GS-2015

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

Written Test in **PHYSICS** - December 14, 2014

Instructions for all candidates appearing for Ph.D. or Integrated Ph.D. Programme in Physics

PLEASE READ THESE INSTRUCTIONS CAREFULLY BEFORE YOU ATTEMPT THE QUESTIONS

1. Please fill in details about name, reference code etc. on the question paper and answer sheet. The Answer Sheet is machine-readable. Use only black/blue ball point pen to fill in the answer sheet.

2. This test consists of three parts, Section A, Section B and Section C. You must answer questions according to the program you are applying for:

Candidates Applying For	Must Answer	Should Not Attempt
Integrated Ph.D.	Section A and Section B	Section C
Ph.D.	Section A and Section C	Section B

Section A contains 25 questions, Section B and Section C contain 15 questions each. Note that the test contains multiple-choice questions as well as numerical-type questions (36-40 in section B and 51-55 in section C) where you have to fill in numbers in the answer-sheet.

4. Indicate your ANSWER ON THE ANSWER SHEET by filling in the appropriate circle or circles completely for each question.

For multiple-choice questions, only ONE of the options given at the end of each question is correct. Do not mark more than one circle for any multiple choice question : this will be treated as a wrong answer.

For number-type questions, the answers should be indicated on the answer sheet by filling in circles for appropriate numbers. All three circles for each answer should be filled to get the credit. Detailed instructions are given inside the question paper.

5. The marking shall be as follows:

	Multiple-choice	Numerical
If the answer is **correct:**	+3	+5
If the answer is **incorrect:**	–1	0
If the answer is **not attempted:**	0	0
If **more than one circle is marked** (only for multiple-choice):	0	Not applicable

Note that there is negative marking for multiple-choice questions, but not for the numerical-type Questions

6. We advise you to first mark the correct answers on the QUESTION PAPER and then to TRANSFER these to the ANSWER SHEET only when you are sure of your choice.

7. Rough work may be done on blank pages of the question paper. If needed, you may ask for extra rough sheets from an Invigilator.

8. Use of calculators is permitted. Calculator which plots graphs is NOT allowed. Multiple-use devices such as cell phones, smartphones etc., CANNOT be used for this purpose.

9. Do NOT ask for clarifications from the invigilators regarding the questions. They have been instructed not to respond to any such inquiries from candidates. In case a correction/clarification is deemed necessary, the invigilator(s) will announce it publicly.

10. List of useful **physical constants** is given on the next page. Make sure to use **only** these values while answering the questions.

USEFUL CONSTANTS

Symbol	Name/Definition	Value
c	speed of light in vacuum	3×10^{8} m s^{-1}
$\hbar$	reduced Planck constant ($=h/2\pi$)	1.04×10^{-34} J s
G_N	gravitational constant	6.67×10^{-11} m^3 kg^{-1} s^{-2}
$M_\odot$	solar mass	1.989×10^{30} kg
ε_0	permittivity of free space	8.85×10^{-12} F m^{-1}
μ_0	permeability of free space	$4\pi\times10^{-7}$ N A^{-2}
e	electron charge (magnitude)	1.6×10^{-19} C
m_e	electron mass	9.1×10^{-31} kg
		$=0.5$ MeV$/c^2$
a_0	Bohr radius	0.51 Å
	ionisation potential of H atom	13.6 eV
N_A	Avogadro number	6.023×10^{23} mol^{-1}
k_B	Boltzmann constant	1.38×10^{-23} J K^{-1}
		$=8.6173\times10^{-5}$ eV K^{-1}
$R=N_Ak_B$	gas constant	8.31 J mol^{-1} K^{-1}
$\gamma=C_p/C_V$	ratio of specific heats: monatomic gas	1.67
	diatomic gas	1.40
σ	Stefan-Boltzmann constant	5.67×10^{-8} W m^{-2} K^{-4}
α	fine structure constant ($=e^2/4\pi\varepsilon_0\hbar c$)	$1/137$
g	acceleration due to gravity	9.8 m s^{-2}
R_E	radius of the Earth	6.4×10^{3} Km
R_S	radius of the Sun	7×10^{5} Km
m_p	proton mass ($\approx 2000\, m_e$)	1.7×10^{-27} kg
		$=938.2$ MeV$/c^2$
m_n	neutron mass ($\approx 2000\, m_e$)	1.7×10^{-27} kg
		$=939.6$ MeV$/c^2$

SECTION A

(For both Int. Ph.D. and Ph.D. candidates)

This section consists of 25 questions. All are of multiple-choice type. Mark only one option on the online interface provided to you. If more than one option is marked, it will be assumed that the question has not been attempted. A correct answer will get +3 marks, an incorrect answer will get –1 mark.

Q1. Which of the following vectors is parallel to the surface $x^2y+2xz=4$ at the point $(2,-2,3)$?

(a) $-6\hat{i}-2\hat{j}+5\hat{k}$ (b) $+6\hat{i}+2\hat{j}+5\hat{k}$

(c) $+6\hat{i}-2\hat{j}+5\hat{k}$ (d) $+6\hat{i}+2\hat{j}+5\hat{k}$

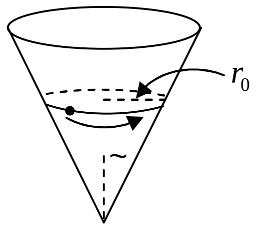
Q2. A random number generator outputs $+1$ or -1 with equal probability every time it is run. After it is run 6 times, what is the probability that the sum of the answers generated is zero? Assume that the individual runs are independent of each other.

