JEST 2016

Part-A: 1-Mark Questions

Q1. Given a matrix $M=\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}$, which of the following represents $\cos\left(\frac{\pi M}{6}\right)$

(a) $\frac{1}{2}\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}$ (b) $\frac{\sqrt{3}}{4}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$ (c) $\frac{\sqrt{3}}{4}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$ (d) $\frac{1}{2}\begin{pmatrix}1 & \sqrt{3}\\ \sqrt{3} & 1\end{pmatrix}$

Q2. The wavefunction of a hydrogen atom is given by the following superposition of energy eigen functions $\psi_{nlm}(\vec{r})$(n,l,m are the usual quantum numbers):

$$\psi(\vec{r})=\frac{\sqrt{2}}{\sqrt{7}}\psi_{100}(\vec{r})-\frac{3}{\sqrt{14}}\psi_{210}(\vec{r})+\frac{1}{\sqrt{14}}\psi_{322}(\vec{r})$$

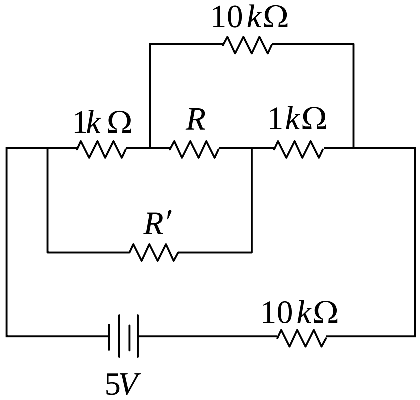
The ratio of expectation value of the energy to the ground state energy and the expectation value of L^2 are, respectively:

(a) $\frac{229}{504}$ and $\frac{12\hbar^2}{7}$ (b) $\frac{101}{504}$ and $\frac{12\hbar^2}{7}$

(c) $\frac{101}{504}$ and $\hbar^2$ (d) $\frac{229}{504}$ and $\hbar^2$

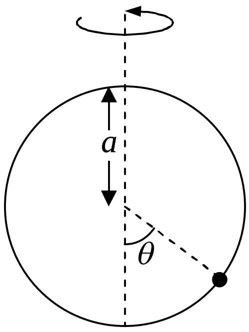
$$\langle E\rangle=\frac{2}{7}\times\frac{E_0}{1}+\frac{9}{14}\times\frac{E_0}{4}+\frac{1}{14}\times\frac{E_0}{9}=\frac{229}{504}E_0 \quad \langle L^2\rangle=\frac{2}{7}\times 0\hbar^2+\frac{9}{14}\times 2\hbar^2+\frac{1}{14}\times 6\hbar^2=\frac{24}{14}\hbar^2=\frac{12}{7}\hbar^2$$

Q3. It is found that when the resistance R indicated in the figure below is changed from $1\ k\Omega$ to $10\ k\Omega$ the current flowing through the resistance R' does not change. What is the value of the resistor R'?



(a) $5\ k\Omega$ (b) $100\ k\Omega$ (c) $10\ k\Omega$ (d) $1\ k\Omega$

Q4. A hoop of radius a rotates with constant angular velocity ω about the vertical axis as shown in the figure. A bead of mass m can slide on the hoop without friction. If $g < \omega^2 a$ at what angle θ apart from 0 and π is the bead stationary (i.e., $\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} = 0$)?



(a) $\tan\theta = \frac{\pi g}{\omega^2 a}$
(b) $\sin\theta = \frac{g}{\omega^2 a}$
(c) $\cos\theta = \frac{g}{\omega^2 a}$
(d) $\tan\theta = \frac{g}{\pi\omega^2 a}$

Q5. A spin-$\frac{1}{2}$ particle in a uniform external magnetic field has energy eigenstates $|1\rangle$ and $|2\rangle$. The system is prepared in ket-state $\frac{(|1\rangle + |2\rangle)}{\sqrt{2}}$ at time t = 0. It evolves to the state described by the ket $\frac{(|1\rangle - |2\rangle)}{\sqrt{2}}$ in time T. The minimum energy difference between two levels is:

