Worksheet Solution
(Equation of Motion & Variable Mass)

MCQ (Multiple Choice Questions)

Ans. 1: (a)

Solution: $F_t - mg = ma \Rightarrow F_t = m(g+a) \Rightarrow F_t = v_r\left(-\frac{dm}{dt}\right) = m(g+a)$

$$\Rightarrow v_r = \frac{m(g+a)}{(-dm/dt)} = \frac{(450+50)(9.0+20)}{7.45} m/s \Rightarrow v_r = 2\,km/s$$

Ans. 2: (b)

Solution: $v = u - gt + v_r \ln\frac{m_0}{m} = 0 - 10\times 40 + 2000\ln\frac{(160+40)}{40}$

$$\Rightarrow v_r = 2818\,m/s = 2.182\,km/s$$

Ans. 3: (a)

Solution: $m\frac{dv}{dt} = mg - kv$ (k is constant)

$$\Rightarrow \int\frac{dv}{g-kv} = \int dt \Rightarrow v = \frac{mg}{k} - \frac{m}{k}e^{-(kt/m)}$$

$t \to \infty, e^{-kt/m} \to 0 \Rightarrow v \approx \frac{mg}{k}$ (which is constant)

Ans. 4: (a)

Solution: As $v = \sqrt{g\times 2\times\frac{l}{4}} = \sqrt{\frac{gl}{2}}$

Weight of chain of table $= \frac{3}{4}mg$

Thrust on the chain vertically hanging on this moment $F_t = \lambda v^2 = \frac{m}{l2}\times g = \frac{mg}{2}$

Total force (thrust) on chain $= \frac{mg}{2} + \frac{3}{4}mg = \frac{5mg}{4}$ (vertically upward)

Ans. 5: (a)

Solution: Initial momentum of truck $P_i = (m_0 - \mu t)v_0$ at any time t.

$(\mu = 2\,kg/\sec, v_0 = 10\,m/\sec)$

Momentum after time dt is $P_f = (m_0 - \mu(t+dt))v_0 + \mu dt v_0$

as $P_i = P_f \Rightarrow F_t = 0$

Ans. 6: (b)

Solution: As $P_i = (m_0 + \mu dt)v_0$

$$P_f = (m_0 - \mu dt)v_0 + \mu dt v_0 + \lambda dt v_0$$

$$P_f - P_i = m_0 v_0 - \mu dt v_0 + \mu dt v_0 + \lambda dt v_0 - m_0 v_0 - \mu dt v_0 = (\lambda - \mu) dt v_0$$

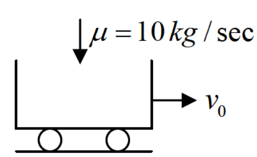
$$F_t = \left|\frac{dp}{dt}\right| = \left|(\lambda - \mu)v_0\right| = \left|(2-5)\times 10\right| = 30\,N$$

Ans. 7: (c)

Solution: $v_0 = 36\,km/h = 10\,m/\sec$

$$P_i = m_0 v_0 + \mu + v_0$$

$$P_f = m_0 v_0 + \mu(t + dt)v_0$$



$dp = \mu dt v_0$ and $F_t = \dfrac{dp}{dt} = \mu v_0 = ma$

$\Rightarrow a = \dfrac{\mu v_0}{m} = \dfrac{10\times 10}{2000} = 0.05\,m/\sec^2$ and $v = u - at = 10 - 0.05\times 5 = 9.75\,m/\sec$

Ans. 8: (c)

Solution: Measuring x vertically downwards, the equation of motion is

$$\frac{d}{dt}\left(m\frac{dx}{dt}\right) = mg \qquad \left[\frac{dp}{dt} = mg,\ P = m\frac{dx}{dt}\right]$$

$m = \dfrac{4}{3}\pi r^3 \rho$ (ρ is density)

$$\Rightarrow \frac{dm}{dt} = \frac{4}{3}\pi \times 3r^2 \times \frac{dr}{dt} = 4\pi\rho r^2 \frac{dr}{dt}$$