(a) $\frac{15}{32}$ (b) $\frac{5}{16}$ (c) $\frac{5}{6}$ (d) $\frac{1}{2}$

Q3. It is required to construct the quantum theory of a particle of mass m moving in one dimension x under the influence of a constant force F . The characteristic length-scale in this problem is

(a) $\frac{mF}{\hbar^2}$ (b) $\frac{\hbar}{mF}$ (c) $\left(\frac{\hbar}{m^2F}\right)^{1/3}$ (d) $\left(\frac{\hbar^2}{mF}\right)^{1/3}$

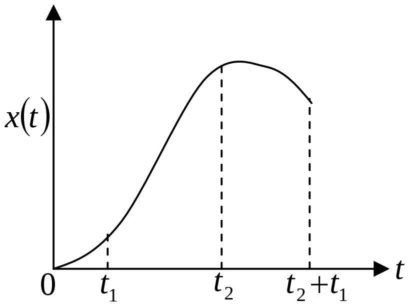
Q4. A particle slides on the inside surface of a frictionless cone. The cone is fixed with its tip on the ground and its axis vertical, as shown in the figure on the right. The semi-vertex angle of the cone is ~ . If the particle moves in a circle of radius r_0, without slipping downwards, the angular frequency μ of this motion will be

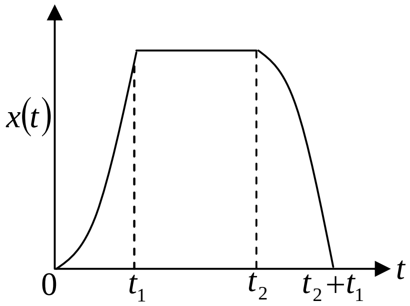


(a) $\sqrt{\frac{g}{r_0 \tan\sim}}$ (b) $\sqrt{\frac{g}{r_0 \cot\sim}}$

(c) $\sqrt{\frac{g}{r_0 \sin\sim}}$ (d) $\sqrt{\frac{g}{r_0 \cos a}}$

Q5. A car starts from rest and accelerates under a force F increasing linearly in time as $F = at$ at where a is a constant. At time $t_1 > 0,$, the force F is suddenly switched off. At a later time $t_2 > t_1$, brakes are applied resulting in a force F' whose magnitude increases linearly with time, $F' = -a(t - t_2)$ where a is the same constant as before. Which of the following graphs would best represent the change in the position of the car $x(t)$ with time?

(a)


(b)


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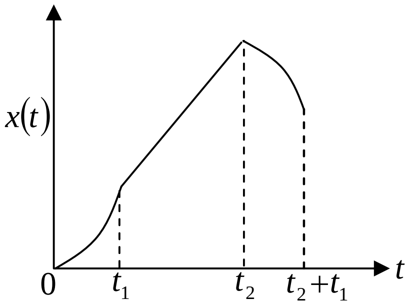

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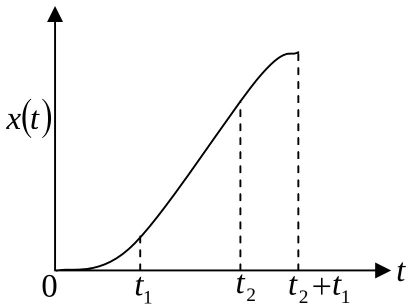
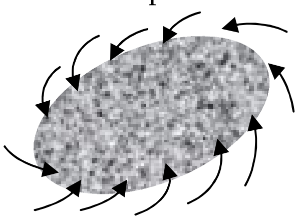
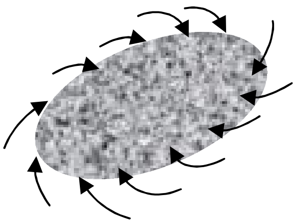
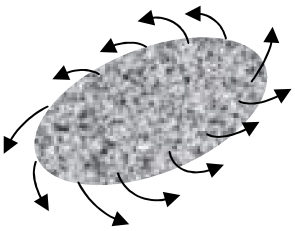


Q6. In the Earth's atmosphere, a localised low-pressure region (shaded in diagrams) develops somewhere in the southern hemisphere. Which one of the following diagrams represents the correct air flow pattern as observed from a satellite?

(a)


(b)


(c)


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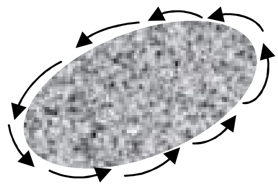


Q7. The focal length in air of a thin lens made of glass of refractive index 1.5 is ℓ. When immersed in water (refractive index $= \frac{4}{3}$), its focal length becomes

(a) $\frac{3\ell}{4}$ (b) $\frac{4\ell}{3}$ (c) $\frac{\ell}{4}$ (d) 4ℓ

Q8. A light beam of intensity I_0 passes at normal incidence through a flat plate of plastic kept in air. If reflection at the interface reduces the intensity by 20% and absorption on passing through the plate reduces the intensity by 2%, the intensity of the emergent beam will be about