(a) $\frac{h}{6T}$
(b) $\frac{h}{4T}$
(c) $\frac{h}{2T}$
(d) $\frac{h}{T}$

Q6. You receive on avenge 5 emails per day during a 365-days year. The number of days on average on which you do not receive any emails in that year are:

(a) more than 5
(b) More than 2
(c) 1
(d) None of the above

Q7. The H_2 molecule has a reduced mass $M = 8.35 \times 10^{-28}\ kg$ and an equilibrium internuclear distance $R = 0.742 \times 10^{-10}\ m$. The rotational energy in terms of the rotational quantum number J is

(a) $E_{rot}(J) = 7J(J-1)\ meV$
(b) $E_{rot}(J) = \frac{5}{2}J(J+1)\ meV$
(c) $E_{rot}(J) = 7J(J+1)\ meV$
(d) $E_{rot}(J) = \frac{5}{2}J(J-1)\ meV$

Q8. The maximum relativistic kinetic energy of β particles from a radioactive nucleus is equal to the rest mass energy of the particle. A magnetic field is applied perpendicular to the beam of β particles, which bends it to a circle of radius R. The field is given by:

(a) $\frac{3m_0c}{eR}$
(b) $\frac{\sqrt{2}m_0c}{eR}$
(c) $\frac{\sqrt{3}m_0c}{eR}$
(d) $\frac{\sqrt{3}m_0c}{2eR}$

Q9. The central force which results in the orbit $r = a(1+\cos\theta)$ for a particle is proportional to:

(a) r (b) r^2 (c) r^{-2} (d) None o the above

Q10. A gas of N molecules of mass m is confined in a cube of volume $V = L^3$ at temperature T. The box is in a uniform gravitational field $-g\hat{z}$. Assume that the potential energy of a molecule is $U = mgz$ where $z \in [0, L]$ is the vertical coordinate inside the box. The pressure $P(z)$ at height z is:

(a) $P(z) = \frac{N}{V}\frac{mgL}{2}\frac{\exp\left(-\frac{mg\left(z-\frac{L}{2}\right)}{k_BT}\right)}{\sinh\left(\frac{mgL}{2k_BT}\right)}$

(b) $P(z) = \frac{N}{V}\frac{mgL}{2}\frac{\exp\left(-\frac{mg\left(z-\frac{L}{2}\right)}{k_BT}\right)}{\cosh\left(\frac{mgL}{2k_BT}\right)}$

(c) $P(z) = \frac{k_BTN}{V}$

(d) $P(z) = \frac{N}{V}mgz$

Q11. A transistor in common base configuration has ratio of collector current to emitter current β and ratio of collector to base current α. Which of the following is true?

(a) $\beta = \frac{\alpha}{(\alpha+1)}$

(b) $\beta = \frac{(\alpha+1)}{\alpha}$

(c) $\beta = \frac{\alpha}{(\alpha-1)}$

(d) $\beta = \frac{(\alpha-1)}{\alpha}$

Q12. The energy of a particle is given by $E = |p| + |q|$ where p and q are the generalized momentum and coordinate, respectively. All the states with $E \le E_0$ are equally probable and states with $E > E_0$ are inaccessible. The probability density of finding the particle at coordinate q, with $q > 0$ is:

(a) $\frac{(E_0+q)}{E_0^2}$ (b) $\frac{q}{E_0^2}$ (c) $\frac{(E_0-q)}{E_0^2}$ (d) $\frac{1}{E_0}$

Q13. Consider a quantum particle of mass m in one dimension in an infinite potential well, i.e., $V(x)=0$ for $\frac{-a}{2}<x<\frac{a}{2}$ and $V(x)=\infty$ for $|x|\geq\frac{a}{2}$. A small perturbation, $V'(x)=\frac{2\in|x|}{a}$ is added. The change in the ground state energy to $O(\in)$ is:

(a) $\frac{\in}{2\pi^2}(\pi^2-4)$ (b) $\frac{\in}{2\pi^2}(\pi^2+4)$

(c) $\frac{\in\pi^2}{2}(\pi^2+4)$ (d) $\frac{\in\pi^2}{2}(\pi^2-4)$

Q14. The strength of magnetic field at the center of a regular hexagon with sides of length a carrying a steady current I is:

(a) $\frac{\mu_0 I}{\sqrt{3}\pi a}$ (b) $\frac{\sqrt{6}\mu_0 I}{\pi a}$ (c) $\frac{3\mu_0 I}{\pi a}$ (d) $\frac{\sqrt{3}\mu_0 I}{\pi a}$

Q15. An ideal gas with adiabatic exponent γ undergoes a process in which its pressure P is related to its volume V by the relation $P=P_0-\alpha V$, where P_0 and α are positive constants. The volume starts from being very close to zero and increases monotonically to $\frac{P_0}{\alpha}$. At what value of the volume during the process does the gas have maximum entropy?

