B then the charge density measured by observer is

(a) λ_0 (b) $\frac{5\lambda_0}{3}$ (c) $\frac{3\lambda_0}{\sqrt{5}}$ (d) $\frac{2\lambda_0}{\sqrt{3}}$

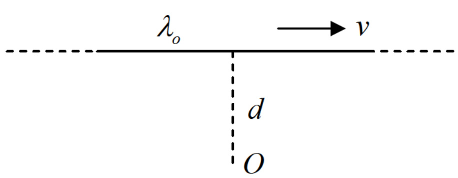
Q5. Consider to long parallel wires separated by a distance $2a$ and bearing equal and opposite uniform charge distributions (see fig). In frame S, at rest with respect to the wires, there is no current flowing and the linear charge density is λ.Frame S' moves at a velocity υ parallel to the length of the wires (the x direction then magnetic fields measured in frame S' at a point midway between the wires is given by



(a) $B_x^{'} = B_y^{'} = 0,\ B_z^{'} = \frac{\mu_0 \lambda v}{2\pi a\sqrt{1-\frac{v^2}{c^2}}}$ (b) $B_x^{'} = B_y^{'} = 0,\ B_z^{'} = -\frac{\mu_0 \lambda v}{2\pi a\sqrt{1-\frac{v^2}{c^2}}}$

(c) $B_x^{'} = B_y^{'} = 0,\ B_z^{'} = B_z^{'} = \frac{\mu_0 \lambda v}{\pi a\sqrt{1-\frac{v^2}{c^2}}}$ (d) $B_x^{'} = B_y^{'} = 0,\ B_z^{'} = -\frac{\mu_0 \lambda v}{\pi a\sqrt{1-\frac{v^2}{c^2}}}$

Q6. Infinite long wire of charge density λ_0 move with speed $v \left(\frac{v}{c} \neq 0\right)$ with respect to point "O" which is distance d from wire as shown in figure. Find the electric field due to wire at point "O".



(a) $\frac{\lambda_0}{2\pi\varepsilon_0 d}$ (b) $\frac{\lambda_0}{2\pi\varepsilon_0 d\sqrt{1-\frac{v^2}{c^2}}}$ (c) $\frac{\lambda_0\sqrt{1-\frac{v^2}{c^2}}}{2\pi\varepsilon_0 d}$ (d) $\frac{\lambda_0\left(1-\frac{v^2}{c^2}\right)}{2\pi\varepsilon_0 d}$

Q7. Which of the following is not invariant under Lorentz transformation?

(a) $\vec{E}.\vec{B}$ where $\vec{E}$ is electric field and $\vec{B}$ is magnetic field

(b) $\left|\vec{E}\right|^2 - \left|\vec{B}\right|^2$ where $\vec{E}$ is electric field and $\vec{B}$ is magnetic field in cgs unit

(c) $\left|\vec{J}\right|^2 - c^2\left|\rho\right|^2$ where $\vec{J}$ is current density and ρ is charge density

(d) $\left|\vec{A}\right|^2 + \left|\phi\right|^2$ where $\vec{A}$ is vector potential and ϕ is scalar potential

Q8. Which of the following questions is Lorentz invariant?

(a) $\left|\vec{E}\times\vec{B}\right|^2$ (b) $\left|\vec{E}\right|^2 - \left|\vec{B}\right|^2$ (c) $\left|\vec{E}\right|^2 + \left|\vec{B}\right|^2$ (d) $\left|\vec{E}\right|^2\left|\vec{B}\right|^2$

Q9. A rod of length L carries a total charge Q distributed uniformly. If this is observed in a frame moving with a speed v along the rod, the charge per unit length (as measured by the moving observer) is

(a) $\frac{Q}{L}\left(1-\frac{v^2}{c^2}\right)$ (b) $\frac{Q}{L}\sqrt{1-\frac{v^2}{c^2}}$ (c) $\frac{Q}{L\sqrt{1-\frac{v^2}{c^2}}}$ (d) $\frac{Q}{L\left(1-\frac{v^2}{c^2}\right)}$

Q10. The value of the electric and magnetic fields in a particular reference frame (in Gaussian units) are $E = 3\hat{x}+4\hat{y}$ and $B = 3\hat{z}$ respectively. An inertial observer moving with respect to this frame measures the magnitude of the electric field to be $\left|E'\right| = 4$. The magnitude of the magnetic field $\left|B'\right|$ measured by him is

(a) 5 (b) 9 (c) 0 (d) 1

Q11. In an inertial frame S, the magnetic vector potential in a region of space is given by $\vec{A} = az\,\hat{i}$ (where a is a constant) and the scalar potential is zero. The electric and magnetic fields seen by an inertial observer moving with a velocity $v\hat{i}$ with respect to S, are, respectively [In the following $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$]

(a) 0 and $\gamma a\hat{j}$ (b) $-va\hat{k}$ and $\gamma a\hat{i}$ (c) $v\gamma a\hat{k}$ and $v\gamma a\hat{j}$ (d) $v\gamma a\hat{k}$ and $\gamma a\hat{j}$

Q12. In the rest frame S_1 of a point particle with electric charge q_1 another point particle with electric charge q_2 moves with a speed v parallel to the x-axis at a perpendicular distance l. The magnitude of the electromagnetic force felt by q_1 due to q_2 when the distance between them is minimum, is [In the following $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$]

(a) $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\gamma l^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{\gamma q_1q_2}{l^2}$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{\gamma q_1q_2}{l^2}\left(1+\frac{v^2}{c^2}\right)$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\gamma l^2}\left(1+\frac{v^2}{c^2}\right)$

Q13. A circular current carrying loop of radius a carries a steady current. A constant electric charge is kept at the centre of the loop. The electric and magnetic fields, $\vec{E}$ and $\vec{B}$ respectively, at a distance d vertically above the centre of the loop satisfy

(a) $\vec{E} \perp \vec{B}$ (b) $\vec{E} = 0$ (c) $\vec{\nabla}\left(\vec{E}\cdot\vec{B}\right)=0$ (d) $\vec{\nabla}\cdot\left(\vec{E}\times\vec{B}\right)=0$

Q14. In an inertial frame uniform electric and magnetic field $\vec{E}$ and $\vec{B}$ are perpendicular to each other and satisfy $\left|\vec{E}\right|^2 - \left|\vec{B}\right|^2 = 29$ (in suitable units). In another inertial frame, which moves at a constant velocity with respect to the first frame, the magnetic field is $2\sqrt{5}\hat{k}$. In the second frame, an electric field consistent with the previous observations is

(a) $\frac{7}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$ (b) $7\left(\hat{i}+\hat{k}\right)$ (c) $\frac{7}{\sqrt{2}}\left(\hat{i}+\hat{k}\right)$ (d) $7\left(\hat{i}+\hat{j}\right)$