Chapter 3 (Wave Nature of Particle)
Solution of Worksheet

MCQ (Multiple Choice Questions)

Ans. 1: (c)

Solution: de-Broglie wavelength $\lambda = \frac{h}{p}$ $\therefore \frac{\lambda_1}{\lambda_2} = \frac{p_1}{P_2}$

Since momentum p is conserved in the decay process, $p_2 = p_1$ $\therefore \frac{\lambda_1}{\lambda_2} = 1$

Ans. 2: (c)

Solution: The de-Broglie wavelength $\lambda = \frac{h}{mv}$ (i)

Where relativistic mass $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ $m_0 \to$ rest mass

$$\Rightarrow \quad \frac{v^2}{c^2} = 1 - \frac{m_0^2}{m^2} \Rightarrow mv = c\sqrt{m^2 - m_0^2} \quad \text{(ii)}$$

Thus, equation (i) and (ii), we get $\Rightarrow$ $\lambda = \frac{hc}{c^2\sqrt{m^2 - m_0^2}}$ (iii)

Now, $c^2\sqrt{m^2 - m_0^2} = \sqrt{c^4(m - m_0)(m + m_0)} = \{(m - m_0)\}\{(m - m_0)c^2 + 2m_0c^2\}^{\frac{1}{2}}$

$= \sqrt{E(E + 2m_0c^2)}$ [Since $E = (m - m_0)c^2$]

So, by equation (iii) $\lambda = \frac{hc}{\sqrt{E(E + 2m_0c^2)}}$

Ans. 3: (d)

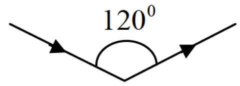
Solution: For photon, $E = \frac{hc}{\lambda_2}$ or $\lambda_2 = \frac{hc}{E}$ (i)

For proton kinetic energy $K = \frac{1}{2}m_p v_p^2$ or $\lambda_1 = \frac{h}{\sqrt{2m_p K}} = \frac{h}{\sqrt{2m_p E}}$ (ii)

From (i) and (ii), $\frac{\lambda_1}{\lambda_2} = \frac{hc}{E} \times \frac{\sqrt{2m_p E}}{h}$ or $\frac{\lambda_2}{\lambda_1} = \frac{c \times \sqrt{2m_p}}{\sqrt{E}} = c\sqrt{2m_p} \times E^{-1/2}$ or $\frac{\lambda_2}{\lambda_1} \propto E^{-1/2}$

Ans. 4: (a)

Solution: (a) The angle of the x-ray beams from the planes is $\theta = \left(90^0 - \dfrac{120^0}{2}\right) = 30^0$.



Strong diffraction peak occurs for $2d\sin\theta = \lambda$

Thus, $\lambda = 2d\sin\theta \Rightarrow \lambda = 2\times 2\times \sin 30^0 = 2\times 2\times \dfrac{1}{2} = 2A^0$

(b) $p = \dfrac{h}{\lambda} = \dfrac{6.6\times 10^{-34}(J-\sec)\times}{2\times 10^{-10}m} = 3.3\times 10^{-24}\, kgm/\sec$

Ans. 5: (a)

Solution: $E = \left(p^2c^2 + m_0^2c^2\right)^{1/2} \Rightarrow v_p = \dfrac{E}{p} = \dfrac{\sqrt{p^2c^2+m_0^2c^2}}{p} = \dfrac{\dfrac{m_0c^2}{\sqrt{1-\dfrac{v^2}{c^2}}}}{\dfrac{m_0 v}{\sqrt{1-\dfrac{v^2}{c^2}}}} = \dfrac{c^2}{v}$

Ans. 6: (b)

Solution: $E^2 = p^2c^2 + m_0^2c^4$

$$2E\frac{dE}{dp} = 2pc^2 \Rightarrow E.v_g = pc^2 \Rightarrow \frac{E}{p} = \frac{c^2}{v_g} = v_p \Rightarrow v_p = \frac{c^2}{0.8c} = \frac{10}{8}c = \frac{5}{4}c$$

Ans. 7: (b)

Solution: From the definition of de-Broglie

$$\lambda = \frac{h}{mV} \quad \text{and} \quad m = \frac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