As per the question $\dfrac{dm}{dt} \propto 4\pi r^2 \rho$ or $\dfrac{dm}{dt} = k \times 4\pi r^2 \rho$

$\Rightarrow \dfrac{dm}{dt} = \rho 4\pi k r^2 = 4\pi\rho r^2 \dfrac{dr}{dt} \Rightarrow \dfrac{dr}{dt} = k \Rightarrow \dfrac{r = kt + c_1}{r = R}$ at $t = 0$, $r = R \Rightarrow c_1 = R$

$$r = kt + R \Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi(kt+R)^3 \rho \frac{dx}{dt}\right) = \frac{4}{3}\pi(kt+R)^3 \rho g$$

$(R+kt)^3 \dfrac{dx}{dt} = \dfrac{(R+kt)^4}{4k} + c_2$ at $t = 0$, $\dfrac{dx}{dt} = 0 \Rightarrow c_2 = -\dfrac{R^4}{4k}g$

So, $v(t) = \dfrac{g}{4k}\left[\dfrac{R+kt}{g} - \dfrac{R^4}{(R+kt)^3}\right]$

Ans. 9: (d)

Solution: As $v=\frac{dx}{dt}=v\tanh(bvt)=v\tanh\left(\frac{gt}{v}\right)$

$$\Rightarrow x=\frac{v^2}{g}\ln\cosh\left(\frac{gt}{v}\right)=\frac{v^2}{g}\ln\left(\frac{e^{gt/v}+e^{-gt/v}}{2}\right)$$

As $t\to\infty,\ v\to V\approx\sqrt{g/k}$ and $e^{bx}=\frac{e^{bvt}+e^{-bvt}}{2}$

Squaring, $4e^{2bx}=e^{2bvt}+e^{-2bvt}+2=\frac{V+v}{V-v}+\frac{V-v}{V+v}+2=\frac{4V^2}{V^2-v^2}$

$$\Rightarrow v^2=V^2\left(1-e^{-2bx}\right)=V^2\left(1-e^{-2gx/v^2}\right)$$

Ans. 10: (a)

Solution: $u^2=25+25s=5^2+2\times12.5\times s\Rightarrow u=5\,m/s$

u^2+2as

$a=12.5\,m/s^2$

Ans. 11: (b)

Solution: $\frac{dv}{dt}=-4v+8$

$$a=\frac{dv}{dt}=-4v+8\Rightarrow\frac{da}{dt}=\frac{d}{dt}(-4v+8)=-\frac{4dv}{dt}=-4(-4+8)=16v^2-32$$

$$\left.\frac{da}{dt}\right|_{t=0}=16\times0-32=-32\,m/s^2$$

NAT (Numerical Answer Type)

Ans. 12: 49

Solution: $v=u-gt+\ln\frac{m_0}{m}$

$$F_t=v_r\left(-\frac{dm}{dt}\right)=mg\Rightarrow-\frac{dm}{dt}=\frac{mg}{v_r}=\frac{5000\times9.8}{100}=49\,kg/\sec$$

Ans. 13: 147

Solution: $F_t=v_r\left(-\frac{dm}{dt}\right)=m(g+a)$

$$\Rightarrow-\frac{dm}{dt}=\frac{5000\times(9.8+2\times9.8)}{100}=147\,m/\sec$$

Ans. 14: 39.2

Solution: $v=\sqrt{2gh}=\sqrt{2\times 9.8\times 10}=14\,m/\sec$

$$F_t=v_r\left(-\frac{dm}{dt}\right)=v\times(\rho av)=\rho av^2=1000\times 0.1\times 14\times 14\,N=19600\,N$$

As $F_t=ma\Rightarrow a=\frac{F_t}{m}=\frac{19600}{1000}m/\sec^2\Rightarrow a=19.6\,m/\sec^2$

as $s=ut+\frac{1}{2}at^2=0+\frac{1}{2}\times 19.6\times 2\times 2=39.2\,m$