GATE 2018 Solution

Section: General Aptitude

Q1. – Q5. carry one mark each.

Q1. "When she fell down the __________, she received many ________ but little help".

The words that best fill the blanks in the above sentence are

(a) Stairs, stares (b) Stairs, stairs

(c) Stares, stairs (d) Stares, stares

Ans. : (a)

Solution: stairs means steps while stares means to look someone continuously.

Q2. "In spite of being warned repeatedly, he failed to correct his___________ behavior"

The word that best fills the blank in the above sentence is

(a) Rational (b) Reasonable (c) Errant (d) Good

Ans. : (c)

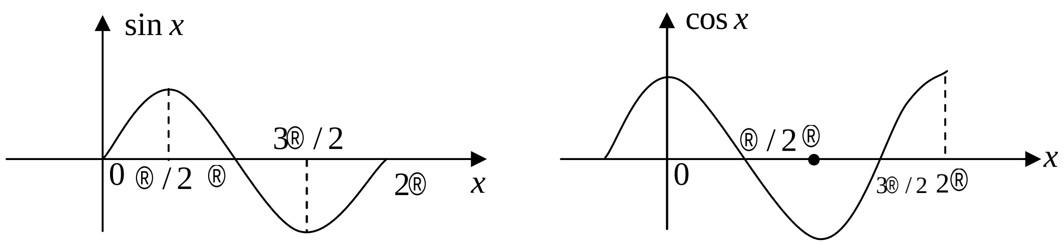
Solution: The most suitable option is errant as errant means irregular.

Q3. For $0 \le x \le 2®$, $\sin x$ and $\cos x$ are both decreasing functions in the interval_________

(a) $\left(0, \frac{®}{2}\right)$ (b) $\left(\frac{®}{2}, ®\right)$ (c) $\left(®, \frac{3®}{2}\right)$ (d) $\left(\frac{3®}{2}, 2®\right)$

Ans.: (b)

Solution: Graph of $\sin x$ and $\cos x$ is shown in the figure below



From the graph we see that $\sin x$ and $\cos x$ are both decreasing function in the interval $\left(\frac{®}{2}, ®\right)$

Q4. The area of an equivalent triangle is $\sqrt{3}$. What is the perimeter of the triangle?

(a) 2 (b) 4 (c) 6 (d) 8

Ans. : (c)

Solution: Let the side of equilateral triangle $=a$,and the area $=\dfrac{\sqrt{3}}{4}a^2$

or $\dfrac{\sqrt{3}}{4}a^2=\sqrt{3}$ or $a^2=4$ or $a=2$

Hence, the perimeter of the equilateral triangle $=3a=3\times 2=6$

Q5. Arrange the following three-dimensional objects in the descending order of their volumes:

(i) A cuboid with dimensions $10\,cm$, $8\,cm$ and $6\,cm$

(ii) A cube of side $8\,cm$

(iii) A cylinder with base radius $7\,cm$ and height $7\,cm$

(iv) A sphere of radius $7\,cm$

(a) (i), (ii), (iii), (iv)   (b) (ii), (i), (iv), (iii)

(c) (iii), (ii), (i), (iv)   (d) (iv), (iii), (ii), (i)

Ans. : (d)

Solution: The volume of cuboid $=10cm\times 8cm\times 6cm=480cm^3$

The volume of cube $=8cm\times 8cm\times 8cm=512cm^3$

The volume of cylinder $=\text{®}r^2h=\dfrac{22}{7}\times 7\times 7\times 7cm^3=1078cm^3$

The volume of sphere $=\dfrac{4}{3}\text{®}r^3=\dfrac{4}{3}\times\dfrac{22}{7}\times 7\times 7\times 7=1437.3cm^3$

Hence the descending orders of volume will be $1437.3cm^3$, $1078cm^3$, $512cm^3$ and $480cm^3$

Q6. – Q10. carry two marks each.

Q6. An automobile travels from city A to city B and returns to city A by the same route. The speed of the vehicle during the onward and return journeys were constant at $60\,km/h$ and $90\,km/h$, respectively. What is the average speed in km/h for the entire journey?

(a) 72   (b) 73   (c) 74   (d) 75

Ans.: (a)

Solution: Let the distance between A and B **is** xkm. Then

$$\text{Average speed}=\frac{\text{Total distance}}{\text{Total time}}=\frac{2xkm}{\left(\frac{x}{60}+\frac{x}{90}\right)\text{ hour}}=2x\times\frac{360}{10x}km/h=72km/h$$

Q7. A set of 4 parallel lines intersect with another set of 5 parallel lines. How many parallelograms are formed?

(a) 20 (b) 48 (c) 60 (d) 72

Ans. (c)

Solution: Any two parallel lines in one direction and any two parallel lines in the other direction can form parallelograms.

So, number of parallelogram formed

$$=5C_2\times 4C_2=\frac{5!}{2!3!}\times\frac{4!}{2!2!}=\frac{4\times 5}{2}\times\frac{3\times 4}{2}=10\times 6=60$$

Q8. To pass a test, a candidate needs to answer at least 2 out of 3 questions correctly. A total of 6,30,000 candidates appeared for the test. Question A was correctly answered by 3,30,000 candidates. Question B was answered correctly by 2,50,000 candidates. Question C was answered correctly by 2,60,000 candidates. Both questions A and B were answered correctly by 1,00,000 candidates. Both questions B and C were answered correctly by 90,000 candidates. Both questions were A and C were answered correctly by 80,000 candidates. If the number of students answering all questions correctly is the same as the number answering none, how many candidates failed to clear the test?

(a) 30,000 (b) 2,70,000 (c) 3,90,000 (d) 4,20,000

Ans. (d)

Solution: Let $n(0)$ denote the number of students answering none of the questions and $n(3)$ be the number of students answering all questions, then

$$n(A\cup B\cup C)-n(0)=n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(A\cap C)+n(3)$$

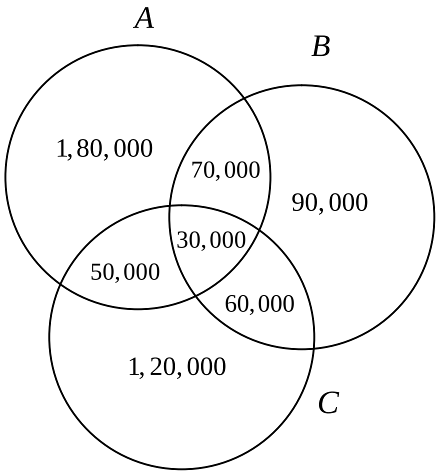
$$6,30,000-n(0)=3,30,000+2,50,000+2,60,000-1,00,000-90,000-80,000+n(3)$$

$$\Rightarrow 6,30,000-n(0)=5,70,000+n(3)$$

Since, $n(0)=n(3)$

Hence, $2n(0)=60,000\Rightarrow n(0)=30,000$

Using this fact and the information given, one fill the Venn-diagram.



It is obvious that the number of failed students will be, the sum of number the students who only passed in one subject and the number of student answering none of the question. Hence, the number of students failed to clear the test

$$=1,80,000+1,20,000+90,000+30,000=4,20,000$$

Q9. If $x^2+x-1=0$, what is the value of $x^4+\frac{1}{x^4}$?

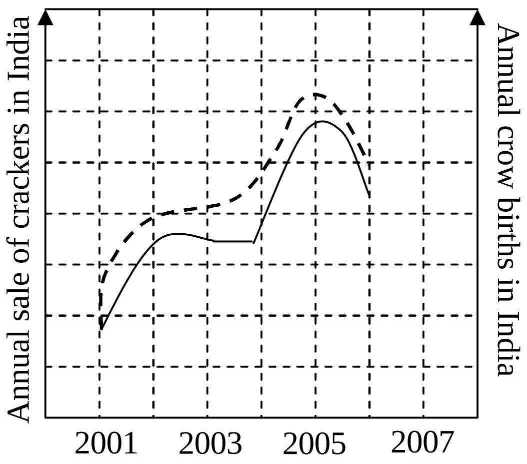
(a) 1 (b) 5 (c) 7 (d) 9

Ans. : (c)

Solution: Given that $x^2+x-1=0 \Rightarrow x(1+x)=1 \Rightarrow 1+x=\frac{1}{x} \Rightarrow x-\frac{1}{x}=-1,$

$$x^2+\frac{1}{x^2}=3 \Rightarrow x^4+\frac{1}{x^4}=9-2=7$$

Q10. In a detailed study of annual crow births in India, it was found that there was relatively no growth during the period 2002 to 2004 and a sudden spike from 2004 to 2005. In another unrelated study, it was found that the revenue from cracker sales in India which remained fairly flat from 2002 to 2004, saw a sudden spike in 2005 before declining again in 2006. The solid line in the graph below refers to annual sale of crackers and the dashed line refers to the annual crow births in India. Choose the most appropriate inference from the above data.