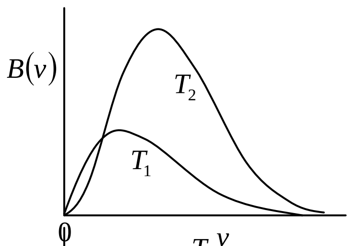
(a) $0.78I_0$ (b) $0.65I_0$ (c) $0.63I_0$ (d) $0.60I_0$

Q9. A light beam is propagating through a medium with index of refraction 1.5. If the medium is moving at constant velocity $0.7c$ in the same direction as the beam, what is the velocity of light in the medium as measured by an observer in the laboratory? ($c =$ velocity of light in vacuum)

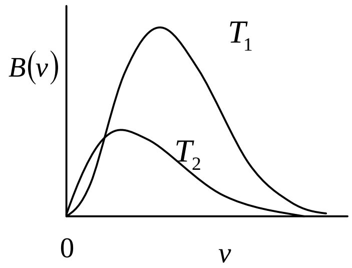
(a) $0.96c$ (b) $0.98c$ (c) $0.93c$ (d) $0.90c$

Q10. Two blackbodies radiate energy at temperatures T_1 and T_2 $(T_1 > T_2)$. The energy emitted per unit time per unit solid angle per unit surface area of a blackbody in the frequency range v to $v + dv$ is given by $B(v)dv$. Which one of the following graphs has the correct form?

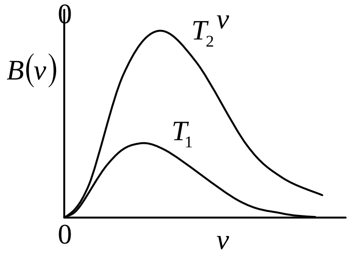
(a)



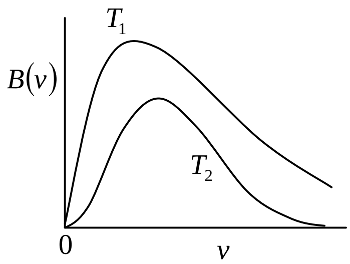
(b)



(c)



(d)

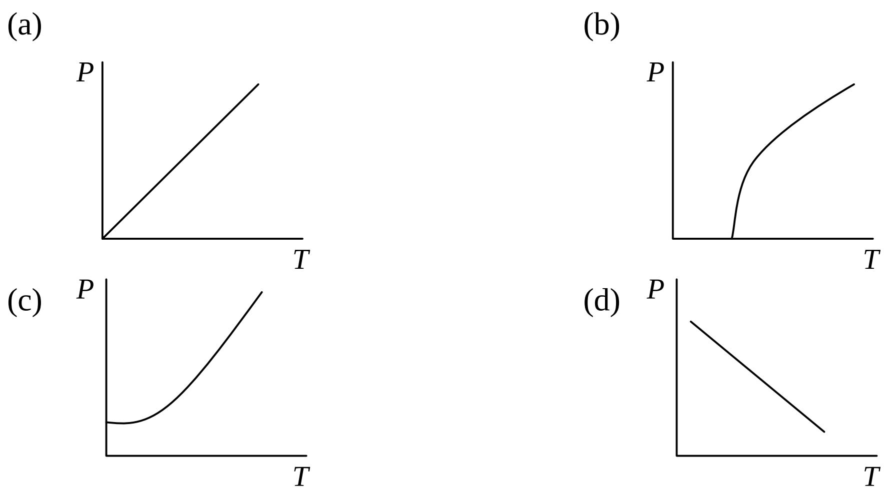


Q11. In a cold country, in winter, a lake was freezing slowly. It was observed that it took 2 hours to form a layer of ice $2\ cm$ thick on the water surface. Assuming a constant thermal conductivity throughout the layer, the thickness of ice would get doubled after

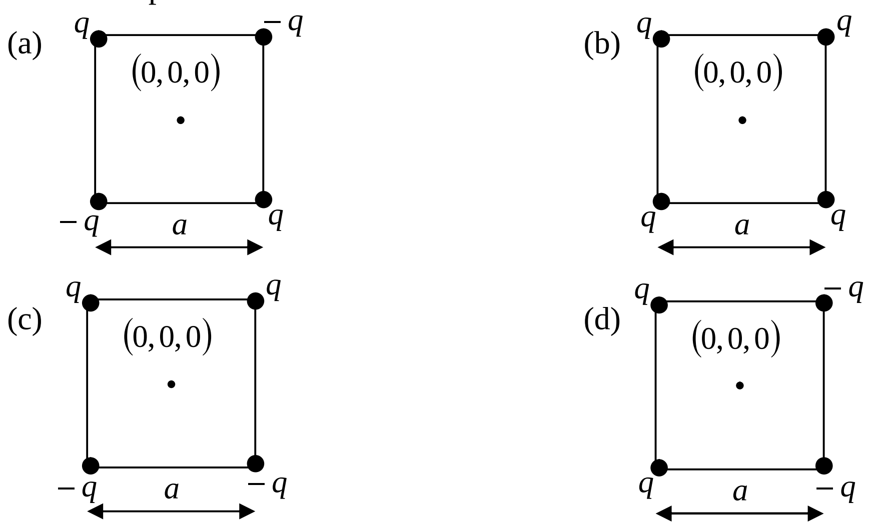
(a) 8 more hours. (b) 4 more hours.

(c) 2 more hours. (d) 6 more hours.