(a) $\frac{P_0}{\alpha(1+\gamma)}$ (b) $\frac{\gamma P_0}{\alpha(1-\gamma)}$ (c) $\frac{\gamma P_0}{\alpha(1+\gamma)}$ (d) $\frac{P_0}{\alpha(1-\gamma)}$

Q16. A point charge q of mass m is released from rest at a distance d from an infinite grounded conducting plane (ignore gravity). How long does it take for the charge to hit the plane?

(a) $\frac{\sqrt{2\pi^3\varepsilon_0 md^3}}{q}$ (b) $\frac{\sqrt{2\pi^3\varepsilon_0 md}}{q}$

(c) $\frac{\sqrt{\pi^3\varepsilon_0 md^3}}{q}$ (d) $\frac{\sqrt{\pi^3\varepsilon_0 md}}{q}$

Q17. A two dimensional box in a uniform magnetic field B contains $\frac{N}{2}$ localised spin-$\frac{1}{2}$ particles with magnetic moment μ, and $\frac{N}{2}$ free spinless particles which do not interact with each other. The average energy of the system at a temperature T is:

(a) $3NkT-\frac{1}{2}N\mu B\sinh\left(\frac{\mu B}{k_B T}\right)$ (b) $NkT-\frac{1}{2}N\mu B\tanh\left(\frac{\mu B}{k_B T}\right)$

(c) $\frac{1}{2}NkT-\frac{1}{2}N\mu B\tanh\left(\frac{\mu B}{k_B T}\right)$ (d) $\frac{3}{2}NkT+\frac{1}{2}N\mu B\cosh\left(\frac{\mu B}{k_B T}\right)$

Q18. If $Y_{xy} = \frac{1}{\sqrt{2}}\left(Y_{22} - Y_{2,-2}\right)$ where $Y_{l,m}$ are spherical harmonics then which of the following is true?

(a) Y_{xy} is an eigenfunction of both L^2 and L_z

(b) Y_{xy} is an eigenfunction of L^2 but not L_z

(c) Y_{xy} is an eigenfunction both of L_z but not L^2

(d) Y_{xy} is not an eigenfunction of either L^2 and L_z

Q19. The value of the integral $\int_0^{\infty} \frac{\ln x}{\left(x^2+1\right)} dx$

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$ (c) π^2 (d) 0

Q20. A spin-1 particle is in a state $|\psi\rangle$ described by the column matrix $\left(\frac{1}{\sqrt{10}}\right)\left\{2, \quad \sqrt{2}, \quad 2i\right\}$ in the S_z basis. What is the probability that a measurement of operator S_z will yield the result h for the state $S_x|\psi\rangle$?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

Q21. Consider N non-interacting electrons $\left(N \sim N_A\right)$ in a box of sides L_x, L_y, L_z Assuming that the dispersion relation is $\in\left(k\right) = Ck^4$ where C is a constant, the ratio of the ground state energy per particle to the Fermi energy is:

(a) $\frac{3}{7}$ (b) $\frac{7}{3}$ (c) $\frac{3}{5}$ (d) $\frac{5}{7}$

Q22. The Hamiltonian of a quantum particle of mass in confined to a ring of unit radius is:

$$H = \frac{\hbar^2}{2m}\left(-i\frac{\partial}{\partial\theta} - \alpha\right)^2$$

where θ is the angular coordinate, α is a constant. The energy eigenvalues and eigenfunctions of the particle are (n is an integer):

(a) $\psi_n\left(\theta\right) = \frac{e^{in\theta}}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}\left(n-\alpha\right)^2$