(a) There is a strong correlation between crow birth and cracker sales

(b) Cracker usage increases crow birth rate

(c) If cracker sale declines, crow birth will decline

(d) Increased birth rate of crows will cause an increase in the sale of crackers

Ans.: (a)

Solution: The growth pattern of crows and the growth in annual sales of fire crackers in nearly the same. The two graphs are almost parallel to each other. Hence there is strong correlation between crow birth and crackers sales.

Section: Physics

Q1. – Q25. carry one mark each.

Q1. The eigenvalues of a Hermitian matrix are all

(a) real (b) imaginary

(c) of modulus one (d) real and positive

Topic- Mathematical Physics

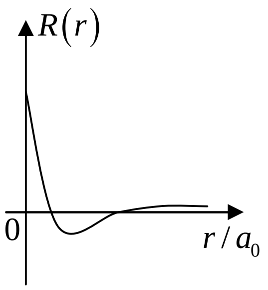
Subtopic- Matrices

Ans. : (a)

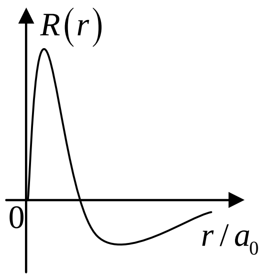
Solution: Eigenvalue of Hermitian matrix and Symmteric matrix must be real.

Q2. Which one of the following represents the $3p$ radial wave function of hydrogen atom? (a_0 is the Bohr radius)

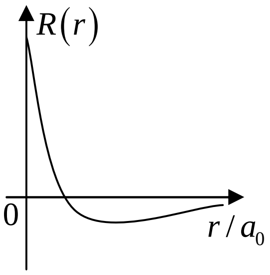
(a)



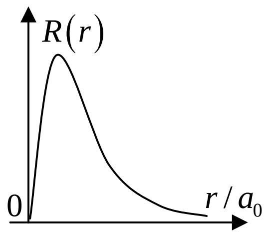
(b)



(c)



(d)



Topic- Quantum Mechanics

Subtopic- Hydrogen Atom

Ans. : (b)

Solution: $3p$ radial wave function is:

$$R_{31} \propto r\left(1-\frac{r}{6a_0}\right)e^{-\frac{r}{3a_0}}$$

Q3. Given the following table,

Group I	Group II
P: Stern-Gerlach experiment	1: Wave nature of particles
Q: Zeeman effect	2: Quantization of energy of electrons in the atoms
R: Frank-Hertz experiment	3: Existence of electron spin
S: Davisson-Germer experiment	4: Space quantization of angular momentum

Which one of the following correctly matches the experiments from Group I to their inferences in Group II?

(a) P-2, Q-3, R-4, S-1 (b) P-1, Q-3, R-2, S-4

(c) P-3, Q-4, R-2, S-1 (d) P-2, Q-1, R-4, S-3

Topic- Experimental Methods

Subtopic- Applications

Ans. : (c)

Q4. In spherical polar coordinates $(r, ¯, ¤)$, the unit vector $\hat{¯}$ at $(10, ® / 4, ® / 2)$ is

(a) $\hat{k}$ (b) $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$ (c) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ (d) $\frac{1}{\sqrt{2}}(\hat{j}-\hat{k})$

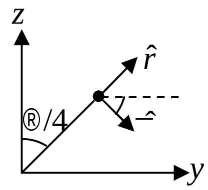
Topic- Mathematical Physics

Subtopic- Vector Analysis

Ans. : (d)

Solution: $\hat{¯} = \cos 45^0 \hat{j} - \sin 45^0 \hat{k}$

$$\Rightarrow \hat{¯} = \frac{1}{\sqrt{2}}(\hat{j}-\hat{k})$$



Q5. The scale factors corresponding to the covariant metric tensor g_{ij} in spherical polar coordinates are

(a) $1, r^2, r^2 \sin^2 ¯$ (b) $1, r^2, \sin^2 ¯$ (c) $1,1,1$ (d) $1, r, r\sin ¯$

Topic- Mathematical Physics

Subtopic-Tensor

Ans. : (d)

Q6. In the context of small oscillations, which one of the following does NOT apply to the normal coordinates?

(a) Each normal coordinate has an eigen-frequency associated with it

(b) The normal coordinates are orthogonal to one another

(c) The normal coordinates are all independent

(d) The potential energy of the system is a sum of squares of the normal coordinates with constant coefficients

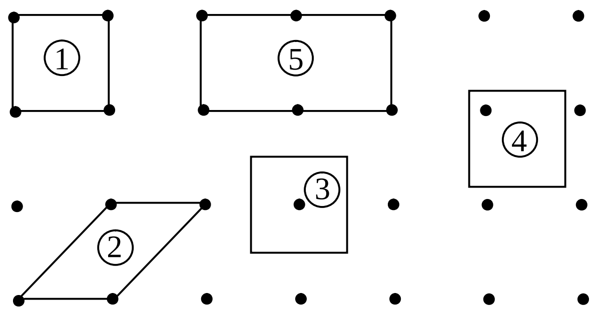
Topic- Classical Mechanics

Subtopic- Small Oscillations

Ans. : (b)

Solution: Normal co-ordinate must be independent. It is not necessary that it should be orthogonal.

Q7. For the given unit cells of a two dimensional square lattice, which option lists all the primitive cells?



(a) (1) and (2)

(b) (1), (2) and (3)

(c) (1), (2), (3) and (4)

(d) (1), (2), (3), (4) and (5)

Topic- Solid state physics

Subtopic- Crystallography

Ans. : (c)

Solution: For primitive cell, N_{eff} should be 1.

In cell (1), (2), (3) and (4) $N_{eff} = 1$, So these are primitive cell

Whereas in cell (5), $N_{eff} = 2$, So this is non-primitive cell.

Q8. Among electric field ($\vec{E}$), magnetic field ($\vec{B}$), angular momentum ($\vec{L}$) and vector potential ($\vec{A}$), which is/are **odd** under parity (space inversion) operation?

(a) $\vec{E}$ only

(b) $\vec{E}$ and $\vec{A}$ only

(c) $\vec{E}$ and $\vec{B}$ only

(d) $\vec{B}$ and $\vec{L}$ only

Topic- Particle Physics

Subtopic- Charge, Parity and time conservation

Ans. : (b)

Solution: Under parity operation $r \to -r$

$$E = -\frac{\partial V}{\partial r} \quad ; \quad E : P \to -E$$

$$B = \vec{I} \times \vec{r} \quad ; \quad B : P \to +B$$

$$L = \vec{r} \times \vec{p} \quad ; \quad L : P \to +L$$

$$E = -\frac{\partial A}{\partial t} \quad ; \quad A : P \to -A$$

Q9. The expression for the second overtone frequency in the vibrational absorption spectra of a diatomic molecule in terms of the harmonic frequency μ_e and anharmonicity constant x_e is

(a) $2\mu_e(1-x_e)$

(b) $2\mu_e(1-3x_e)$

(c) $3\mu_e(1-2x_e)$

(d) $3\mu_e(1-4x_e)$

Topic- Molecular Physics

Subtopic- Vibrational Spectra

Ans. : (d)

Solution: $£_V = \mu_e\left(v+\frac{1}{2}\right) - \mu_e x_e\left(v+\frac{1}{2}\right)^2$

Second overtone $v = 0 \to 3$

$$\therefore \ \overline{v} = £_{v=3} - £_{v=0} = \frac{7}{2}\mu_e - \mu_e x_e\left(\frac{7}{2}\right)^2 - \frac{\mu_e}{2} + \mu_e x_e\left(\frac{1}{2}\right)^2 = 3\mu_e - 12\mu_e x_e = 3w_e(1-4x_e)$$

Q10. Match the physical effects and order of magnitude of their energy scales given below, where $\sim = \frac{e^2}{4\circledR \in_0 \hbar c}$ is fine structure constant; m_e and m_p are electron and proton mass, respectively.