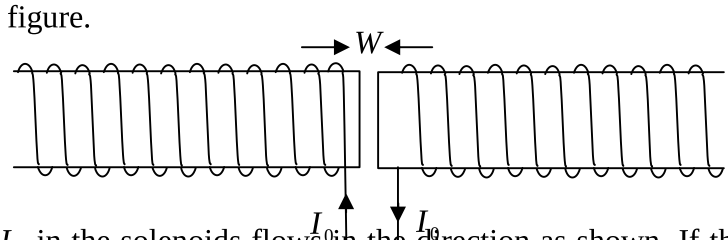
Q12. Which of the following graphs qualitatively describes the pressure P of a gas of non-interacting fermions in thermal equilibrium at a constant volume as a function of temperature?



Q13. The electrostatic potential $\phi(r)$ of a distribution of point charges has the form $\phi(r) \propto r^{-3}$ at a distance r from the origin $(0,0,0)$, where $r >> a$. Which of the following distributions can give rise to this potential?



Q14. Two semi-infinite solenoids placed next to each other are separated by a small gap of width W as shown in the figure.



The current I_0 in the solenoids flows in the direction as shown. If the solenoids have a circular cross-section of radius R and are filled with a magnetic material of permeability $\mu(\mu > \mu_0)$ then the magnetic energy densities u_i inside the solenoid and u_g in the gap are best related by

(a) $u_g > cu_i$ (b) $u_g = cu_i$ (c) $u_g < u_i$ (d) $u_g > u_i$

Q15. A light source has a small filament at the centre of a spherical glass bulb of radius $5cm$ and negligible thickness. If this source emits 100 Watts of power in the form of spherical electromagnetic waves, the $r.m.s$ electric field E at the surface of the bulb (in units of Volt/m) will be approximately

(a) 1094 (b) 109.4 (c) 15.47 (d) 1547

Q16. A particle is moving in one dimension under a potential $V(x)$ such that, for large positive values of $x, V(x) \approx kx$ where $k > 0$ and ≥ 1. If the wavefunction in this region has the form $\psi(x) \sim \exp\left(-x^{\alpha}\right)$ which of the following is true?

(a) $\alpha =$ (b) $\alpha = \frac{}{2} + 1$ (c) $\alpha = 2 \quad - 2$ (d) $\alpha = \frac{2}{2}$

Q17, The ground state energy of a particle of mass m in a three-dimensional cubical box of side ℓ is not zero but $\frac{3h^2}{8m\ell^2}$ This is because

(a) the potential at the boundaries is not really infinite, but just very large.

(b) this is the most convenient choice of the zero level of potential energy.

(c) position and momentum cannot be exactly determined simultaneously.

(d) the ground state has no nodes in the interior of the box.

Q18. A one-dimensional box contains a particle whose ground state energy is $\in$. It is observed that a small disturbance causes the particle to emit a photon of energy $hv = 8\in$, after which it is stable. Just before emission, a possible state of the particle in terms of the energy eigenstates $\{\psi_1, \psi_2, ...\}$ would be

(a) $\frac{\sqrt{2}\psi_1 - 3\psi_2 + 5\psi_5}{6}$ (b) $\frac{\psi_2 + 2\psi_3}{\sqrt{5}}$

(c) $\frac{-4\psi_4 + 5\psi_5}{\sqrt{41}}$ (d) $\frac{\psi_1 - \psi l2}{\sqrt{2}}$

Q19. A sample of ordinary hydrogen $\left({}_1^1H\right)$ gas in a discharge tube was seen to emit the usual Balmer spectrum. On careful examination, however, it was found that the H_a line in the spectrum was split into two fine lines, one an intense line at $656.28\,nm$, and the other a faint line at $656.04\ nm$. From this, one can conclude that the gas sample had a small impurity of

(a) ${}_1^3H$ (b) ${}_1^2H$ (c) ${}_2^4He$ (d) H_2O

Q20. An electron makes a transition from the valence band to the conduction band in an indirect band gap semiconductor. Which of the following is NOT true?

(a) There is no momentum change in the electron.

(b) A phonon is involved in the process.

(c) A photon is absorbed in the process.

(d) The energy of the electron increases.

Q21. Which of the following radioactive decay chains is it possible to observe?

(a) ${}^{206}_{82}Pb \rightarrow {}^{202}_{80}Hg \rightarrow {}^{202}_{79}Au$ (b) ${}^{206}_{82}Pb \rightarrow {}^{206}_{83}Bi \rightarrow {}^{206}_{82}Pb$

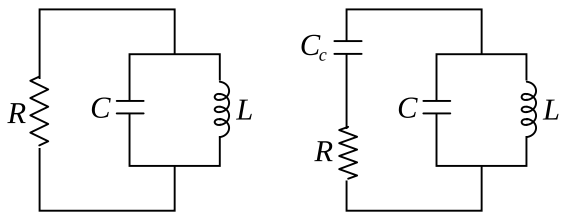
(c) ${}^{210}_{83}Bi \rightarrow {}^{210}_{84}Po \rightarrow {}^{206}_{82}Pb$ (d) ${}^{214}_{88}Ra \rightarrow {}^{210}_{86}Rn \rightarrow {}^{207}_{82}Pb$

Q22. Which of the following is the best technique for measuring the effective mass of an electron in a semiconductor?

