

Chapter 2 (Mass Energy Equivalence) Solution of Worksheet

Ans. 1: (c)

$$\text{Solution: } E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow 1 - \frac{v^2}{c^2} = \left(\frac{mc^2}{E}\right)^2 \Rightarrow \frac{v^2}{c^2} = 1 - \frac{m^2 c^4}{E^2} \Rightarrow v = c \sqrt{1 - \left(\frac{mc^2}{E}\right)^2}$$

Ans. 2: (a)

$$\text{Solution: } K.E. = mc^2 - m_0 c^2, \text{ rest mass energy} = m_0 c^2$$

$$K.E. = \text{rest mass energy} \Rightarrow mc^2 - m_0 c^2 = m_0 c^2 \Rightarrow mc^2 = 2m_0 c^2$$

$$\frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} c^2 = 2m_0 c^2 \Rightarrow \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 2 \Rightarrow 4 \left(1 - \frac{v^2}{c^2}\right) = 1 \Rightarrow 4 \frac{v^2}{c^2} = 3 \Rightarrow v = \frac{\sqrt{3}}{2} c$$

Ans. 3: (b)

$$\text{Solution: } E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{5}{4} m_0 c^2$$

Ans. 4: (a)

$$\text{Solution: Kinetic energy } T = mc^2 \text{ and } T = E - mc^2, E = 2mc^2$$

$$E^2 = p^2 c^2 + m^2 c^4$$

$$p = \sqrt{3} mc$$

Ans. 5: (b)

$$\text{Solution: } T = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 c^2 = \frac{2}{3} m_0 c^2$$

Ans. 6: (a)

$$\text{Solution: } E = \sqrt{p^2 c^2 + m_0^2 c^2} \quad \frac{dE}{dP} = \frac{pc}{\sqrt{m_0^2 c^2 + p^2}} = \frac{pc^2}{c \sqrt{m_0^2 c^2 + p^2}} = \frac{pc^2}{E}$$

Ans. 7: (d)

$$\text{Solution: } E = T + m_0 c^2 = 2m_0 c^2$$

$$E^2 = p^2 c^2 + m_0^2 c^4$$

$$p = \sqrt{3} m_0 c$$

Ans. 8: (a)

$$\text{Solution: } E = \sqrt{p^2 c^2 + m_0^2 c^4} \Rightarrow \frac{dE}{dP} = \frac{pc^2}{\sqrt{m_0^2 c^4 + p^2 c^2}} = \frac{pc^2}{E} = \frac{pc^2}{T + m_0 c^2}$$

Ans. 9: (c)

$$\begin{aligned} \text{Solution: } P &= \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow F = \frac{dP}{dt} = m \frac{dv}{dt} \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} + mv \left(-\frac{1}{2} \right) \cdot \frac{1}{\left(1 - \frac{v^2}{c^2} \right)^{3/2}} \cdot \frac{-2v}{c^2} \frac{dv}{dt} \\ &\Rightarrow F = m \frac{dv}{dt} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{\cancel{v^2/c^2}}{\left(1 - \frac{v^2}{c^2} \right)} \right) = \frac{mc^3}{\left(c^2 - v^2 \right)^{3/2}} \frac{dv}{dt} \end{aligned}$$

Ans. 10: (d)

Solution: From conservation of energy

$$K + m_0 c^2 = \sqrt{p^2 c^2 + m_0^2 c^4}, \text{ so momentum } p = \frac{\sqrt{K(K + 2m_0 c^2)}}{c}$$

If particle flux (number of particles per unit area per unit time) is J , then pressure $P = \frac{F}{A}$

$$P = \frac{J \sqrt{K(K + 2m_0 c^2)}}{c}$$

Ans. 11: (c)

Solution: Since $E = 210 \text{ MeV}$ and $m_0 = 105 \text{ MeV}/c^2$.