(b) $\psi_n\left(\theta\right) = \frac{\sin\left(n\theta\right)}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}\left(n-\alpha\right)^2$

(c) $\psi_n\left(\theta\right) = \frac{\cos\left(n\theta\right)}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}\left(n-\alpha\right)^2$

(d) $\psi_n\left(\theta\right) = \frac{e^{in\theta}}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}\left(n+\alpha\right)^2$

Q23. The sum of the infinite series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots$ is

(a) 2π (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

Q24. Light takes approximately 8 minutes to travel from the Sun to the Earth. Suppose in the frame of the Sun an event occurs at $t=0$ at the Sun and another event occurs on Earth at $t=1$ minute. The velocity of the inertial frame in which both these events are simultaneous is:

(a) $\frac{c}{8}$ with the velocity vector pointing from Earth to Sun

(b) $\frac{c}{8}$ with the velocity vector pointing from Sun to Earth

(c) The events can never be simultaneous - no such frame exists

(d) $c\sqrt{1-\left(\frac{1}{8}\right)^2}$ with velocity vector Pointing from to Earth

Q25. A spherical shell of radius R carries a constant surface charge density σ and is rotating about one of its diameters with an angular velocity ω. The magnitude of the magnetic moment of the shell is:

(a) $4\pi\sigma\omega R^4$ (b) $\frac{4\pi\sigma\omega R^4}{3}$ (c) $\frac{4\pi\sigma\omega R^4}{15}$ (d) $\frac{4\pi\sigma\omega R^4}{9}$

Part-B: 1-Mark Questions

Q1. The ad joint of a differential operator $\frac{d}{dx}$ acting on a wavefunction $\psi(x)$ for a quantum mechanical system is:

(a) $\frac{d}{dx}$ (b) $-i\hbar\frac{d}{dx}$ (c) $-\frac{d}{dx}$ (d) $i\hbar\frac{d}{dx}$

Q2. In Millikan's oil-drop experiment an oil drop of radius r, mass m and charge $q=\frac{6\pi\eta r\left(v_1+v_2\right)}{E}$ is moving upwards with a terminal velocity v_2 due to an applied electric field of magnitude E, where η is the coefficient of viscosity. The acceleration due to gravity is given

(a) $g=\frac{6\pi\eta r v_1}{m}$ (b) $g=\frac{3\pi\eta r v_1}{m}$

(c) $g=\frac{6\pi\eta r v_2}{m}$ (d) $g=\frac{3\pi\eta r v_2}{m}$

Q3. The electric field $\vec{E} = E_0 \sin(\omega t - kz)\hat{x} + 2E_0 \sin\left(\omega t - kz + \frac{\pi}{2}\right)\hat{y}$ represents:

(a) a linwearly polarized wave

(b) a right-hand circularly polarized wave

(c) a left-hand circularly polarized wave

(d) an elliptically polarized wave

Q4. An ideal gas has a specific heat ratio $\frac{C_P}{C_V} = 2$. Starting at a temperature T_1 the gas under goes an isothermal compression to increase its density by a factor of two. After this an adiabatic compression increases its pressure by a factor of two. The temperature of the gas at the end of the second process would be:

(a) $\frac{T_1}{2}$ (b) $\sqrt{2}T_1$ (c) $2T_1$ (d) $\frac{T_1}{\sqrt{2}}$

Q5. Suppose yz plane forms the boundary between two linear dielectric media I and II with dielectric constant $\in_I = 3$ and $\in_{II} = 4$, respectively. If the electric field in region I at the interface is given by $\vec{E}_I = 4\hat{x} + 3\hat{y} + 5\hat{z}$, then the electric field $\vec{E}_{II}$ at the interface in region II is:

(a) $4\hat{x} + 3\hat{y} + 5\hat{z}$ (b) $4\hat{x} + 0.75\hat{y} - 1.25\hat{z}$

(c) $-3\hat{x} + 3\hat{y} + 5\hat{z}$ (d) $3\hat{x} + 3\hat{y} + 5\hat{z}$

Q6. Given the condition $\nabla^2\phi = 0$, the solution of the equation $\nabla^2\psi = k\vec{\nabla}\phi.\vec{\nabla}\phi$ is given by

(a) $\psi = \frac{k\phi^2}{2}$ (b) $\psi = k\phi^2$ (c) $\psi = \frac{k\phi \ln\phi}{2}$ (d) $\psi = \frac{k\phi \ln\phi}{2}$

Q7. Circular fringes are obtained with a Michelson interferometer using $600\ nm$ laser light. What minimum displacement of one mirror will make the central fringe from bright to dark?