(a) Millikan's oil drop experiment (b) Cyclotron resonance

(c) X-ray diffraction experiment (d) Resistivity measurements

Q23. Two LCR circuits (A) and (B) are shown below where $C_c << C$. At time $t = 0$, a charge Q is put on the capacitor C.



(A) (B)

Which of the following statements is correct?

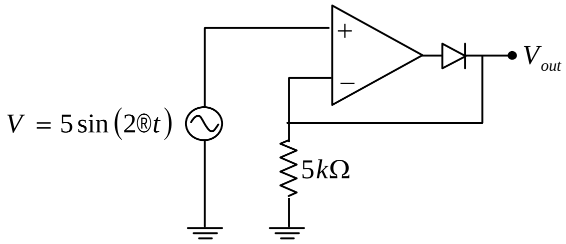
(a) The charge Q will decay at the same rate in (A) and (B)

(b) The charge Q will decay faster in (B)

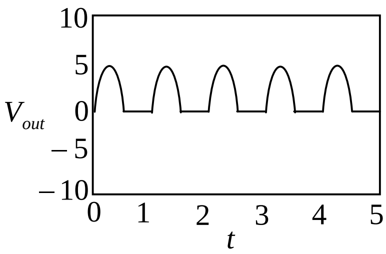
(c) The charge Q will decay faster in (A)

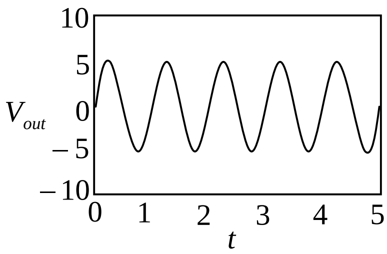
(d) The relative decay rates cannot be predicted without knowing the exact values of L, C, R and C_c

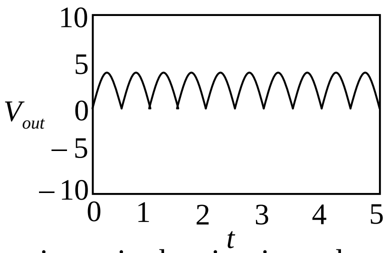
Q24. In the circuit shown below, the op-amp is powered by a bipolar supply of $\pm 10V$.

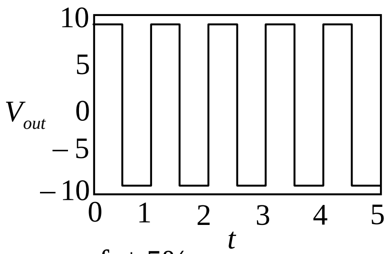


Which one of the following graphs represents V_{out} correctly?

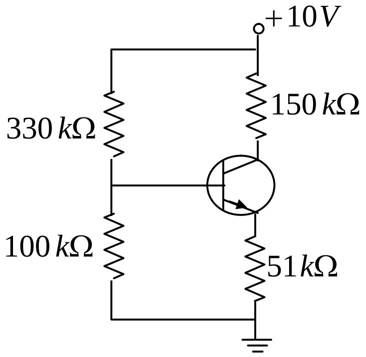
(a)


(b)


(c)


(d)


Q25. All resistors in the circuit on the right have a tolerance of $\pm 5\%$. Assuming a diode drop of $0.7V$, which of the following is the lowest possible value of the collector voltage?



(a) $4.7V$ (b) $5.2V$

(c) $3.1V$ (d) $4.1V$

Section B

Q26. Consider the differential equation

$$\frac{d^2y}{dx^2} = -4\left(y + \frac{dy}{dx}\right)$$

with the boundary condition that $y(x) = 0$ at $x = \frac{1}{5}$. When plotted as a function of x, for $x \geq 0$, we can say with certainty that the value of y

(a) first increases, then decreases to zero

(b) first decreases, then increases to zero

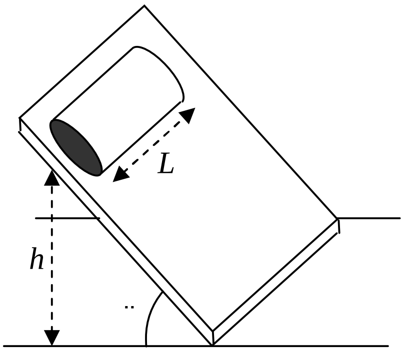
(c) has an extremum in the range $0 < x < 1$

(d) oscillates from positive to negative with amplitude decreasing to zero

Q27. In a transmission diffraction grating, there are 10^4 lines/mm. Which of the following ranges of wavelength (in nm) will produce at least one principal maximum?

(a) $1001-5000$ (b) $501-1000$ (c) $201-500$ (d) $1-200$

Q28. Two cylinders A and B of the same length L and outer radius R were placed at the same height h on an inclined plane at an angle ¨ with the horizontal (see figure). Starting from rest, each cylinder was allowed to roll down the plane without slipping. It was found that A reached the end of the inclined plane earlier than B Which of the following possibilities could be true?



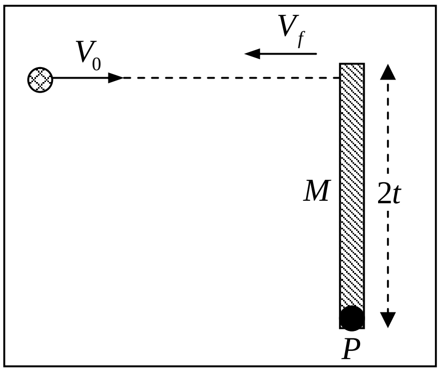
(a) A is solid and made of copper; B is hollow and made of copper.