$$E = mc^2 \Rightarrow E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} 210 = \frac{105}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \sqrt{1 - \frac{v^2}{c^2}} = \frac{105}{210} = \frac{1}{2}$$

$$\text{Now, } t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2}{0.5} = 4 \mu s$$

Ans. 12: (c)

Solution: $E = T + m_0 c^2 = 2m_0 c^2 + m_0 c^2 = 3m_0 c^2$

$$E^2 = p^2 c^2 + m_0^2 c^4 \Rightarrow 9m_0^2 c^4 = p^2 c^2 + m_0^2 c^4 = p^2 c^2 = 8m_0 c^4$$

$$p = 2\sqrt{2}m_0 c$$

Ans. 13: (b)

$$\text{Solution: } K \rightarrow \mu + \nu, \quad E_\nu = \frac{(m_k^2 - m_\mu^2)c^2}{2m_k} = \frac{\left(\frac{494}{c^2} \times \frac{494}{c^2} - \frac{106}{c^2} \times \frac{106}{c^2}\right)c^2}{2 \times \frac{494}{c^2}}$$

$$\Rightarrow E_\nu = 244036 - \frac{11236}{988} = 235.6275 \approx 236 \text{ MeV}$$

Ans. 14: (b)

$$\text{Solution: } E_\nu = \frac{(m_\pi^2 - m_\mu^2)c^2}{2m_\pi} = p \times c \Rightarrow p = \frac{(m_\pi^2 - m_\mu^2)c}{2m_\pi} = \frac{19321 - 11025}{2 \times 139} \approx 30 \text{ MeV/c}$$

Ans. 15: (c)

Solution: From conservation of energy

$$\sqrt{\frac{mc^2}{1 - \frac{v^2}{c^2}}} + \sqrt{\frac{mc^2}{1 - \frac{v^2}{c^2}}} = m_1 c^2 \Rightarrow \sqrt{\frac{2mc^2}{1 - \frac{v^2}{c^2}}} = m_1 c^2 \quad [\text{Since } v = 0.6c \Rightarrow m_1 = 5m/2]$$

Ans. 16: (c)

Solution: From conservation of momentum, massless particle and particle of mass m have same momentum p and from conservation of energy, $Mc^2 = \sqrt{p^2 c^2 + m^2 c^4} + pc$

$$p = \frac{c}{2M} (M^2 - m^2)$$

Ans. 17: (c)

Solution: From conservation of momentum

$$\sqrt{\frac{m_0 v}{1 - \frac{v^2}{c^2}}} + 0 = p$$

From conservation of energy

$$\begin{aligned} \sqrt{\frac{m_0 c^2}{1 - \frac{v^2}{c^2}}} + m_0 c^2 &= \sqrt{p^2 c^2 + M^2 c^4} \Rightarrow \left(\sqrt{\frac{m_0 c^2}{1 - \frac{v^2}{c^2}}} + m_0 c^2 \right)^2 = p^2 c^2 + M^2 c^4 = \frac{m_0^2 v^2 c^2}{1 - \frac{v^2}{c^2}} + M^2 c^4 \\ &\Rightarrow \frac{m_0^2}{1 - \frac{v^2}{c^2}} + m_0^2 + 2 \sqrt{\frac{m_0^2}{1 - \frac{v^2}{c^2}}} = \frac{m_0^2 v^2}{c^2 - v^2} + M^2 \Rightarrow \frac{m_0^2 c^2}{c^2 - v^2} + m_0^2 + 2 \sqrt{\frac{m_0^2}{1 - \frac{v^2}{c^2}}} = \frac{m_0^2 v^2}{c^2 - v^2} + M^2 \end{aligned}$$

$$\Rightarrow \frac{m_0^2 c^2}{c^2 - v^2} - \frac{m_0^2 v^2}{c^2 - v^2} + m_0^2 + 2 \frac{m_0^2}{\sqrt{1 - \frac{v^2}{c^2}}} = M^2 \Rightarrow \frac{m_0^2 (c^2 - v^2)}{c^2 - v^2} + m_0^2 + 2 \frac{m_0^2}{\sqrt{1 - \frac{v^2}{c^2}}} = M^2$$

$$2m_0^2 + \frac{2m_0^2}{\sqrt{1 - \frac{v^2}{c^2}}} = M^2 \Rightarrow M = m_0 \sqrt{2 \left(1 + \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right)} = m_0 \sqrt{2(1 + \gamma)}$$