Group I	Group II
P: Lamb shift	1: $\sim O\left(\sim^2 m_e c^2\right)$
Q: Fine structure	2: $\sim O\left(\sim^4 m_e c^2\right)$
R: Bohr energy	3: $\sim O\left(\sim^4 m_e^2 c^2 / m_p\right)$
S: Hyperfine structure	4: $\sim O\left(\sim^5 m_e c^2\right)$

(a) P-3, Q-1, R-2, S-4 (b) P-2, Q-3, R-1, S-4

(c) P-4, Q-2, R-1, S-3 (d) P-2, Q-4, R-1, S-3

Topic- Atomic Physics

Subtopic- Spectra

Ans. : (c)

Solution:- Bohr energy $\Delta E \propto \sim^2 m_e c^2$

Fine structure $\Delta E \propto \sim^4 m_e c^2$

Lamb straight $\Delta E \propto \sim^5 m_e c^2$

Hyperfine structure $\Delta E \propto \frac{\sim^4 m_e c^2}{m_p}$

Q11. The logic expression $\bar{A}BC + \bar{A}\bar{B}C + ABC + AB\bar{C}$ can be simplified to

(a) A XOR C (b) A AND C (c) 0 (d) 1

Topic- Electronics

Subtopic- Boolean Algebra

Ans. : (a)

Solution: $Y = \bar{A}BC + \bar{A}\bar{B}C + AB\bar{C} + A\bar{B}\bar{C} = \bar{A}C\left(B + \bar{B}\right) + A\bar{C}\left(B + \bar{B}\right)$

$\Rightarrow Y = \bar{A}C + A\bar{C} = A \text{ XOR } C$

Q12. At low temperatures (T), the specific heat of common metals is described by (with $\sim$ and as constants)

(a) $\sim T + \ T^3$ (b) $\ T^3$ (c) $\exp\left(-\sim / T\right)$ (d) $\sim T + \ T^5$

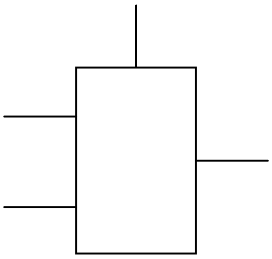
Topic- Solid state physics

Subtopic- Specific heat

Ans. : (a)

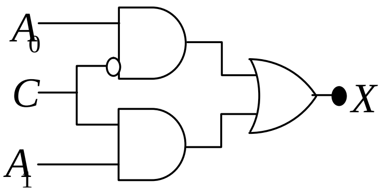
Solution: $C = C_e + C_{ph} = \frac{\text{®}^2 N_A k_B^2 T}{2E_F} + \frac{12\text{®}^4 RT^3}{5\,^{-3}_{D}} = \sim T + \quad T^3$

Q13. In a 2-to-1 multiplexer as shown below, the output $X = A_0$ if $C = 0$ and $X = A_1$ if $C = 1$.

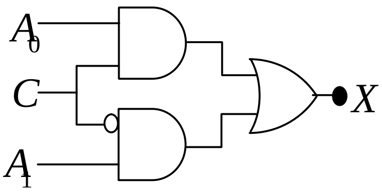


Which one of the following is the correct implementation of this multiplexer?

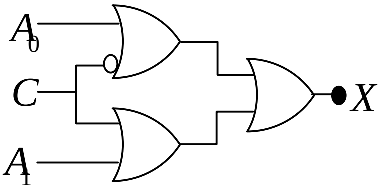
(a)



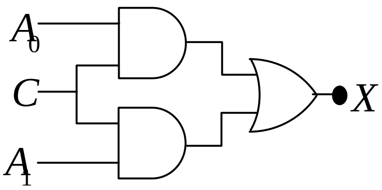
(b)



(c)



(d)



Topic- Electronics

Subtopic- Multiplexer

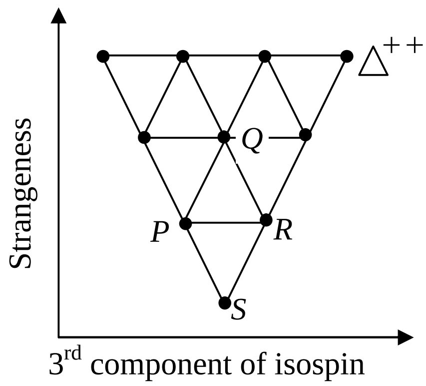
Ans. : (a)

Solution: Check option (a),

$X = A_0\bar{C} + A_1 C$

If $C = 0 \Rightarrow X = A_0$, and if $C = 1 \Rightarrow X = A_1$

Q14. The elementary particle Ξ^0 is placed in the baryon decouplet, shown below, at

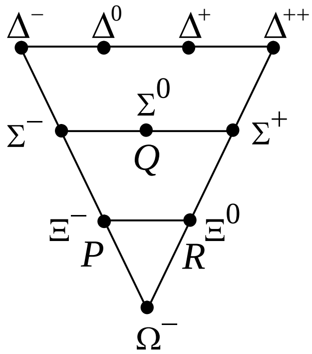


(a) P (b) Q (c) R (d) S

Topic- Particle Physics

Subtopic- Elementary particle

Ans. : (c)



Q15. The intrinsic/permanent electric dipole moment in the ground state of hydrogen atom is (a_0 is the Bohr radius)

(a) $-3ea_0$ (b) zero (c) ea_0 (d) $3ea_0$

Topic- Quantum Mechanics

Subtopic- Hydrogen Atom

Ans. : (b)

Solution: For dipole moment energy is $-eEr\cos\theta$

$$E_1^1 = \langle -eEr\cos\theta \rangle = eE\langle r\rangle\langle\cos\theta\rangle = 0 \qquad [\because \langle\cos\theta\rangle = 0]$$

Q16. The high temperature magnetic susceptibility of solids having ions with magnetic moments can be described by $\chi \propto \frac{1}{T+\theta}$ with T as absolute temperature and θ as constant. The three behaviours i.e., paramagnetic, ferromagnetic and anti-ferromagnetic are described, respectively, by

(a) $\theta < 0, \theta > 0, \theta = 0$ (b) $\theta > 0, \theta < 0, \theta = 0$

(c) $\theta = 0, \theta < 0, \theta > 0$ (d) $\theta = 0, \theta > 0, \theta < 0$

Topic- Solid state physics

Subtopic- Magnetism

Ans. : (c)

Solution: Paramagnetism: $\chi = \frac{C}{T}$

Ferromagnetism: $\chi = \frac{C}{T - T_C}$

Anti-ferromagnetism: $\chi = \frac{C}{T + T_C}$

Q17. Which one of the following is an allowed electric dipole transition?

(a) ${}^1S_0 \to {}^3S_1$ (b) ${}^2P_{3/2} \to {}^2D_{5/2}$ (c) ${}^2D_{5/2} \to {}^2P_{1/2}$ (d) ${}^3P_0 \to {}^5D_0$

Topic- Atomic Physics

Subtopic- Transition and Selection rule

Ans. : (b)

Solution: For electric dipole transition

$$\Delta L = 0, \pm 1 \quad (0 \not\to 0),\ \Delta J = 0.\pm 1,\ \Delta S = 0$$

Only option (b) satisfies above selection rules

Q18. In the decay, $«^+ \to e^+ + ¬_e + X$, what is X ?

(a) ¥ (b) $\bar{¬}_e$ (c) $¬_«$ (d) $\bar{¬}_«$

Topic- Particle Physics

Subtopic- Conservation

Ans. : (d)

Solution:- $u^+ \to e^+ + ¬_e + X$

$L_u :\ -1 \quad 0 \quad 0 \quad -1$

$L_e :\ 0 \quad -1 \quad +1 \quad 0$

$q :\ +1 \quad +1 \quad 0 \quad 0$

So X should be $\bar{¬}_«$.