Where v is velocity of particle and equal to group velocity v_g

$$\lambda = \frac{h}{\dfrac{m_0 v}{\sqrt{1-\dfrac{v^2}{c^2}}}};\quad V = V_g = \left(\frac{h^2c^2}{m_0^2c^2\lambda^2 + h^2}\right)^{1/2}$$

$$v_p \cdot v_g = {}^2 \Rightarrow v_p = \frac{c^2}{v_g} \quad \Rightarrow v_p = c\left[1 + \left(\frac{mc\lambda}{h}\right)^2\right]^{1/2}$$

Ans. 8: (a)

Solution: $v_g = -\lambda^2 \frac{d\nu}{d\lambda} = \frac{3}{2}\sqrt{\frac{2\pi T}{\rho\lambda}}$

Ans. 9: (d)

Solution: $\omega = ck$, $v_g = \frac{d\omega}{dk} = c$, $v_P = \frac{\omega}{k} = c$

Ans. 10: (d)

Solution: Group velocity is $v_g = d\omega / dk$. So, take the derivative of the quantity $\omega = \sqrt{c^2k^2 + m^2}$ to get

$$v_g = d\omega / dk = \frac{c^2 k}{\sqrt{c^2k^2 + m^2}}$$

Use the above equation to test the choices:

(a) As $k \to 0 \Rightarrow v_g = 0$, not infinity. The first condition doesn't work, no need to test the second (don't have to remember L'Hopital's rule).

(b) Wrong for the same reason as (a).

(c) As $k \to \infty, v_g \approx \frac{kc^2}{\sqrt{c^2k^2}} = c$, since $m << (ck)^2$. So, v_g doesn't tend towards ∞. This choice is wrong.

(d) This is it. The conditions work (and it's the only choice left).

Ans. 11: (b)

Solution: $E^2 = p^2c^2 + m_0^2c^4$

$$2E\frac{dE}{dp} = 2pc^2 \Rightarrow E.v_g = pc^2 \Rightarrow \frac{E}{p} = \frac{c^2}{v_g} = v_p \Rightarrow v_p = \frac{c^2}{\beta c} \Rightarrow \frac{c}{\beta}$$

Ans. 12: (c)

Solution: $P = \frac{E}{c} \Rightarrow \frac{E}{p} = c = v_p$ and $\frac{dE}{dp} = c = v_g$

MSQ (Multiple Select Questions)

Ans. 13: (a), (b) and (d)

Solution: $T = E - m_0c^2 \Rightarrow E = 3m_0c^2$

$$2m_0c^2 = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2 \Rightarrow 3 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \frac{1}{9} = 1 - \frac{v^2}{c^2} \Rightarrow \frac{v^2}{c^2} = 1 - \frac{1}{9} = \frac{8}{9} \Rightarrow v = \sqrt{\frac{8}{9}}c \Rightarrow \frac{2\sqrt{2}}{3}c$$

The de-Broglie wavelength $\lambda = \frac{h}{mv}$

$$E^2 = p^2c^2 + m_0^2c^4 \Rightarrow 9m_0^2c^4 = p^2c^2 + m_0^2c^4 \Rightarrow p^2c^2 = 8m_0^2c^4 \Rightarrow p = \sqrt{8}m_0c$$

$$E = pc = (2.000\,MeV/c)c = 2.000\,MeV$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{8}m_0c}$$

Ans. 14: (a), (b), (c), (d)

Solution: Here $m = 9.1\times10^{-31}kg$ and $L = 0.10\,nm = 1.0\times10^{-10}m$, so that the permitted electron energies are

$$E_n = \frac{(n^2)(6.63\times10^{-34}J.s)^2}{(8)(9.1\times10^{-31}kg)(1.0\times10^{-10}m)^2} = 6.0\times10^{-18}n^2J = 38n^2eV$$

The minimum energy the electron can have is $38eV$, corresponding to $n=1$. The sequence of energy levels continues $E_2 = 152eV, E_3 = 342eV, E_4 = 608eV$, and so If such a box existed, the quantization of trapped electrons energy would be a prominent feature of the system. (And indeed energy quantization is prominent in the case of an atomic electron).