Ans. 18: (c)

Solution: $M \rightarrow \frac{2}{5}M + \frac{2}{5}M$

From momentum conservation

$$0 = \vec{P}_1 + \vec{P}_2 \Rightarrow \vec{P}_1 = -\vec{P}_2 \Rightarrow |P_1| = |P_2|$$

From energy conservation

$$E = E_1 + E_2$$

$$\Rightarrow Mc^2 = \frac{2}{5} \frac{Mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{2}{5} \frac{Mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow Mc^2 = \frac{4}{5} \frac{Mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\left(1 - \frac{v^2}{c^2} \right) = \frac{16}{25} \Rightarrow \frac{v^2}{c^2} = \frac{9}{25} \Rightarrow v = \frac{3}{5}c = 0.6c$$

Ans. 19: (a)

Solution: Using the conservation of momentum and energy respectively

$$E^2 = p^2 c^2 + m_0 c^2 \Rightarrow p = \frac{3}{4} m_0 c, \text{ conservation of momentum } p_1 - p_2 = \frac{3}{4} m_0 c$$

$$\text{from conservation of energy } p_1 c + p_2 c = \frac{5}{4} m_0 c^2 \Rightarrow E_1 = m_0 c^2, E_2 = \frac{m_0 c^2}{4}$$

Ans. 20: (d)

Solution: Energy before collision is $E = 2m_0 c^2$ so momentum before and after collision is

$$p = \frac{\sqrt{E^2 - m_0^2 c^4}}{c} = \sqrt{3} m_0 c^2$$

$$\text{From conservation of energy } 2m_0 c^2 + m_0 c^2 = \sqrt{p^2 c^2 + M^2 c^4} \Rightarrow 3m_0 c^2 = \sqrt{3m_0^2 c^4 + M^2 c^4}$$

$$M = \sqrt{6} m_0$$

Ans. 21: (a)

Solution: Conversations of energy gives

$$m_k c^2 = E_\mu + E_\nu = \sqrt{p_\mu^2 c^2 + m_\mu^2 c^4} + p_\nu c$$

as $P_\mu = -P_\nu$, or $p_\mu = p_\nu$ for momentum conservation. Hence

$$p_\mu = \left(\frac{m_k^2 - m_\mu^2}{2m_k} \right) c .$$

Thus

$$E_\nu = p_\nu c = p_\mu c = \left(\frac{m_k^2 - m_\mu^2}{2m_k} \right) c^2 = 235.6 \text{ MeV} ,$$

$$E_\mu = \sqrt{p_\mu^2 c^2 + m_\mu^2 c^4} = \left(\frac{m_k^2 + m_\mu^2}{2m_k} \right) c^2 = 258.4 \text{ MeV} .$$

Therefore the kinetic energy of the neutrino is 235.6 MeV , and that of the muon is $258.4 - 106 = 152.4 \text{ MeV}$.

Ans. 22: (a)

Solution: The electron's total energy is

$$\begin{aligned} E &= \sqrt{m^2 c^4 + p^2 c^2} = \sqrt{\left(0.511 \text{ MeV}/c^2\right)^2 c^4 + \left(2.000 \text{ MeV}/c\right)^2 c^2} \\ &= \sqrt{\left(0.511 \text{ MeV}\right)^2 + \left(2.000 \text{ MeV}\right)^2} = 2.064 \text{ MeV} \end{aligned}$$

The photon's energy is $pc = 2.000 \text{ MeV}$