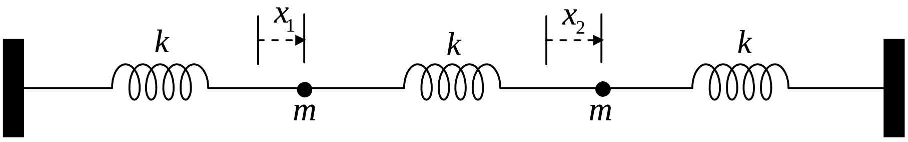
(a) $600\ nm$ (b) $300\ nm$ (c) $150\ nm$ (d) $120\ \mathring{A}$

Q8. If $\vec{k}$ is the wavevector of incident light ($|\vec{k}| = \frac{2\pi}{\lambda}$, λ is the wavelength of light) and $\vec{G}$ is a reciprocal lattice vector, then the Bragg's law can be written as:

(a) $\vec{k} + \vec{G} = 0$ (b) $2\vec{k}.\vec{G} + G^2 = 0$

(c) $2\vec{k}.\vec{G} + k^2 = 0$ (d) $\vec{k}.\vec{G} = 0$

Q9. For the coupled system shown in the figure, the normal coordinates are $x_1 + x_2$ and $x_1 - x_2$ corresponding to the normal frequencies ω_0 and $\sqrt{3}\omega_0$ respectively.



At $t = 0$, the displacements are $x_1 = A$, $x_2 = 0$, and the velocities are $v_1 = v_2 = 0$. The displacement of the second particle at time t is given by:

(a) $x_2(t) = \frac{A}{2}\left(\cos(\omega_0 t) + \cos\left(\sqrt{3}\omega_0 t\right)\right)$ (b) $x_2(t) = \frac{A}{2}\left(\cos(\omega_0 t) - \cos\left(\sqrt{3}\omega_0 t\right)\right)$

(c) $x_2(t) = \frac{A}{2}\left(\sin(\omega_0 t) - \sin\left(\sqrt{3}\omega_0 t\right)\right)$ (d) $x_2(t) = \frac{A}{2}\left(\sin(\omega_0 t) - \frac{1}{\sqrt{3}}\sin\left(\sqrt{3}\omega_0 t\right)\right)$

Q10. How much force does light from a $1.8\ W$ laser exert when it is totally absorbed by an object?

(a) $6.0\times10^{-9}\ N$ (b) $0.6\times10^{-9}\ N$ (c) $0.6\times10^{-8}\ N$ (d) $4.8\times10^{-9}\ N$

Q11. An electron confined within a thin layer of semiconductor may be treated as a free particle inside an infinitely deep one-dimensional potential well. If the difference in energies between the first and the second energy levels is δE, then the thickness of the layer is:

(a) $\sqrt{\frac{3\hbar^2\pi^2}{2m\delta E}}$ (b) $\sqrt{\frac{2\hbar^2\pi^2}{3m\delta E}}$ (c) $\sqrt{\frac{\hbar^2\pi^2}{2m\delta E}}$ (d) $\sqrt{\frac{\hbar^2\pi^2}{m\delta E}}$

Q12. The half-life of a radioactive nuclear source is 9 days. The fraction of nuclei which are left under cayed after 3 days is:

(a) $\frac{7}{8}$ (b) $\frac{1}{3}$ (c) $\frac{5}{6}$ (d) $\frac{1}{2^{\frac{1}{3}}}$

Q13. Self inductance per unit length of a long solenoid of radius R with n turns per unit length is:

(a) $\mu_0\pi R^2 n^2$ (b) $2\mu_0\pi R^2 n$

(c) $2\mu_0\pi R^2 n^2$ (d) $\mu_0\pi R^2 n$

Q14. A gas contains particles of type A with fraction 0.8, and particles of type B with fraction 0.2. The probability that among 3 randomly chosen particles at least one is of type A is:

