Chapter 1 (Black Body Radiation)
Solution of Worksheet

MCQ (Multiple Choice Questions)

Ans. 1: (b)

Ans. 2: (a)

Solution: $\dfrac{P_1}{P_2}=\dfrac{T_1^4}{T_2^4}=\dfrac{(10+273)^4}{(20+273)^4}=\left(\dfrac{283}{293}\right)^4=0.87=\dfrac{1}{1.15}\Rightarrow 1:1.15$

Ans. 3: (c)

Ans. 4: (b)

Solution: Electric power = power radiated $W=\sigma T^4 A$

$$A=2\pi rl=2\pi\times10^{-3}\times1.0=6.283\times10^{-3}m^2$$

$$T=\left[\frac{W}{\sigma A}\right]^{1/4}=\left[\frac{1,000}{5.67\times10^{-8}\times6.283\times10^{-3}}\right]=1,294\,K$$

Ans. 5: (b)

Solution: Power, $P=\sigma AT^4=4\pi R^2\sigma T^4$

$$=4\pi\left(7\times10^8\right)^2\left(5.67\times10^{-8}\right)\left(5,700\right)^4=3.68\times10^{26}W$$

$$r=\frac{dm}{dt}=\frac{P}{c^2}=\frac{3.68\times10^{26}}{\left(3\times10^8\right)^2}=4.1\times10^9\,kg/\sec$$

Ans. 6: (d)

Solution 6: Power radiated, $P=\sigma AT^4=4\pi R^2\sigma T^4$

$$\frac{P_2}{P_1}=\frac{R_2^2}{R_1^2}\cdot\frac{T_2^4}{T_1^4}=\frac{(4R_1)^2}{R_1^2}\cdot\frac{(2T_1)^4}{T_1^4}=256$$

Ans. 7: (b)

Solution 11: Power radiated by the sun, $P_s=\sigma4\pi R_s^2T_s^4$

Power received by $1m^2$ of earth's surface,

$$S=\frac{\sigma4\pi R_s^2T_s^4}{4\pi r^2}=\frac{\left(5.7\times10^{-8}\right)\left(7\times10^8\right)^2\left(5,800\right)^4}{\left(1.5\times10^{11}\right)^2}=1,400\,W/m^2$$

Ans. 8: (d)

Solution: $\frac{1}{\sqrt{\varepsilon_0\mu_0}}=c \Rightarrow \frac{1}{\varepsilon_0\mu_0}=c^2$

Ans. 9: (b)

Solution: $\langle E\rangle=\langle T\rangle+\langle V\rangle=\left\langle\frac{p^2}{2m}\right\rangle+\left\langle\frac{1}{2}m\omega^2x^2\right\rangle=\frac{kT}{2}+\frac{kT}{2}=kT$

Ans. 10: (b)

Solution: $\rho(\nu)d\nu=\frac{8\pi\nu^2}{c^3}d\nu$

Ans. 11: (d)

Solution: $\rho(\lambda)d\lambda=\frac{8\pi V}{\lambda^4}d\lambda$

Ans. 12: (d)

Solution: $\frac{8\pi kT}{c^3}\nu^2 d\nu$

Ans. 13: (c)

Solution: $u(\nu)d\nu=\frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{\exp\left(\frac{h\nu}{kT}\right)-1}$

Ans. 14: (b)

Solution: The Wien's displacement law is $\lambda_{\max}T=b$. The knowledge of $\lambda_{\max}$ or T of the given sources will help in getting the answer. Typical temperature of tungsten filament in lamp is $3000\,K$. The surface temperature of the sun is $5800\,K$. The temperature of welding arc varies from $6000\,K$ to $30000\,K$.

Ans. 15: (c)

Solution: The temperature of a black body and the wavelength corresponding to its maximum intensity are related by the Wien's displacement law,

$$\lambda_m T=b.$$

The power radiated by a black body at temperature T having surface area A and emissivity e is given by the Stefan's law,

$$Q=\sigma eAT^4.$$

For a thin disc of radius r emitting from both surfaces, $A=2\pi r^2$. Eliminate T from above equations and substitute for A to get,

$$Q = 2\pi\sigma e b^4 \left(r^2 / \lambda_m^4 \right),$$

which gives,

$$Q_A : Q_B : Q_C = \frac{r_A^2}{\lambda_A^4} : \frac{r_B^2}{\lambda_B^4} : \frac{r_C^2}{\lambda_C^4}$$