(b) A is solid and made of copper; B is solid and made of aluminium.

(c) A is hollow and made of aluminium; B is solid and made of aluminium.

(d) A is hollow and made of copper; B is hollow and made of copper; B is heavier than A.

Q29. A thin uniform rod of length $2l$ and mass M is pivoted at one end P on a horizontal plane (see figure). A ball of mass $m << M$ and speed v_0 strikes the free end of the rod perpendicularly and bounces back with velocity v_f along the original line of motion as shown in the figure. If the collision is perfectly elastic the magnitude of v_f is



(a) $\frac{M+3m}{M-3m}v_0$

(b) $\frac{M-3m}{M+3m}v_0$

(c) $\frac{M-4m}{M+4m}v_0$

(d) $\frac{M+4m}{M-4m}v_0$

Q30. A collimated beam of pions originate from an accelerator and propagates in vacuum along a long straight beam pipe. The intensity of this beam was measured in the laboratory after a distance of $75\,m$ and found to have dropped to one-fourth of its intensity at the point of origin. If the proper half- life of a pion is $1.77\times10^{-8}\,s$, the speed of the pions in the beam, as measured in the laboratory, must be

(a) $0.99c$ (b) $0.98c$ (c) $0.96c$ (d) $0.97c$

Q31. The equation of state of a gas is given by

$$V=\frac{RT}{P}-\frac{b}{T}$$

where R is the gas constant and b is another constant parameter. The specific heat at constant pressure C_P and the specific heat at constant volume C_V for this gas is related by $C_P-C_V=$

(a) R (b) $R\left(1+\frac{RT^2}{bP}\right)^2$

(c) $R\left(1+\frac{bP}{RT^2}\right)^2$ (d) $R\left(1-\frac{bP}{RT^2}\right)^2$

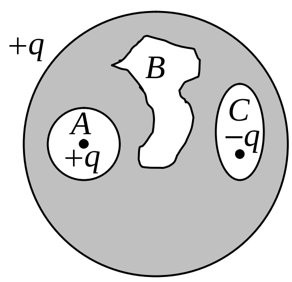
Q32. An ideal diatomic gas is initially at a temperature $T=0^{o}C$. Then it expands reversibly and adiabatically to 5 times its volume. Its final temperature will be approximately

(a) $0^{o}C$ (b) $-150^{0}C$ (c) $-130^{o}C$ (d) $-180^{o}C$

Q33. Consider an infinitely long cylinder of radius R, placed along the z-axis, which carries a static charge density °$(r)=kr$, where r is the perpendicular distance from the axis of the cylinder and k is a constant. The electrostatic potential ¤(r) inside the cylinder is proportional to

(a) $-\frac{2}{3}\left(\frac{r^3}{R^3}-1\right)$ (b) $-2\ln\left(\frac{r}{R}\right)$ (c) $-\frac{2}{3}\left(\frac{r^3}{R^3}+1\right)$ (d) $-2\ln\left(\frac{R}{r}\right)$

Q34. A solid spherical conductor encloses 3 cavities, a cross-section of which are as shown in the figure. A net charge $+q$ resides on the outer surface of the conductor. Cavities A and C contain point charges $+q$ and $-q$, respectively.



The net charges on the surfaces of these cavities are

(a) $A=-q, B=0, C=-q$ (b) $A=-q, B=q, C=0$

(c) $A=+q, B=0, C=-q$ (d) $A=-q, B=0, C=+q$

Q35. 1000 neutral spinless particles are confined in a one-dimensional box of length $100\,nm$. At a given instant of time, if 100 of these particle have energy $4\epsilon_0$ and the remaining 900 have energy $225\epsilon_0$ then the number of particles in the left half of the box will be approximately

(a) 441 (b) 100 (c) 500 (d) 625

Q36. A one-dimensional quantum harmonic oscillator is in its ground state

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{\frac{-m\omega x^2}{2\hbar}}$$

Two experiments, $[A]$ and $[B]$ are performed on the system. In $[A]$ the frequency ω of the oscillator is suddenly doubled, while in $[B]$ the frequency ω is suddenly halved. If p_A and p_B denote the probability in each case that the system is found in its new ground state immediately after the frequency change, which of the following is true?

(a) $p_A = 2p_B$ (b) $p_A = p_B$ (c) $2p_A = p_B$ (d) $p_A = \sqrt{2}p_B$

Q37. In the basic band structure theory of crystalline solids, which of the following leads to energy gaps in the allowed electronic energy values?

(a) Bragg reflection (b) Electron spin

(c) Electron-phonon interaction (d) Electron-electron interaction

Q38. The material inside a box is either a metal or a semiconductor. If $R(=1\Omega)$ is the resistance of the material, which of the following experiments CANNOT distinguish whether it is a metal or a semiconductor?

(a) Measurement of R at different temperatures.

(b) Measurement of R using power supplies of different frequencies.

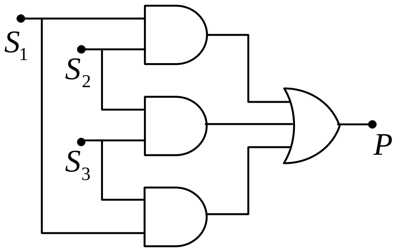
(c) Measurement of absorption spectrum in the energy range $0.1-2\,eV$.

