GS-2020

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

Written Test in **PHYSICS** - December 8, 2019

Instructions for all candidates appearing for Ph.D. or Integrated Ph.D. Programme in Physics

PLEASE READ THESE INSTRUCTIONS CAREFULLY BEFORE YOU ATTEMPT THE QUESTIONS

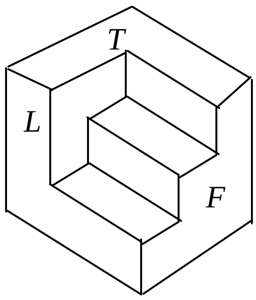
- You may NOT keep with you any books, papers, mobile phones or any electronic devices which can be used to get/store information. Use of scientific, non-programmable calculators is permitted. Calculators which plot graphs are NOT allowed. Multiple-use devices, such as smart phones, etc. CANNOT be used as calculators.
- This test consists of TWO sections.
  - SECTION A comprises 25 questions, numbered Q1-Q25. These are questions on basic topics.
  - SECTION B comprises 15 questions, numbered Q1-Q15. These may require somewhat more thought/knowledge.
- All questions are Multiple-Choice Type. In each case, ONLY ONE option is correct. Answer them by clicking the radio button next to the relevant option.
- If your calculated answer does not match any of the given options exactly, you may mark the closest one if it is reasonably close.
- The grading scheme will be as follows:
  Section A: +3 marks if correct; -1 mark if incorrect; 0 marks if not attempted
  Section B: +5 marks if correct; 0 marks if incorrect or not attempted, i.e. NO negative marks.
- The Invigilators will supply you with paper sheets for rough work.
- Do NOT ask the invigilators for clarifications regarding the questions. They have been instructed not to respond to any such queries. In case a correction/clarification is deemed necessary, it will be announced in the examination hall.
- You can get a list of useful physical constants by clicking on the link Useful Data. Make sure to use only these values in answering the questions, especially where the options are numerical.

SECTION A

(For both Int. Ph.D. and Ph.D. candidates)

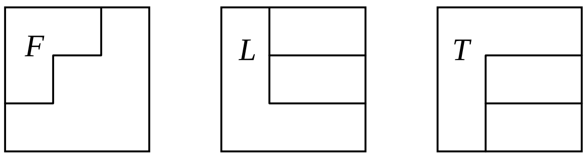
This section consists of 25 questions. All are of multiple-choice type. Mark only one option on the online interface provided to you. If more than one option is marked, it will be assumed that the question has not been attempted. A correct answer will get +3 marks, an incorrect answer will get –1 mark.

Q1. A three-dimension view of a solid is sketched below:

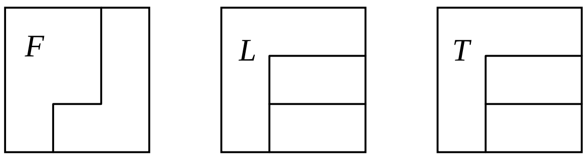


The three projections below are each intended to show the solid from its front (F), left side (L) and top (T), as marked in the figure. Which one is correct?

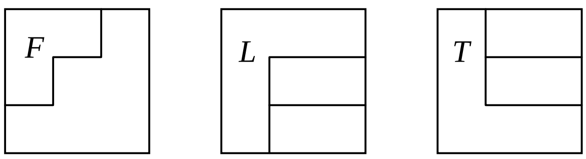
(a)



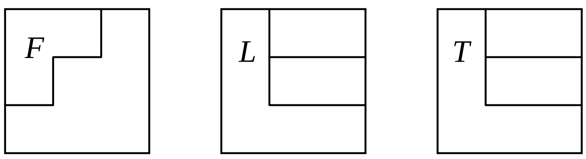
(b)



(c)



(d)



Q2. The limit $\lim\limits_{x\to\infty} x\log\dfrac{x+1}{x-1}$

Evaluate to

(a) 2 (b) 0 (c) ∞ (d) 1

Q3. The eigenvector e_1 corresponding to the smallest eigenvalue of the matrix

$$\begin{pmatrix} 2a^2 & a & 0 \\ a & 1 & a \\ 0 & a & 2a^2 \end{pmatrix}$$

where $a=\sqrt{\dfrac{3}{2}}$ is given (in terms of its transpose) by

(a) $e_1^T=\dfrac{1}{2}\left(\dfrac{1}{\sqrt{2}} -\sqrt{3} \dfrac{1}{\sqrt{2}}\right)$

(b) $e_1^T=\dfrac{1}{2}\left(\sqrt{\dfrac{3}{2}} \quad 1 \quad \sqrt{\dfrac{3}{2}}\right)$

(c) $e_1^T=\dfrac{1}{\sqrt{2}}\begin{pmatrix}1 & 0 & -1\end{pmatrix}$

(d) $e_1^T=\dfrac{1}{\sqrt{2}}\begin{pmatrix}1 & 0 & 1\end{pmatrix}$

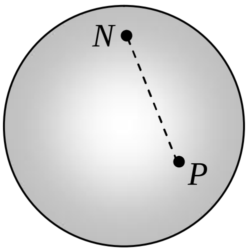
Q4. Consider the improper differential

$$ds=\left(1+y^2\right)dx+xydy$$

An integrating factor for this is

(a) $-x$ (b) $1+x^2$ (c) xy (d) $-1+y^2$

Q5. Consider a sphere of radius R, with the north pole N marked as shown in the figure below. The r.m.s. distance (straight line cutting through the sphere) of a point P on the sphere from this north pole N is given by