Q19. A spaceship is travelling with a velocity of $0.7c$ away from a space station. The spaceship ejects a probe with a velocity $0.59c$ opposite to its own velocity. A person in the space station would see the probe moving at a speed Xc, where the value of X is ____________ (up to three decimal places).

Topic- Classical Mechanics

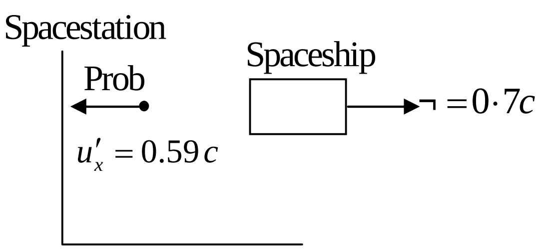
Subtopic- STR

Ans.: $0.187c$

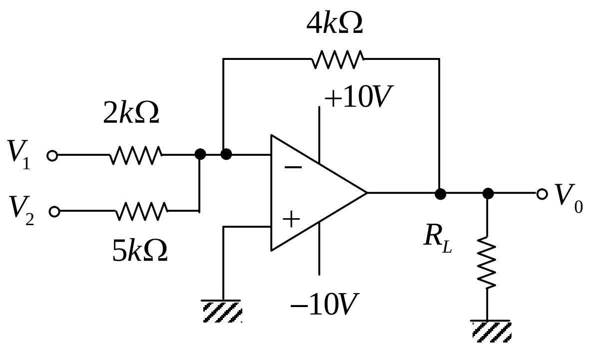
Solution: $¬ = 0\cdot 7c$, $u'_x = -0\cdot 59c$,

$$u_x = \frac{u'_x + ¬}{1 + \frac{u'_x ¬}{c_2}}$$

$$u_x = \frac{-0\cdot 59c + 0\cdot 7c}{1 - 0\cdot 7 \times 0\cdot 59} = \frac{0.11c}{1-0.413} = \frac{0.11c}{0.587} = 0.187c$$



Q20. For an operational amplifier (ideal) circuit shown below,



If $V_1 = 1V$ and $V_2 = 2V$, the value of V_0 is __________ V (up to one decimal place).

Topic- Electronics

Subtopic- Operational Amplifier

Ans. : -3.6

Solution: By superposition principle:

$$V_0 = V_{01} + V_{02} = -\frac{4}{2}\times 1V - \frac{4}{5}\times 2V$$

$$V_0 = -2 - 1.6 = -3.6V$$

Q21. An infinitely long straight wire is carrying a steady current I. The ratio of magnetic energy density at distance r_1 to that at $r_2 \left(=2r_1\right)$ from the wire is ___________.

Topic- Electromagnetic Theory

Subtopic- Energy density

Ans. : 4

Solution: $u_B = \frac{B^2}{2«_0} \propto \frac{1}{r^2} \Rightarrow \frac{u_{B1}}{u_{B2}} = \frac{r_2^2}{r_1^2} = \frac{\left(2r_1\right)}{r_1^2} = 4$

Q22. A light beam of intensity I_0 is falling normally on a surface. The surface absorbs 20% of the intensity and the rest is reflected. The radiation pressure on the surface is given by $X\, I_0 / c$, where X is __________ (up to one decimal place). Here c is the speed of light.

Topic- Electromagnetic Theory

Subtopic- Radiation Pressure

Ans. : 1.8

Solution: Radiation pressure $= \frac{I_0}{c} - \left(-0.8\frac{I_0}{c}\right) = 1.8\frac{I_0}{c}$

Q23. The number of independent components of a general electromagnetic field tensor is__________

Topic- Electromagnetic Field

Subtopic- Tensor

Ans. : 6

Solution: In Cartesian co-ordinate, three Independent coordinate for electric field, (E_x, E_y, E_z) and three Independent co-ordinate for magnetic field (B_x, B_y, B_z).

Q24. If X is the dimensionality of a free electron gas, the energy (E) dependence of density of states is given by $E^{\frac{1}{2}X-Y}$, where Y is_________.

Topic- Solid state physics

Subtopic- Density of states

Ans. : 1

Solution: For free electron gas: $E = \frac{\hbar^2 k^2}{2m}$ $\Rightarrow k = \frac{\sqrt{2mE}}{\hbar}$, $dk = \frac{\sqrt{2m}}{2\hbar\sqrt{E}}$

For one dimension:

$$g(k)dk = \frac{L}{®}dk = \frac{L}{®}\frac{\sqrt{2m}}{2\hbar\sqrt{E}} \propto E^{\frac{-1}{2}}$$

So for $X = 1, Y$ will be 1.

OR

$° \propto E^{\left(\frac{d}{2}-1\right)}$ where d is the dimension.

Q25. For nucleus ${}^{164}Er$, a $J^{®} = 2^+$ state is at $90\ keV$. Assuming ${}^{164}Er$ to be a rigid rotor, the energy of its 4^+ state is __________ keV (up to one decimal place)

Topic- Nuclear Physics

Subtopic- Rotational Energy

Ans. : 300

Solution: $E_J = hcBJ(J+1)$ _________ 4^+

$E_{2^+} = hc\ B\ 2(2+1)$ and $E_{4^+} = hc\ B\ 4(4+1)$ _________ 2^+

Then, $\frac{E_{4^+}}{E_{2^+}} = \frac{20}{6} \Rightarrow E_{4^+} = \frac{20}{6} \times 90\ keV = 300\, keV$

Q26. – Q55. carry two marks each.

Q26. Given $\vec{V}_1 = \hat{i} - \hat{j}$ and $\vec{V}_2 = -2\hat{i} + 3\hat{j} + 2\hat{k}$, which one of the following $\vec{V}_3$ makes $(\vec{V}_1, \vec{V}_2, \vec{V}_3)$ a complete set for a three dimensional real linear vector space?

(a) $\vec{V}_3 = \hat{i} + \hat{j} + 4\hat{k}$ (b) $\vec{V}_3 = 2\hat{i} - \hat{j} + 2\hat{k}$

(c) $\vec{V}_3 = \hat{i} + 2\hat{j} + 6\hat{k}$ (d) $\vec{V}_3 = 2\hat{i} + \hat{j} + 4\hat{k}$

Topic- Mathematical Physics

Subtopic- Vector Analysis

Ans. : (d)

Solution: Let A be the matrix formed by taking $\vec{V}_1$, $\vec{V}_2$ and $\vec{V}_3$ as column matrix i.e.,

$$A = [V_1 \quad V_2 \quad V_3] = \begin{bmatrix} 1 & -2 & 2 \\ -1 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix} \Rightarrow |A| = -2 \text{. Here } V_3 = \left(2\hat{i} + \hat{j} + 4\hat{k}\right)$$

Since, $|A| \neq 0$, hence, $\vec{V}_1$, $\vec{V}_2$ and $\vec{V}_3$ form a three dimensional real vector space.

Hence, option (d) is correct.

Q27. An interstellar object has speed v at the point of its shortest distance R from a star of much larger mass M. Given $v^2 = 2GM/R$, the trajectory of the object is

(a) circle (b) ellipse (c) parabola (d) hyperbola

Topic- Classical Mechanics

Subtopic- Central Motion

Ans. : (c)

Solution: At shortest distance $E = \dfrac{J^2}{2mR^2} - \dfrac{GMm}{R}$

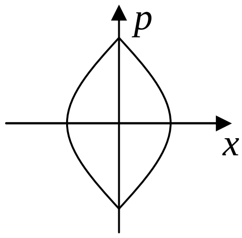
Since, $mvR = J \Rightarrow J^2 = m^2v^2R^2$

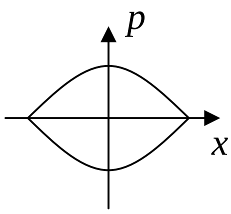
Now, $J^2 = m^2 2GMR = 2GMm^2R$ (Given that $v^2 = \dfrac{2GM}{R}$)

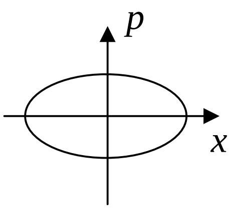
$$E = \frac{2GMm^2R}{2mR^2} - \frac{GMm}{R} = \frac{GMm}{R} - \frac{GMm}{R} = 0$$

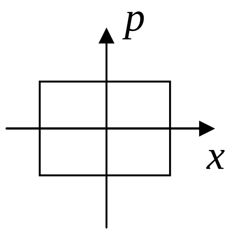
For Kepler's potential, if energy is zero, then the shape is parabola.