(a) 0.8 (b) 0.25 (c) 0.33 (d) 0.992

Q15. The number of different Bravais lattices possible in two dimensions is:

(a) 2 (b) 3 (c) 5 (d) 6

Part-C: 3-Mark Questions

Q1. The output intensity I of radiation from a single mode of resonant cavity obeys

$$\frac{d}{dt}I = -\frac{\omega_0}{Q}I$$

where Q is the quality factor of the cavity ω_0 is the resonant frequency. The form of the frequency spectrum of the output is:

(a) Delta function (b) Gaussian (c) Lorentzian (d) Exponential

Q2. For a quantum mechanical harmonic oscillator with energies, $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$, where $n = 0, 1, 2...$, the partition function is:

(a) $\dfrac{e^{\frac{\hbar\omega}{k_BT}}}{e^{\frac{\hbar\omega}{2k_BT-1}}}$ (b) $e^{\frac{\hbar\omega}{2k_BT}} - 1$ (c) $e^{\frac{\hbar\omega}{2k_BT}} + 1$ (d) $\dfrac{e^{\frac{\hbar\omega}{2k_BT}}}{e^{\frac{\hbar\omega}{k_BT-1}}}$

Q3. If the direction with respect to a right-handed cartesian coordinate system of the ket vector $|z,+\rangle$ is (0. u. I), then the direction of the ket vector obtained by application of rotations: $\exp\left(-i\sigma_z \frac{\pi}{2}\right)\exp\left(i\sigma_y \frac{\pi}{4}\right)$, on the ket $|z,+\rangle$ is (σ_y, σ_z are the Pauli matrices):

(a) $(0,\ 1,\ 0)$ (b) $(1,\ 0,\ 0)$

(c) $\dfrac{(1,\ 1,\ 0)}{\sqrt{2}}$ (d) $\dfrac{(1,\ 1,\ 1)}{\sqrt{3}}$

Q4. In the ground state of hydrogen atom, the most probable distance of the electron from the nucleus, in units of Bohr radius a_0 is:

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) $\frac{3}{2}$

Q5. For operators P and Q, the commutator $\left[P, Q^{-1}\right]$ is

(a) $Q^{-1}[P,Q]Q^{-1}$ (b) $-Q^{-1}[P,Q]Q^{-1}$ (c) $Q^{-1}[P,Q]Q$ (d) $-Q[P,Q]Q^{-1}$

Q6. The mean value of random variable x with probability density $p(x) = \frac{1}{\sigma\sqrt{2\pi}}.\exp\left[-\frac{(x^2+\mu x)}{(2\sigma^2)}\right]$ is:

(a) 0 (b) $\frac{\mu}{2}$ (c) $\frac{-\mu}{2}$ (d) σ

Q7. A spin $\frac{1}{2}$ particle is in a state $\frac{(|\uparrow\rangle+|\downarrow\rangle)}{\sqrt{2}}$ where $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of S_z operator. The expectation value of the spin angular momentum measured along x direction is:

(a) $\hbar$ (b) $-\hbar$ (c) 0 (d) $\frac{\hbar}{2}$

Q8. A semicircular piece of paper is folded to make a cone with the centre of the semicircle as the apex. The half-angle of the resulting cone would be:

(a) 90^{o} (b) 60^{o} (c) 45^{o} (d) 30^{o}

Q9. If the Rydberg constant of an atom of finite nuclear mass is αR_{∞}, where R_{∞} the Rydberg constant corresponding to an infinite nuclear mass, the ratio of the electronic to nuclear mass of the atom is:-

(a) $\frac{(1-\alpha)}{\alpha}$ (b) $\frac{(\alpha-1)}{\alpha}$ (c) $(1-\alpha)$ (d) $\frac{1}{\alpha}$

Q10. A cylindrical shell of mass in has an outer radius b and an inner radius a. The moment of inertia of the shell about the axis of the cylinder is:

(a) $\frac{1}{2}m(b^2-a^2)$ (b) $\frac{1}{2}m(b^2+a^2)$ (c) $m(b^2+a^2)$ (d) $m(b^2-a^2)$