$$= 0.049 : 0.0625 : 0.057$$

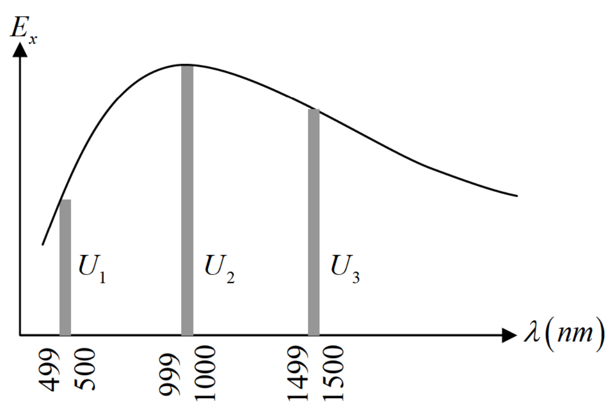
Ans. 16: (b)

Solution: The Wien's displacement law relates wavelength at which intensity attains maximum to the temperature of the body, $\lambda_m T = b$. In given cases, $\lambda_{m,1} < \lambda_{m,3} < \lambda_{m,2}$ and hence $T_1 > T_3 > T_2$.

Ans. 17: (d)

Solution: The Wien's displacement law gives,

$$\lambda_m = \frac{b}{T} = \frac{2.88 \times 10^6}{2880} = 1000\,nm$$



Thus radiated energy E_λ attains its maximum at $\lambda_m = 1000\,nm$. The variation of E_λ with λ is shown in figure. The area under the curve for given wavelength range gives radiated energy in that wavelength range i.e., $U_\lambda = E_\lambda \Delta_\lambda$. The figure clearly shows that $U_1 < U_3 < U_2$.

MSQ (Multiple Select Questions)

Ans. 18: (c), (d)

Ans. 19: (a), (c)

Solution: $\lambda T = k \Rightarrow \frac{T}{\nu} = k \Rightarrow T = k\nu$ if absolute temperature is doubled then frequency will also double.

$P \propto T^4$, if temperature will be doubled, then power will get $2^4 = 16$ times

Ans. 20: (a), (b)

Solution: Surface area, temperature and material of both sphere are same in both the case so both will emit and absorbed same amount in the beginning.

Ans. 21: (a), (b), (d)

Solution: (a) is correct $\lambda_m \cdot T = b \quad T = \dfrac{b}{\lambda_m} = \dfrac{2.897 \times 10^{-3}}{1 \times 10^{-6}} = 2,896\,K$

(b) is correct $\dfrac{P_2}{P_1} = \dfrac{T_2^4}{T_1^4} = 2 \; T_2 = T_1 \times 2^{1/4} = 2,897 \times 1.189 = 3,445\,K$

(d) is correct The wavelength at which the radiation has maximum intensity

$$\lambda_m = \frac{2.897 \times 10^{-3}}{3445} = 0.84 \times 10^{-6}\,m = 0.84\,\mu m$$

Ans. 22: (a), (c), (d)

Solution: (a) $\rho(\lambda, T) = \dfrac{8\pi hc}{\lambda^5} \dfrac{1}{\exp(hc/\lambda k_B T) - 1}$

(b) for $hc/\lambda k_B T \ll 1 \quad = \dfrac{8\pi hc}{\lambda^5} \dfrac{1}{\left[1 + \dfrac{hc}{\lambda k_B T} + \left(\dfrac{hc}{\lambda k_B T}\right)^2 + \cdots\right] - 1}$

In the linear approximation of the Taylor series expansion,

$$\rho_\lambda(\lambda, T) = \frac{8\pi hc}{\lambda^5} \frac{\lambda k_B T}{hc} = \frac{8\pi}{\lambda^4} k_B T$$

(c) For $hc/\lambda k_B T \gg 1$ we have $\exp(hc/\lambda k_B T) \gg 1$ and

$$\rho_\lambda(\lambda, T) \approx \frac{8\pi hc}{\lambda^5} \frac{1}{\exp(hc/\lambda k_B T)} = \frac{8\pi hc}{\lambda^5} \exp(-hc/\lambda k_B T)$$

In this case $hc/\lambda \gg k_B T$ and the discreteness of the cavity energy is obvious; the classical theory gives completely wrong predictions.

Ans. 23: (b), (c)

Solution: Rayleigh-Jeans theory for intensity of radiation is correct for smaller frequency or greater wavelength.

Wien's Distribution Law intensity of radiation is is correct for greater frequency or small wavelength.

Ans. 24: (a), (b), (c), (d)

Ans. 25: (a) and (b)

Solution: When a photon of frequency ν incident on a target metal of threshold frequency ν_0 and if $\nu > \nu_0$ the electron from metal is ejected. These electron are known as photoelectrons and this effect is called photo electric effect.