Q28. A particle moves in one dimension under a potential $V(x) = \sim |x|$ with some non-zero total energy. Which one of the following best describes the particle trajectory in the phase space?

(a) 

(b) 

(c) 

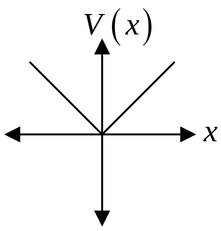
(d) 

Topic- Classical Mechanics

Subtopic- Small Oscillations

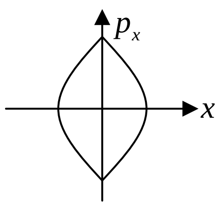
Ans.: (a)

Solution: $E = \frac{p^2}{2m} + \sim |x|$



For $x > 0$, $E = \frac{p^2}{2m} + \sim x$

$\Rightarrow p^2 = 2m(E - \sim x)$



For $x < 0$, $E = \frac{p^2}{2m} - \sim x$

$\Rightarrow p^2 = 2m(E + \sim x)$

Q29. Consider an infinitely long solenoid with N turns per unit length, radius R and carrying a current $I(t) = \sim \cos \mu t$, where $\sim$ is a constant and μ is the angular frequency. The magnitude of electric field at the surface of the solenoid is

(a) $\frac{1}{2} «_0 NR\mu \sim \sin \mu t$

(b) $\frac{1}{2} «_0 \mu NR \cos \mu t$

(c) $«_0 NR\mu \sim \sin \mu t$

(d) $«_0 \mu NR \cos \mu t$

Topic- Electromagnetic Theory

Subtopic- Solenoid

Ans. : (a)

Solution: $\vec{B}=\begin{cases}«_0 NI(t)\hat{z}, \text{ inside}\\ 0 \qquad\qquad , \text{ outside}\end{cases}$

By Maxwell relation:

Since, $\oint_{line} \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{a}$

$\Rightarrow \left|\vec{E}\right| \times 2® R = -«_0 N\left(-\text{~}\mu \sin \mu t\right)\times ® R^2 \Rightarrow \left|\vec{E}\right| = \frac{1}{2}«_0 NR\mu\text{~} \sin \mu t$

Q30. A constant and uniform magnetic field $\vec{B}=B_0\hat{k}$ pervades all space. Which one of the following is the correct choice for the vector potential in Coulomb gauge?

(a) $-B_0(x+y)\hat{i}$ (b) $B_0(x+y)\hat{j}$ (c) $B_0 x\hat{j}$ (d) $-\frac{1}{2}B_0\left(x\hat{i}-y\hat{j}\right)$

Topic- Electromagnetic Theory

Subtopic- Coulomb gauge

Ans. : (c)

Solution: In Coulomb gauge condition: $\vec{\nabla}\cdot\vec{A}=0$

For $\vec{A}=B_0 x\hat{j} \quad \Rightarrow \vec{\nabla}\cdot\vec{A}=0$

And $\vec{\nabla}\times\vec{A}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & B_0 x & 0\end{vmatrix} = B_0\hat{k}=\vec{B}$

Q31. If H is the Hamiltonian for a free particle with mass m, the commutator $[x,[x,H]]$ is

(a) $\hbar^2/m$ (b) $-\hbar^2/m$ (c) $-\hbar^2/(2m)$ (d) $\hbar^2/(2m)$

Topic- Quantum Mechanics

Subtopic- Commutation Relation

Ans. : (b)

Solution: For free particle, potential is zero.

$\Rightarrow H=\frac{P_x^2}{2m}$

Now, $[x,H]=\left[x,\frac{P_x^2}{2m}\right]=\frac{2i\hbar}{2m}P_x$

$[x,[x,H]]=\frac{2i\hbar}{2m}[x,P_x]=\frac{i\hbar}{m}(i\hbar)=-\frac{\hbar^2}{m}$

Q32. A long straight wire, having radius a and resistance per unit length r, carries a current I. The magnitude and direction of the Poynting vector on the surface of the wire is

(a) $I^2r/2®a$, perpendicular to axis of the wire and pointing inwards

(b) $I^2r/2®a$, perpendicular to axis of the wire and pointing outwards

(c) $I^2r/®a$, perpendicular to axis of the wire and pointing inwards

(d) $I^2r/®a$, perpendicular to axis of the wire and pointing outwards

Topic- Electromagnetic Theory

Subtopic- Poynting vector

Ans. : (a)

Solution: $\left|\vec{S}\right| = \frac{1}{«_0}\left|\left(\vec{E}\times\vec{B}\right)\right| = \frac{1}{«_0}\frac{V}{l}\times\frac{«_0 I}{2®a} = \frac{IR}{l}\times\frac{I}{2®a}$

$\because V = IR,\ r = \frac{R}{l} \Rightarrow \left|\vec{S}\right| = \frac{I^2 r}{2®a}$

Q33. Three particles are to be distributed in four non-degenerate energy levels. The possible number of ways of distribution: (i) for distinguishable particles, and (ii) for identical Boson, respectively, is

(a) (i) 24, (ii) 4 (b) (i) 24, (ii) 20 (c) (i) 64, (ii) 20 (d) (i) 64, (ii) 16

Topic- Statistical Mechanics

Subtopic- Number of microstats

Ans. : (c)

Solution: Number of particles, $N = 3$

Number of state, $g = 4$

For distinguishable particle, $w = g^N = 4^3 = 64$

For identical Bosons, $w = \frac{\lfloor N+g-1}{\lfloor N \lfloor g-1} = \frac{\lfloor 6}{\lfloor 3 \lfloor 3} = \frac{6\times5\times4}{3\times2} = 20$

Q34. The term symbol for the electronic ground state of oxygen atom is

(a) $^1S_0$ (b) $^1D_2$ (c) $^3P_0$ (d) $^3P_2$

Topic- Atomic Physics

Subtopic- LS Coupling

Ans. : (d)

Solution: O: $1s^2, 2s^2, 2p^4$

$M_L = -1$	0	$+1$
↑↓	↑	↑

Here, $S = 1$, $L = 2$

According to Hund's rule, for ground state energy

$J = (L + S) = 2 \qquad \therefore \quad {}^{2S+1}L_J = {}^3P_2$

Q35. The energy dispersion for electrons in one dimensional lattice with lattice parameter a is given by $E(k) = E_0 - \frac{1}{2}W \cos ka$, where W and E_0 are constants. The effective mass of the electron near the bottom of the band is

(a) $\frac{2\hbar^2}{Wa^2}$ (b) $\frac{\hbar^2}{Wa^2}$ (c) $\frac{\hbar^2}{2Wa^2}$ (d) $\frac{\hbar^2}{4Wa^2}$

Topic- Solid state physics

Subtopic- Free electron Theory

Ans. : (a)

Solution: $E(k) = E_0 - \frac{1}{2}W \cos(ka)$

$$\frac{dE}{dk} = \frac{aW}{2}\sin(ka) \Rightarrow \frac{d^2E}{dk^2} = \frac{a^2W}{2}\cos(ka)$$

$$\therefore \ m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}} = \frac{\hbar^2}{\frac{a^2W}{2}\cos(ka)} = \frac{2\hbar^2}{Wa^2}$$ [At bottom of the band, $k = 0$]

Q36. Amongst electrical resistivity (°), thermal conductivity (©), specific heat (C), Young's modulus (Y) and magnetic susceptibility (¡), which quantities show a sharp change at the superconducting transition temperature?