Ans. 15: (a), (b)

Solution: Bragg's angle $\theta = \frac{\pi}{2} - \frac{\phi}{2}$

Ans. 16: (a), (c), (d)

Solution: (a) First calculate the wavelength of the electron. The kinetic energy K of an electron accelerated through a potential of $100V$ is $100\,eV$. So, using the equation above we have:

$$\lambda = \frac{hc}{\sqrt{2mc^2K}} = \frac{1240\,eV\cdot nm}{\sqrt{2(5.11\times10^5eV)(V\,eV)}} = 0.123\,nm \Rightarrow V = 100V$$

Now we use the Davisson-Germer formula for first-order $(n=1)$ diffraction, $2d\sin\theta = \lambda = 0.123\,nm$, where $d = 0.091\,nm$ is the distance between adjacent crystal planes in Ni. Now we can solve for θ, the angle between the original direction of the electron beam and scattered direction:

$$\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right) = \sin^{-1}\left(\frac{0.123}{2\times0.091}\right) = 0.74 \text{ radians} = 42.52^0$$

Now θ is not the scattering angle measured by Davisson and Germer. Rather, they measured angle ϕ.

$\theta = \frac{\pi}{2} - \frac{\phi}{2}$, so in this case they would have measured. $\phi = 180^0 - 85.04 \simeq 95^0$.

Ans. 17: (a), (b), (c), (d)

Solution: $\omega^2 = \alpha k^2 + \beta k^3 \Rightarrow \omega = k\sqrt{\alpha + \beta k}$

$\therefore$ Phase velocity $v_p = \frac{\omega}{k} = \sqrt{\alpha + \beta k}$

and Group velocity $v_g = \frac{d\omega}{dk} = \frac{d}{dk}\left[\sqrt{\alpha k^2 + \beta k^3}\right]$

$$\Rightarrow v_g = \frac{1}{2}\left(\alpha k^2 + \beta k^3\right)^{-1/2}\left(2\alpha k + 3\beta k^2\right) = \frac{2\alpha + 3\beta k}{2(\alpha + \beta k)^{1/2}} \Rightarrow v_p . v_g = \alpha + 1.5\beta k$$

let at $k = k_0$, $v_p = v_g$

$$\Rightarrow \sqrt{\alpha + \beta k_0} = \frac{2\alpha + 3\beta k_0}{2(\alpha + \beta k_0)^{1/2}} \Rightarrow 2\alpha + 2\beta k_0 = 2\alpha + 3\beta k_0 \Rightarrow \beta k_0 = 0 \therefore k_0 = 0$$

Ans. 18: (b), (d)

Ans. 19: (a), (d)

NAT (Numerical Answer Type)

Ans. 20: 4

Solution: Every particle of mass m moving with velocity v is associated with a wave of wavelength

given as $\lambda = \frac{h}{mv}$ $\quad h \rightarrow$ Planck's constant

the wavelength for proton $\lambda_p = \frac{h}{m_p v_p}$ (i)

and the wavelength for He $\lambda_\alpha = \frac{h}{m_\alpha v_\alpha}$ (ii)

Since, $\lambda_\alpha = \lambda_p \Rightarrow \frac{h}{m_p v_p} = \frac{h}{m_\alpha v_\alpha} \Rightarrow \frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p}$ (iii)

Since, ${}_2He^4$ has four nucleons so $m_\alpha = 4m_p$

Thus, (iii) $= \frac{v_p}{v_\alpha} = 4 \quad \Rightarrow v_p : v_\alpha = 4:1$

Ans. 21: 1.414

Solution: de Broglie wavelength $\lambda = \frac{h}{\sqrt{2mK}}$ where K denotes kinetic energy of particle

Case (I): $0 \le x \le 1$

Given: potential energy $= E_0$, Given: Total energy $= 2E_0$

$\therefore$ Kinetic energy $= 2E_0 - E_0 = E_0$ $\therefore \lambda_1 = \dfrac{h}{\sqrt{2mE_0}}$

Case (II): $x > 1$

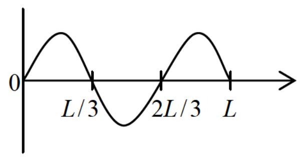
Given: potential energy $V(x) = 0$, given: Total energy $= 2E_0$