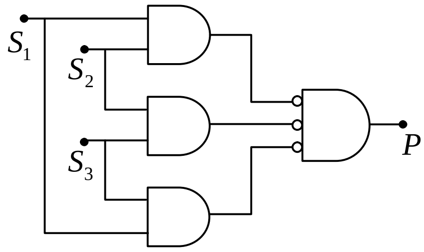
(d) Measurement of R in the presence of different magnetic fields.

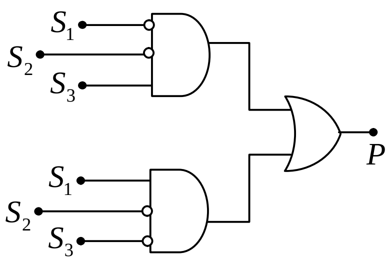
Q39. To measure the voltage in the range $0-5V$ with a precision of $5\,mV$, the minimum number of bits required in a digital voltmeter is

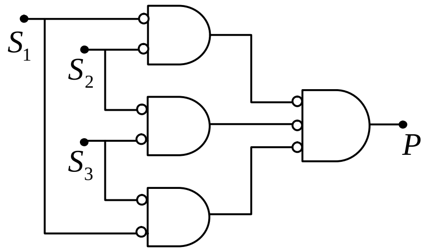
(a) 12 (b) 9 (c) 11 (d) 10

Q40. A building has three overhead water tanks, each fitted with a sensor (S_1, S_2, S_3) which goes to 0 when the water level in the tank falls below a set value and remains 1 otherwise. A common pump is used to raise water from an underground storage tank to these overhead tanks. Of the following circuits, which one will turn on $(P = 1)$ the pump only when at least two of the tanks have water level below the set value?

(a) 

(b) 

(c) 

(d) 

Section C

Q41. The integral $\int_0^{2\pi} \frac{d\theta}{1 - 2a\cos\theta + a^2}$ where $0 < a < 1$, evaluate to

(a) $\frac{2\pi}{1-a^2}$ (b) $\frac{2\pi}{1+a^2}$ (c) 2π (d) $\frac{4\pi}{1-a^2}$

Q42. The generating function for a set of polynomials in x is given by

$$f(x,t) = \left(1 - 2xt + t^2\right)^{-1}$$

The third polynomial (order x^2) in this set is

(a) $2x^2 + 1$ (b) $4x^2 + 1$ (c) $2x^2 - x$ (d) $4x^2 - 1$

Q43. A particle moves under the influence of a central potential in an orbit $r = k\theta^4$,where k is a constant and r is the distance from the origin. It follows that the angle θ varies with time t as

(a) $\theta \propto t^{\frac{1}{7}}$ (b) $\theta \propto t^{\frac{1}{8}}$ (c) $\theta \propto t^{\frac{1}{9}}$ (d) $\theta \propto t^{\frac{1}{6}}$

Q44. In a system with two degrees of freedom, if (p,q) are the canonical coordinates, then which of the following transformations to (P,Q) is canonical?

(a) $P=\frac{1}{2}\left(p^2+q^2\right), Q=\tan^{-1}\frac{2q}{p}$

(b) $P=\frac{1}{2}\left(p^2+q^2\right), Q=\cot^{-1}\frac{p}{q}$

(c) $P=\frac{1}{2}\left(p^2+q^2\right), Q=\sin^{-1}\frac{q}{2p}$

(d) $P=\frac{1}{2}\left(p^2+q^2\right), Q=\cos^{-1}\frac{p}{q}$

Q45. In a monatornic gas, the first excited state is only $1.5eV$ above the ground state, while the other excited states are much higher up. The ground state is doubly-degenerate, while the first excited state has a four-fold degeneracy. If now, the gas is heated to a temperature of $7000K$, the fraction of atoms in the excited state will be approximately

(a) 0.42 (b) 0.3 (c) 0.14 (d) 0.07

Q46. Measurement of the magnitudes of the electric field (E) and the magnetic field (B) in a plane-polarised electromagnetic wave in vacuum leads to the following results

$$\frac{\partial E}{\partial y}=-\frac{\partial B}{\partial t} \qquad \frac{\partial B}{\partial y}=-\frac{1}{c^2}\frac{\partial E}{\partial t}$$

at all points where the measurement is made. In this case the electric vector $\vec{E}$, the magnetic vector $\vec{B}$ and the wave vector $\vec{k}$ (with magnitude k) can be written in terms of the unit vectors $(\hat{x},\hat{y},\hat{z})$ along the Cartesian axes as

(a) $\vec{E}=E\hat{x},\ \vec{B}=B\hat{z},\ \vec{k}=-k\hat{y}$

(b) $\vec{E}=E\hat{x},\ \vec{B}=-B\hat{z},\ \vec{k}=k\hat{y}$

(c) $\vec{E}=E\hat{x},\ \vec{B}=B\hat{y},\ \vec{k}=k\hat{z}$

(d) $\vec{E}=-E\hat{y},\ \vec{B}=-B\hat{z},\ \vec{k}=-k\hat{x}$

Q47. A two-state quantum system has two observables A and B. It is known that the observable A has eigenstates $|\sim_1\rangle$ and $|\sim_2\rangle$ with eigenvalues a_1 and a_2 respectively, while B has eigenstates $|\ _1\rangle$ and $|\ _2\rangle$ with eigenvalues b_1 and b_2 respectively, and that these eigenstates are related by

$$|\ _1\rangle=\frac{3}{5}|\sim_1\rangle-\frac{4}{5}|\sim_2\rangle \qquad |\ _2\rangle=\frac{4}{5}|\sim_1\rangle+\frac{3}{5}|\sim_2\rangle$$