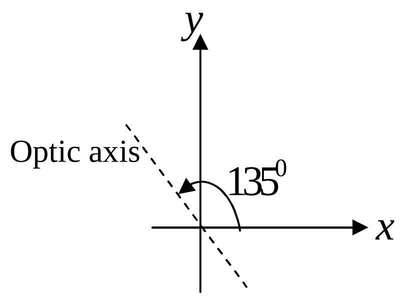
(a) °, ©, C, Y (b) °, C, ¡ (c) °, ©, C, ¡ (d) ©, Y, ¡

Topic- Solid state physics

Subtopic- Superconductivity

Ans. : (b)

Q37. A quarter wave plate introduces a path difference of $\lambda/4$ between the two components of polarization parallel and perpendicular to the optic axis. An electromagnetic wave with $\vec{E}=(\hat{x}+\hat{y})E_0 e^{i(kz-\omega t)}$ is incident normally on a quarter wave plate which has its optic axis making an angle 135^0 with the x- axis as shown.



The emergent electromagnetic wave would be

(a) elliptically polarized

(b) circularly polarized

(c) linearly polarized with polarization as that of incident wave

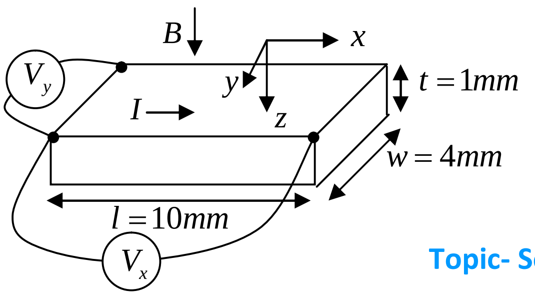
(d) linearly polarized but with polarization at 90^0 to that of the incident wave

Topic- Electromagnetic Theory

Subtopic- EM Wave

Ans. : (c)

Q38. A p- doped semiconductor slab carries a current $I=100\,mA$ in a magnetic field $B=0.2T$ as shown. One measures $V_y=0.25\,mV$ and $V_x=2\,mV$. The mobility of holes in the semiconductor is___________ $m^2V^{-1}s^{-1}$ (up to two decimal places)

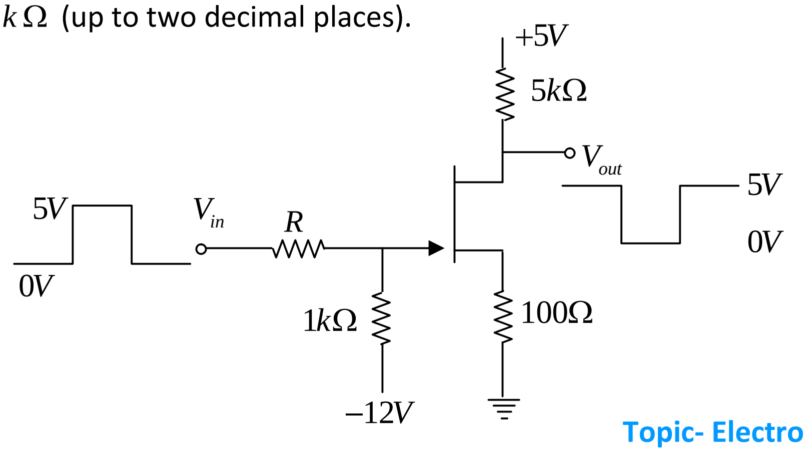


Topic- Solid state physics

Subtopic- Hall effect

Ans. : 1.55

Q39. An n- channel FET having Gate-Source switch-off voltage $V_{GS(\mathrm{OFF})} = -2V$ is used to invert a $0-5V$ square-wave signal as shown. The maximum allowed value of R would be _________ $k\Omega$ (up to two decimal places).



Topic- Electronics

Subtopic- Field effect transistor

Ans. : 0.70

Q40. Inside a large nucleus, a nucleon with mass $939\,MeVc^{-2}$ has Fermi momentum $1.40\ \mathrm{fm}^{-1}$ at absolute zero temperature. Its velocity is Xc, where the value of X is__________ (up to two decimal places).

($\hbar c = 197\,MeV$-fm)

Topic- Solid state physics

Subtopic- Free electron theory

Ans. : 0.29

Solution: Here, fermi – momentum or fermi radius, $k_F = 1.40\,fm^{-1}$

Now $P = mV_F = \hbar k_F \qquad \Rightarrow \frac{\hbar k_F}{m} = \frac{(\hbar c)k_F}{mc}$

$$= \frac{(197)\,MeV\text{-fm} \times 1\cdot 40\mathrm{fm}^{-1}}{939 MeVc^{-2} \times c} = \frac{275\cdot 8c}{939} = 0.29c$$

Q41. $4\,MeV$ ¥ - rays emitted by the de-excitation of ^{19}F are attributed, assuming spherical symmetry, to the transition of protons from $1d_{3/2}$ state to $1d_{5/2}$ state. If the contribution of spin-orbit term to the total energy is written as $C\langle \vec{l}\cdot\vec{s}\rangle$, the magnitude of C is ______ MeV (up to one decimal place).

Topic- Atomic Physics

Subtopic- LS Coupling

Ans. : 1.6

Solution: $l = 1, s = \frac{1}{2}, \hat{j}_1 = \frac{3}{2}, \hat{j}_2 = \frac{5}{2}$

$$\vec{j} = (\vec{l} + \vec{s}) \Rightarrow j^2 = l^2 + s^2 + 2\vec{l}\cdot\vec{s} \Rightarrow \vec{l}\cdot\vec{s} = \frac{(j^2 + l^2 - s^2)}{2}$$

$$\langle \vec{l}\cdot\vec{s} \rangle = \frac{[j(j+1) - (l+1) - s(s+1)]\hbar^2}{2}$$

$$\Delta E = \sim \left[\langle \vec{l}\cdot\vec{s} \rangle_{5/2} - \langle \vec{l}\cdot\vec{s} \rangle_{3/2} \right] = \sim \left[\frac{5}{2}\cdot\frac{7}{2} - \frac{3}{2}\cdot\frac{5}{2} \right] \frac{\hbar^2}{2} = \sim \cdot \left(\frac{20}{8} \right) \hbar^2 = \frac{20}{8} \cdot C$$

$$\Delta E = \frac{20}{8} C = 4MeV \Rightarrow C = \frac{32}{20} MeV, C = 1.6\, MeV.$$

Q42. An $\sim$ particle is emitted by a $^{230}_{90}Th$ nucleus. Assuming the potential to be purely Coulombic beyond the point of separation, the height of the Coulomb barrier is________ MeV (up to two decimal places). ($\frac{e^2}{4® \in_0} = 1.44\, MeV$-fm, $r_0 = 1.30$ fm)

Topic- Nuclear Physics

Subtopic- binding energy

Ans. : 25.995

Solution: The height of coulomb barrier for $\sim$ particle from

$${}_{90}Th^{230} \rightarrow {}_{88}X^{226} + 2He^4 \quad (\sim \text{ - particle})$$

$$V_C = \frac{1}{4® \in_0} \left(\frac{2ze^2}{R} \right)$$

Here, $R_0 = 1.3\ fm$, $\frac{e^2}{4® \in_0} = 1.44\ MeV\ fm$ and $R = R_0 A^{1/3}$

Here, we consider pure Coulombic interaction

$$A_{Th}^{1/3} = A_X^{1/3} + A_{\sim}^{1/3} = (226)^{1/3} + (4)^{1/3} = (6.09 + 1.58) = 7.67$$

$$R = R_0 A_{Th}^{1/3} = 1.3(7.67)$$

Hence, $V_C = \left(\frac{e^2}{4® \in_0} \right) \frac{2 \times 90}{1.3(7.67)} = \frac{180 \times 1.44}{1.3 \times 7.67} \frac{MeV}{fm}$

$$V_C = 25.995\ MeV$$

Q43. For the transformation

$$Q = \sqrt{2q}\, e^{-1+2\sim} \cos p, P = \sqrt{2q}\, e^{-\sim-1} \sin p$$

(where ~ is a constant) to be canonical, the value of ~ is __________.