The kinetic energy of the ejected electron $= h\nu - h\nu_0$

$\Rightarrow KE$ depends on incident frequency ν and ν_0 i.e., nature of photo emitter.

NAT (Numerical Answer Type)

Ans. 26: 955

Solution: Let the temperature of the coil be T. The coil will emit radiation at a rate $A\sigma T^4$. Thus

$$1000W \approx \left(0.020m^2\right)\times\left(6.0\times10^{-8}Wm^{-2}K^{-4}\right)\times T^4$$

or, $T^4 \approx \dfrac{1000}{0.020\times6.00\times10^{-8}}K^4 \approx 8.33\times10^{11}K^4$ or, $T = 955K$.

Ans. 27: 22

Solution: $r = 1cm = 1\times10^{-2}m$, $A = 4\pi\left(10^{-2}\right)^2 = 4\pi\times10^{-4}m^2$

$\varepsilon = 0.3, \sigma = 6\times10^{-8}$

$$\frac{E}{t} = A\times\varepsilon\times\sigma\times\left(T_1^4 - T_2^4\right) = 4\pi\times10^{-4}\times0.3\times6\times10^{-8}\times\left[(1000)^4-(300)^4\right]$$

$$= 0.3\times6\times4\pi\times10^{-12}\times9919\times10^8 = 0.3\times6\times4\times3.14\times9919\times10^{-4}$$

$$= 1.8\times4\times3.14\times9919\times10^{-4} = 22.4 = 22W$$

Ans. 28: 2

Solution 4: The energy density u and pressure p of radiation are related by $p = \dfrac{u}{3}$

Furthermore, $u = 4\sigma T^4/c$

Eliminating u,

$$T = \left(\frac{3cp}{4\sigma}\right)^{1/4} = \left(\frac{3\times3\times10^8\times4\times1.013\times10^5}{4\times5.67\times10^{-8}}\right)^{1/4} = 2\times10^7K$$

Ans. 29: 1.55

Solution: Power, $P = \sigma AT^4 = 4\pi R^2\sigma T^4$

$$= 4\pi\left(7\times10^8\right)^2\left(5.67\times10^{-8}\right)(5,700)^4 = 3.68\times10^{26}W$$

$$r=\frac{dm}{dt}=\frac{P}{c^2}=\frac{3.68\times10^{26}}{\left(3\times10^8\right)^2}=4.1\times10^9\,kg/\sec$$

$$t=\frac{M}{100}\times\frac{1}{r}=\frac{2\times10^{30}}{100}\times\frac{1}{4.1\times10^9}=4.88\times10^{18}\,s=\frac{4.88\times10^{18}}{3.15\times10^7}=1.55\times10^{11}\text{ years}$$

Ans. 30: 4

Solution: Power radiated, $P=\sigma AT^4=4\pi R^2\sigma T^4$

$$\frac{P_2}{P_1}=\frac{R_2^2}{R_1^2}\cdot\frac{T_2^4}{T_1^4}=\frac{\left(4R_1\right)^2}{R_1^2}\cdot\frac{\left(2T_1\right)^4}{T_1^4}=256$$

Furthermore, $\dfrac{P_2}{P_1}=\dfrac{dQ_2/dt}{dQ_1/dt}=\dfrac{m_2 s\left(dT/dt\right)_2}{m_1 s\left(dT/dt\right)_1}$

where s is the specific heat

But $m_2\propto R_2^3$ and $m_1\propto R_1^3\ \therefore\ \dfrac{\left(dT/dt\right)_2}{\left(dT/dt\right)_1}=\dfrac{P_2}{P_1}\cdot\dfrac{R_1^3}{R_2^3}=\dfrac{256}{4^3}=4$

Ans. 31: 60

Solution: $H=eA\sigma T^4=0.4\times5.67\times10^{-8}\times0.3\times10^{-4}\times\left(3000\right)^4=60\,watt$

Ans. 32: 2.9

Ans. 33: 9.4

Solution: $T=37+273=310$

Applying Wien's relation $\lambda_{\max}T=2.9\times10^{-3}\,mK\Rightarrow\lambda_{\max}=\dfrac{2.9\times10^{-3}}{310}=0.0093\times10^{-3}=9.4\times10^{-6}\,m$.

Ans. 34: 0.69

Solution: The Wien's displacement law relates wavelength at which intensity attains maximum to the temperature of the body, $\lambda_m T=b$. Thus,

$$\frac{T_{sum}}{T_{star}}=\frac{\lambda_{m,star}}{\lambda_{m,sun}}=\frac{350}{510}=0.69$$