$\therefore$ Kinetic energy $= 2E_0$

$\therefore \lambda_2 = \dfrac{h}{\sqrt{2m(2E_0)}}$ $\quad \therefore \dfrac{\lambda_1}{\lambda_2} = \sqrt{\dfrac{2E_0}{E_0}} = \sqrt{2}$ $\quad \therefore \dfrac{\lambda_1}{\lambda_2} = \sqrt{2}$

Ans. 22: 1.5

Solution:



If wavelength of standing wave is λ and length of wall is L From the figure $\dfrac{3\lambda}{2} = L$ so $\lambda = \dfrac{2L}{3}$

If p is momentum and λ is wavelength then from De-Broglie hypothesis $p = \dfrac{h}{\lambda}$ where

So $p = \dfrac{3h}{2L}$.

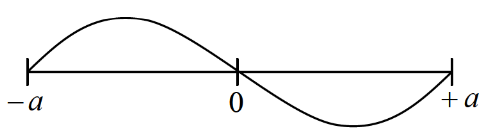
Ans. 23: 1.43

Solution: $E^2 = p^2c^2 + (m_0c^2)^2 \Rightarrow p = \sqrt{\dfrac{E^2 - (m_0c^2)^2}{c^2}} = \dfrac{\sqrt{1-0.25}}{c} = \dfrac{\sqrt{0.75}MeV}{c} =$

$$\lambda = \frac{h}{p} = \frac{6.6\times10^{-34}\times3\times10^{8}}{\sqrt{0.75}\times1.6\times10^{-13}} = \frac{19.8\times10^{-13}}{1.38} = 14.34\times10^{-13}m = 1.43\times10^{-12}m$$

Ans. 24: 0.5

Solution: For A particle is confined in a one dimensional box with impenetrable walls at $x = \pm a$.



The given state is representation of first exited state whose energy is $2eV$

If E_n is energy of nth state and E_0 is energy of ground state then $E_n = n^2E_0$

so $E_2 = 4E_0$ and $E_0 = 0.5eV$

Ans. 25: 1

Solution: De-Broglie wavelength, $\lambda = \frac{h}{p}$ $\quad \therefore \frac{\lambda_1}{\lambda_2} = \frac{p_2}{P_1}$ Since momentum p is conserved in the decay process, $p_2 = p_1$ $\quad \therefore \frac{\lambda_1}{\lambda_2} = 1$

Ans. 26: 3

Solution: The particle is in an infinite well of length $2a$. (It's stuck forever bouncing around between the two walls.)

The number of nodes in the wave function determines the energy level. In this case, there is one load, thus this is E_2. The lowest state would be E_1.

$E_n = kn^2 eV$ for particle in a box. Given that $E_2 = 2eV$ one determines $k = 1/2\,eV$. For one particle system. Thus, $E_1 = 0.5\,eV$ for three electron system $E = 2\times 0.5\,eV + 1\times 2\,eV = 3\,eV$.

Ans. 27: 0.72

Solution: $E^2 = p^2c^2 + \left(m_0c^2\right)^2 \Rightarrow p = \sqrt{\frac{E^2 - \left(m_0c^2\right)^2}{c^2}} = \frac{\sqrt{4-1}}{c} = \frac{\sqrt{3}MeV}{c}$

As, $\lambda = \frac{h}{p} = \frac{6.63\times 10^{-34}\times 3\times 10^8}{\sqrt{3}\times 1.6\times 10^{-13}} = \frac{19.8\times 10^{-13}}{2.72} = 7.17\times 10^{-13}m = 0.72\times 10^{-12}m$

Ans. 28: 152

Solution: Here $m = 9.1\times 10^{-31}kg$ and $L = 0.10\,nm = 1.0\times 10^{-10}m$, so that the permitted electron energies are

$$E_n = \frac{\left(n^2\right)\left(6.63\times 10^{-34}J.s\right)^2}{(8)\left(9.1\times 10^{-31}kg\right)\left(1.0\times 10^{-10}m\right)^2} = 6.0\times 10^{-18}n^2J = 38n^2eV$$

The minimum energy the electron can have is $38eV$, corresponding to $n=1$ for fundamental wave and $n=2$ the energy is $E_2 = 152eV$ for first overtone.