Topic- Classical Mechanics

Subtopic- Canonical Transformation

Ans. : 2

Solution: $Q = \sqrt{2qe^{-1+2\sim}} \cos p, P = \sqrt{2qe^{-\sim-1}}.\sin p$

Since, $\left[Q, P\right] = 1 \Rightarrow \frac{\partial Q}{\partial q}\frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p}\frac{\partial P}{\partial q} = 1$

$$\Rightarrow \left(\frac{1}{2}\sqrt{2}q^{-\frac{1}{2}}.e^{-1+2\sim}\cos p\right)\left(\sqrt{2qe^{-\sim-1}}\cos p\right) - \sqrt{2qe^{-1+2\sim}}\left(-\sin p\right).\frac{\sqrt{2}}{2}q^{-\frac{1}{2}}e^{-\sim.1}\sin p = 1$$

$$\Rightarrow e^{\sim-2}.\left[\cos^2 p + \sin^2 p\right] = 1 = e^0 \Rightarrow \sim = 2$$

Q44. Given

$$\frac{d^2 f(x)}{dx^2} - 2\frac{df(x)}{dx} + f(x) = 0,$$

and boundary conditions $f(0) = 1$ and $f(1) = 0$, the value of $f(0.5)$ is __________ (up to two decimal places) .

Topic- Mathematical Physics

Subtopic- Differential Equation

Ans. : 0.81

Solution: $\frac{d^2 f(x)}{dx^2} - 2\frac{df(x)}{dx} + f(x) = 0$

Auxiliary equation is,

$\left(m^2 - 2m + 1\right) = 0$

$\left(m-1\right)^2 = 0 \Rightarrow m = 1,1$

Hence, the solution is

$f(x) = \left(c_1 + c_2 x\right)e^x$

using boundary condition,

$f(0) = c_1 e^0 \Rightarrow c_1 = 1$ (i)

$f(1) = (c_1 + c_2)e = 0$ (ii)

From (i) and (ii), $c_2 = -1$

Hence, $f(x) = (1-x)e^x \Rightarrow f(0.5) = (1-0.5)e^{0.5} = 0.81$

Q45. The absolute value of the integral

$$\int \frac{5z^3 + 3z^2}{z^2 - 4} dz\,,$$

over the circle $|z - 1.5| = 1$ in complex plane, is __________ (up to two decimal places).

Topic- Mathematical Physics

Subtopic- Complex Analysis

Ans. : 81.64

Solution: $f(z) = \dfrac{5z^3 + 3z^2}{(z-2)(z+2)}$

Pole, $z = 2, -2$

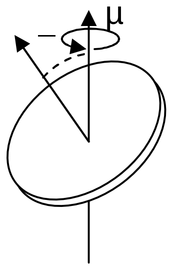
$z = -2$ is outside the center

$|-2 - 1.5| > 1$ So, will not be considered

Now, $\text{Res}(2) = \lim_{z \to 2}(z-2)\dfrac{(5z^3 + 3z^2)}{(z-2)(z+2)} = \dfrac{52^3 + 32^2}{4} = \dfrac{40+12}{4} = 13$

$I = 2®i \times residue = 2®i \times 13 = 26 \times 3.14 \Rightarrow I = 81.64$

Q46. A uniform circular disc of mass m and radius R is rotating with angular speed μ about an axis passing through its centre and making an angle $¯ = 30^0$ with the axis of the disc. If the kinetic energy of the disc is $\sim m\mu^2 R^2$, the value of $\sim$ is__________ (up to two decimal places).



Topic- Classical Mechanics

Subtopic- Angular Momenta and MOI

Ans. : 0.21

Solution: The kinetic energy of the disc is,

$$T = \frac{1}{2}\vec{L} \cdot \vec{\mu}$$

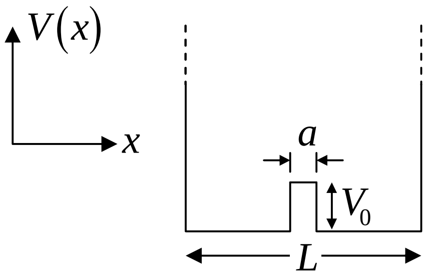
Where $\vec{L}$ is angular momentum and μ is angular velocity

$$T = \frac{1}{2}|\vec{L}||\vec{\mu}|\cos 30^0 = \frac{1}{2} I\mu \cdot \mu \frac{\sqrt{3}}{2} = \frac{1}{2}\left(\frac{mR^2}{2}\right)\mu^2 \times \frac{\sqrt{3}}{2}$$

$$T = \frac{\sqrt{3}}{8} m\mu^2 R^2 = 0.21 m\mu^2 R^2 \Rightarrow \sim m\mu^2 R^2 = 0.21 m\mu^2 R^2$$

Hence, $\sim = 0.21$

Q47. The ground state energy of a particle of mass m in an infinite potential well is E_0. It changes to $E_0\left(1+\sim\times 10^{-3}\right)$, when there is a small potential pump of height $V_0 = \frac{®^2\hbar^2}{50\,mL^2}$ and width $a = L/100$, as shown in the figure. The value of $\sim$ is ________ (up to two decimal places).



Topic- Quantum Mechanics

Subtopic- Perturbation

Ans. : 0.81

Solution: $\sim_1 = \left(\frac{L}{2}-\frac{a}{2}\right), \sim_2 = \left(\frac{L}{2}+\frac{a}{2}\right), \quad a = \frac{L}{100}$

$$E_1 = V_0 \int_{\sim_1}^{\sim_2} \left(\sqrt{\frac{2}{L}}\right)^2 \sin^2\left(\frac{® x}{L}\right) dx$$

$$= \frac{V_0}{L}\int_{\sim_1}^{\sim_2}\left[1-\cos\frac{2® x}{L}\right]dx = \frac{V_0}{L}\left[x - \frac{L}{2®}\sin\frac{2® x}{L}\right]_{\sim_1}^{\sim_2}$$

$$= \frac{V_0}{L}\left[a - \frac{L}{2®}\left(\sin\frac{2®\left(L+a\right)}{2L} - \sin\frac{2®\left(L-a\right)}{2L}\right)\right]$$

$$= \frac{V_0}{L}\left[\frac{L}{100} - \frac{L}{2®}\left(\sin\left(® + \frac{® a}{L}\right) - \sin\left(® - \frac{® a}{L}\right)\right)\right]$$

$$= V_0\left[\frac{1}{100} + \frac{1}{2®}\left(0.0314 + 0.0314\right)\right]$$

$$= V_0 \times 10^{-3}\left(10+10\right) = E_0 \times 10^{-3} \times \left(\frac{20}{25}\right) \Rightarrow \sim E_0 \times 10^{-3} = 0.81 \times E_0 \times 10^{-3}$$

Hence, $\sim = 0.81$

Q48. An electromagnetic plane wave is propagating with an intensity $I = 1.0\times10^{5}\,Wm^{-2}$ in a medium with $\epsilon = 3\epsilon_0$ and $\mu = \mu_0$. The amplitude of the electric field inside the medium is _________ $\times10^{3}\,Vm^{-1}$ (up to one decimal place).

($\epsilon_0 = 8.85\times10^{-12}\,C^2N^{-1}m^{-2}$, $\mu_0 = 4\pi\times10^{-7}\,NA^{-2}, c = 3\times10^{8}\,ms^{-1}$)

Topic- Electromagnetic Theory

Subtopic- EM Wave

Ans. : 6.6

Solution: $I = \frac{1}{2}v\epsilon E^2 \Rightarrow E^2 = \frac{2I}{v\epsilon} = \frac{2I}{\frac{1}{\sqrt{\mu\epsilon}}\epsilon} = 2I\sqrt{\frac{\mu}{\epsilon}}$

$$\Rightarrow E^2 = 2\times10^{5}\sqrt{\frac{\mu_0}{3\epsilon_0}} = 2\times10^{5}\sqrt{\frac{4\pi\times10^{-7}}{3\times8.8\times10^{-12}}} \approx 4363.4\times10^{4}$$

$$\Rightarrow E \approx 66\times10^{2} \approx 6.6\times10^{3}\,V/m$$

Q49. A microcanonical ensemble consists of 12 atoms with each taking either energy 0 state, or energy ϵ state. Both states are non-degenerate. If the total energy of this ensemble is 4ϵ, its entropy will be _________ k_B (up to one decimal place), where k_B is the Boltzmann constant.