Suppose a measurement is made of the observable A and a value a_1 is obtained. If the observable B is now measured, the probability of obtaining the value b_1 will be

(a) 0.80 (b) 0.64 (c) 0.60 (d) 0.36

Q48. An rigid rotator has the wave function

$$\psi(\theta,\phi)=N\left[2iY_{1,0}(\theta,\phi)+(2+i)Y_{2,-1}(\theta,\phi)+3iY_{1,1}(\theta,\phi)\right]$$

where $Y_{l,m}(\theta,\phi)$ are the spherical harmonics, and N is a normalization constant. If $\vec{L}$ is the orbital angular momentum operator, and $L_{\pm}=L_x\pm iL_y$ the expectation value of L_+L_- is

(a) $\frac{21\hbar^2}{9}$ (b) $\frac{23\hbar^2}{9}$ (c) $\frac{25\hbar^2}{9}$ (d) 0

Q49. In the ground state electronic configuration of nitrogen $\left({}^{14}_{7}N\right)$ the L,S and J quantum numbers are

(a) $L=1, S=\frac{1}{2}, J=\frac{3}{2}$ (b) $L=0, S=\frac{3}{2}, J=\frac{3}{2}$

(c) $L=0, S=\frac{1}{2}, J=\frac{1}{2}$ (d) $L=1, S=\frac{1}{2}, J=\frac{1}{2}$

Q50. Solar radiation tends to push any particle inside solar system away from the Sun. Consider a spherical dust particle of specific gravity 6.0 and no angular momentum about the Sun. What should be its minimum radius so that it does not escape from the solar system? Take the solar luminosity to be $3.8\times10^{26}W$.

(a) $0.1\,\mu m$ (b) $10\,\mu m$ (c) $0.01\,\mu m$ (d) $10^{-6}\,\mu m$

Q51. Bosonic excitations of ferromagnets have a dispersion relation $e=\alpha k^2$ where $\in$ is the energy and k is the wavevector of the excitation. Assuming a system of such non-interacting bosonic excitations, at low temperature T, the specific heat C_V of a three-dimensional ferromagnet will be proportional to

(a) T (b) $T^{\frac{5}{2}}$ (c) T^3 (d) $T^{\frac{3}{2}}$

Q52. In the semi-empirical mass formula, the volume (V), surface (S), coulomb (C), and pairing (P) contributions to the binding energy of a nucleus ${}^{A}_{Z}X$ vary with mass number A as

(a) $V\propto A,\quad S\propto A^{\frac{-2}{3}},\quad C\propto A^{\frac{1}{3}},\quad P\propto A^{\frac{-3}{4}}$

(b) $V\propto A,\quad S\propto A^{\frac{1}{3}},\quad C\propto A^{\frac{-1}{3}},\quad P\propto A^{\frac{-3}{4}}$

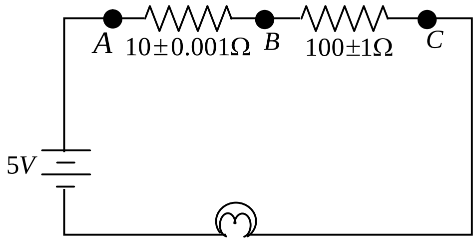
(c) $V\propto A,\quad S\propto A^{\frac{2}{3}},\quad C\propto A^{\frac{-1}{3}},\quad P\propto A^{\frac{-3}{4}}$

(d) $V\propto A^2,\quad S\propto A^{\frac{2}{3}},\quad C\propto A^{\frac{-1}{3}},\quad P\propto A^{\frac{-3}{4}}$

Q53. In an experiment, ${}^{197}_{79}Au$ nuclei were bombarded with neutrons leading to formation of ${}^{198}_{79}Au$, which is unstable. The half-life of ${}^{198}_{79}Au$ was measured to be 2.25 days and it was found later that this measured half-life was an underestimate by 10%. The corresponding percentage error in the estimated population of ${}^{198}_{79}Au$ after 9 day's is

(a) 2.5% (b) 10% (c) 15% (d) 25%

Q54. You are given the following circuit and two instruments: a voltmeter and an ammeter both with 0.001% accuracy in their readings.



Which of the following methods will result in the most accurate reading for the current without interrupting the current in the circuit?

(a) Use voltmeter to measure voltage across points A and B

(b) Use the ammeter to measure current at point B

(c) Use voltmeter to measure voltage across points B and C

(d) Use voltmeter to measure voltage across points A and C

Q55. Consider the following reactions involving elementary particles:

(A) $® ^{-} + p \rightarrow K^{-} + \Sigma^{+}$

(B) $K^{-} + p \rightarrow K^{-} + p^{+}$

Which of the following statements is true for strong interactions?

(a) (A) and (B) are both forbidden

(b) (B) is allowed but (A) is forbidden

(c) (A) is allowed but (B) is forbidden

(d) (A) and (B) are both allowed