Ans. 29: 720

Solution: $\lambda = \frac{h}{p} = \frac{h}{mv}$ Or, $v = \frac{h}{m\lambda} = \left(\frac{h}{mc}\right)\frac{c}{\lambda}$ For an electron, $\left(\frac{h}{mc}\right) = 2.4$ pm.

So, $v = \frac{(2.4\,\text{pm})}{(1\mu m)}\times c = 2.4\times 10^{-6}c = \left(2.4\times 10^6\right)\times\left(3\times 10^8 m/s\right) = 720m/s$.

Ans. 30: 12.4

Solution: For a highly relativistic electron, the kinetic energy and the linear momentum are related by $E = pc$, a relation strictly true for a photon. Thus the de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{hc}{E} = \frac{1240eVnm}{E} = \frac{12.4keV\overset{0}{\mathrm{A}}}{E}$$

If E is in λ is angstrom, $\lambda = \frac{12.4}{E}$. Thus $\alpha = 12.4$

Ans. 31: 12.25

Solution: As an electron is accelerated through a potential difference V, its potential energy is decreased by eV. The kinetic energy gained is equal to this value. So

$$eV = mv^2/2 = p^2/(2m) \text{ or, } p = \sqrt{2meV}$$

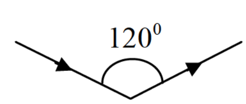
de Broglie wavelength is $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} = \sqrt{\frac{h^2}{2m}\frac{1}{eV}}$

For an electron, $\frac{h^2}{2m} = 1.5eVnm^2$. So, $\lambda = \sqrt{\frac{1.5eVnm^2}{eV}}$

If V is put in volts, the energy eV is the same as V electron volts. Canceling the unit eV from the numerator and the denominator, $\lambda = \frac{\sqrt{1.5}}{\sqrt{V}}nm \approx \frac{12.25}{\sqrt{V}}\overset{0}{\mathrm{A}}$ So, $\alpha = 12.25$

Ans. 32: 2

Solution: (a) The angle of the x-ray beams from the planes is $\theta = \left(90^0 - \frac{120^0}{2}\right) = 30^0$.



Strong diffraction peak occurs for $2d\sin\theta = \lambda$

Thus, $\lambda = 2d\sin\theta \Rightarrow \lambda = 2\times 2\times \sin 30^0 = 2\times 2\times\frac{1}{2} = 2A^0$

(b) $E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}(J-\sec)\times 3\times10^8 m/\sec}{1.5\times10^{-10}m} = 13.2\times10^{-16} Joule$

Ans. 33: 0.5

Solution: $v_g = v$

Ans. 34: 2

Solution: $v_p = \frac{c^2}{v} = \frac{c^2}{0.5c} = 2c$

Ans. 35: 1

Solution: Group velocity, $V_g = \frac{d\omega}{dk}$ and phase velocity is $V_p = \frac{\omega}{k}$

$\omega^2 = \alpha k + \beta k^3$ ....(A)

Differentiating both sides we get $2\omega . \frac{d\omega}{dk} = \alpha + 3\beta k^2$

Now dividing both sides by k we will get

$2\frac{\omega}{k} . \frac{d\omega}{dk} = \frac{\alpha}{k} + 3\beta k \Rightarrow 2V_p . V_g = \frac{\alpha}{k} + 3\beta k$

For $k = k_0$ and $V_p = V_g$

$$2V_p^2 = \frac{\alpha}{k_0} + 3\beta k_0 \Rightarrow V_p = \left(\frac{\alpha}{2k_0} + \frac{3\beta k_0}{2}\right)^{1/2}$$

From equation (A) $V_p = \frac{\omega}{k} = \left(\frac{\alpha}{k_0} + \beta k_0\right)^{1/2}$

Thus, $\left(\frac{\alpha}{2k_0} + \frac{3\beta k_0}{2}\right)^{1/2} = \left(\frac{\alpha}{k_0} + \beta k_0\right)^{1/2} \Rightarrow \frac{\alpha}{2k_0} - \frac{\beta k_0}{2} = 0 \Rightarrow k_0 = \sqrt{\frac{\alpha}{\beta}}$