Topic- Statistical Mechanics

Subtopic- Number of ways

Ans. : 6.204

Solution: The number of ways having total energy 4ϵ, out of 12 atom is

$$= {}^{12}C_4 = \frac{12!}{4!\,8!} = \frac{12\times11\times10\times9}{4\times3\times2} = 495$$

Hence, entropy, $S = k_B \ln w = k_B \ln(495) = k_B(6.204) = 6.204\,k_B$

Q50. A two-state quantum system has energy eigenvalues $\pm\epsilon$ corresponding to the normalized states $|\psi_\pm\rangle$. At time $t=0$, the system is in quantum state $\frac{1}{\sqrt{2}}\left[|\psi_+\rangle + |\psi_-\rangle\right]$. The probability that the system will be in the same state at $t = h/(6\epsilon)$ is _________ (up to two decimal places).

Topic- Quantum Mechanics

Subtopic- Postulates

Ans. : 0.25

Solution: $\left|\cdot\ (0)\right\rangle = \frac{1}{\sqrt{2}}\left[\left|\cdot\ _{+}\right\rangle + \left|\cdot\ _{-}\right\rangle\right]$

And $\left|\cdot\ (t)\right\rangle = \frac{1}{\sqrt{2}}\left[\left|\cdot\ _{+}\right\rangle e^{-\frac{i\in t}{\hbar}} + \left|\cdot\ _{-}\right\rangle e^{\frac{i\in t}{\hbar}}\right]$

At $t = \frac{\hbar}{6\in}$,

$$\left|\cdot\ (t)\right\rangle = \frac{1}{\sqrt{2}}\left[\left|\cdot\ _{+}\right\rangle e^{-\frac{i\in h\times 2®}{6\in h}} + \left|\cdot\ \right\rangle e^{\frac{i\in h\times 2®}{6\in h}}\right] = \frac{1}{\sqrt{2}}\left[\left|\cdot\ _{+}\right\rangle e^{\frac{-i®}{3}} + \left|\cdot\ _{-}\right\rangle e^{\frac{i®}{3}}\right]$$

Now, probability in same state

$$P = \frac{\left|\left\langle\cdot\ (t)\right|\cdot\ (0)\right\rangle\right|^2}{\left\langle\cdot\ \middle|\ \cdot\right\rangle} = \frac{1}{4}\left|e^{-i®/3} + e^{i®/3}\right|^2 = \frac{1}{4}\left|2\cos\frac{®}{3}\right|^2 = \frac{1}{4}\times\left|2\times\frac{1}{2}\right|^2 = 0.25$$

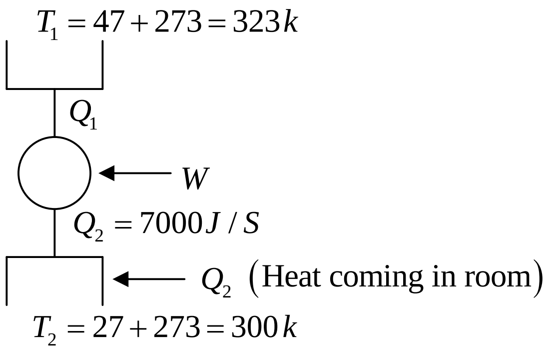
Q51. An air-conditioner maintains the room temperature at 27^0C while the outside temperature is 47^0C. The heat conducted through the walls of the room from outside to inside due to temperature difference is $7000\ W$. The minimum work done by the compressor of the air-conditioner per unit time is__________ W.

Topic- Thermodynamics

Subtopic- Refrigerator

Ans. : 466.67

Solution: $Q_2 + W = Q_1$



Coefficient of performance of refrigerator $(AC) = \frac{Q_2}{W}$

Also, coefficient of performance of refrigerator, $= \frac{T_2}{T_1 - T_2} \Rightarrow \frac{300}{47-27} = \frac{7000}{W}$

$$\Rightarrow W = \frac{7000\times 20}{300}\ J/s = \frac{1400}{3} = 466.67\ W$$

Q52. Two solid spheres A and B have same emissivity. The radius of A is four times the radius of B and temperature of A is twice the temperature of B. The ratio of the rate of heat radiated from A to that from B is __________.

Topic- Statistical Mechanics

Subtopic- Black Body Radiation

Ans. : 256

Solution: $\dfrac{\text{Rate of heat radiation from solid sphere}(A)}{\text{Rate of heat radiation from solid sphere}(B)}=\dfrac{4® R_A^2T_A^4}{4® R_B^2T_B^4}$

$\because\ R_A=4R_B$ and $T_A=2T_B$

$$=\frac{4® R_A^2T_A^4}{4® R_B^2T_B^4}=\frac{(4R_B)^2\times(2T_B)^4}{(R_B)^2\times(T_B)^4}=16\times16=256$$

Q53. The partition function of an ensemble at a temperature T is

$$Z=\left(2\cosh\frac{£}{k_BT}\right)^N$$

where k_B is the Boltzmann constant. The heat capacity of this ensemble at $T=\dfrac{£}{k_B}$ is $X\,Nk_B$, where the value of X is __________ (up to two decimal places).

Topic- Statistical Mechanics

Subtopic- Canonical Ensemble

Ans. : 0.42

Solution: The partition function, $z=\left[2\cosh\left(\dfrac{£}{k_BT}\right)\right]^N$

The average energy, $\langle E\rangle=k_BT^2\dfrac{\partial(\ln z)}{\partial T}$

$$=\frac{Nk_BT^2\left[2\sinh\left(\frac{£}{k_BT}\right)\right]\left(\frac{-£}{k_BT^2}\right)}{2\cosh\left(\frac{£}{k_BT}\right)}=-N£\tanh\left(\frac{£}{k_BT}\right)$$

$$C=\frac{d\langle E\rangle}{dT}=-N£\,\mathrm{sec}\,h^2\left(\frac{£}{k_BT}\right)\cdot\left(\frac{-£}{k_BT^2}\right)$$

At $T=\dfrac{£}{k}$, $C=\dfrac{N£^2}{k\cdot(£^2/k^2)}\mathrm{sec}\,h^2(1)=Nk\,\mathrm{sec}\,h^2(1)=0.42Nk_B$

Q54. An atom in its singlet state is subjected to a magnetic field. The Zeeman splitting of its $650\ nm$ spectral line is $0.03\ nm$. The magnitude of the field is ___________ Tesla (up to two decimal places).

$(e=1.60\times10^{-19}C, m_e=9.11\times10^{-31}kg, c=3.0\times10^8ms^{-1})$

Topic- Atomic Physics

Subtopic- Zeeman effect

Ans. : 1.52

Solution: $\Delta\lambda = \frac{\lambda^2}{c} \times \frac{eB}{4\pi m}$

$$\Rightarrow B = \frac{c}{\lambda^2} \cdot \frac{4\pi m}{e} \Delta\lambda = \frac{3\times10^8}{\left(650\times10^{-9}\right)^2} \cdot \frac{4\pi \times 9.1\times10^{-31}}{1.6\times10^{-19}} \cdot \left(0.03\times10^{-9}\right) = 1.52T$$

Q55. The quantum effects in an ideal gas become important below a certain temperature T_Q when de Broglie wavelength corresponding to the root mean square thermal speed becomes equal to the inter-atomic separation. For such a gas of atoms of mass $2\times10^{-26}\,kg$ and number density $6.4\times10^{25}\,m^{-3}$, $T_Q =$ ____________ $\times10^{-3}\,K$ (up to one decimal place).

($k_B = 1.38\times10^{-23}\,J/K, h = 6.6\times10^{-34}\,J\text{-s}$)

Topic- Quantum Mechanics

Subtopic- De-Broglie Hypothesis

Ans. : 84.2

Solution: $\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2m\frac{3}{2}kT}} = \frac{h}{\sqrt{3mkT}}$

At $T = T_Q$, $\lambda = a$

$$\therefore \frac{h}{\sqrt{3mkT_Q}} = a \Rightarrow T_Q = \frac{h^2}{3mka^2}$$

where $\frac{1}{a^3} = 6.4\times10^{25}\,m^{-3} \Rightarrow a = 2.5\times10^{-9}\ m$

$$\therefore T_Q = \frac{\left(6.6\times10^{-34}\,J\text{-}s\right)^2}{3\times2\times10^{-26}\,kg\times1.38\times10^{-23}\,J/k\times\left(2.5\times10^{-9}\,m\right)^2}$$

$$= 0.0842\,K = 84.2\times10^{-3}\ K$$