(a) R (b) $2\sqrt{\dfrac{2}{5}}R$ (c) $\sqrt{4\circledR}\,R$ (d) $\sqrt{2}R$

Q6. Consider a satellite orbiting the Earth in a circular orbit, as sketched in the figure on the right (not to scale). The satellite has four small thruster rockets, whose exhaust gases come out along

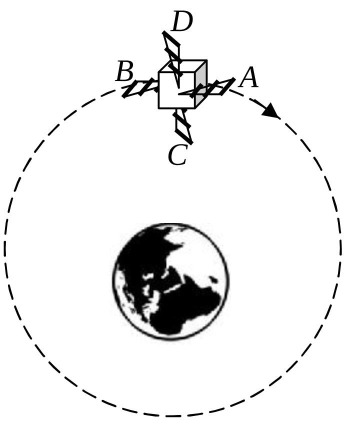
(A) the forward direction,

(B) the backward direction,

(C) radially inward towards the Earth's centre, and

(D) radially outward from the Earth's centre,

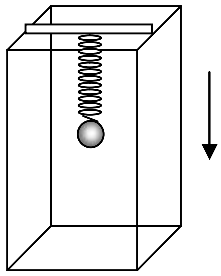
as indicated in the figure.



If the satellite wants to increase its speed, while remaining in a circular orbit, and has fuel enough to keep only one thruster rocket in operation, it should fire the rocket marked

(a) A (b) B (c) C (d) D

Q7. A particle of mass m hangs from a light spring inside a lift (see figure). When the lift is at rest, the mass oscillates in the vertical direction with an angular frequency $2.5\, rad/s$. Now consider the following situation.



The suspended mass is at rest inside the lift which is descending vertically at a speed of $0.5\ m/s$. If the lift suddenly stops, the amplitude of oscillations of the mass will be

(a) $0.20\, m$ (b) $0.25\, m$ (c) $0.05\ m$ (d) $1.25\ m$

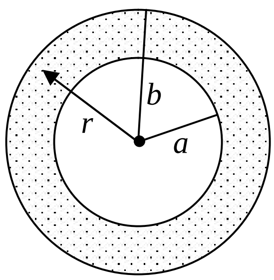
Q8. Consider two planets P_1 and P_2 which can be modelled as uniform spheres of radii R_1 and R_2 respectively, and of the same material with the same density and other physical properties. If the maximum possible height of a conical mountain (of the same material) on these planets is denoted by h_1 and h_2 respectively $(h_1 \ll R_1, h_2 \ll R_2)$, then the ratio $\frac{h_1}{h_2}$ is

(a) $\frac{R_2}{R_1}$ (b) $\frac{R_1}{R_2}$ (c) $\frac{R_2^{2/3}}{R_1^{2/3}}$ (d) $\frac{R_1^{2/3}}{R_2^{2/3}}$

Q9. A particle of rest mass $\sqrt{3}g$ emerges from a gun with a velocity $v = c/4$. If the rest mass of the gun is $1\ kg$, its approximate speed of recoil will be

(a) $\frac{c}{1000}$ (b) $\frac{c}{2236}$ (c) $\frac{c}{1732}$ (d) $\frac{c}{2309}$

Q10. Consider two concentric spheres of radii a and b, where $a < b$ (see figure). The (shaded) space between thee two spheres is filled uniformly with total charge Q. The electric field at any point between thee two spheres at distance r from the centre is given by



(a) $\frac{Q}{4\pi\in_0}\frac{r^3 - a^3}{r^2\left(b^3 - a^3\right)}$ (b) $\frac{Q}{4\pi\in_0}\frac{1}{r^2}$

(c) $\frac{Q}{4\pi\in_0}\left(\frac{b}{r^4} - \frac{a}{r^4}\right)^{2/3}$ (d) zero

Q11. A metallic wire of uniform cross- section and resistance R is bent into a circle of radius a. The circular loop is placed in a magnetic field $\vec{B}(t)$ which is perpendicular to the plane of the wire. This magnetic field is uniform over space, but its magnitude decreases with time at a constant rate k, where

$$k = -\frac{d\left|\vec{B}(t)\right|}{dt}$$

The tension in the metallic wire is

(a) $\frac{\text{®} a^3 k}{2R}\left|\vec{B}(t)\right|$ (b) $\frac{\text{®} a^3 k}{R}\left|\vec{B}(t)\right|$

(c) (d) zero

Q12. Four students were asked to write down possible forms for the magnetic vector potential $\vec{A}(\vec{x})$ corresponding to a uniform magnetic field of magnitude B along the positive z direction. Three returned correct answers and one returned an incorrect answers. Their answers are reproduced below. Which was the incorrect answer?

(a) $Bx\hat{j}$ (b) $-By\hat{i}$

(c) $\frac{1}{2}\left(Bx\hat{i} - By\hat{j}\right)$ (d) $\frac{1}{2}\left(-By\hat{i} + Bx\hat{j}\right)$

Q13. The components of the electric and magnetic fields corresponding to a plane electromagnetic field propagating in vacuum satisfy

$$E_x = E_y = -E_z = \frac{\left|\vec{E}\right|}{\sqrt{3}} \qquad B_x = -B_y = \frac{\left|\vec{B}\right|}{\sqrt{2}} \qquad B_z = 0$$

A unit vector along the direction of propagation of the plane wave is

(a) $\frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}}$ (b) $-\frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}}$

(c) $\frac{2\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{3}}$ (d) $-\frac{2\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{3}}$

Q14. A gas has the following equation of state

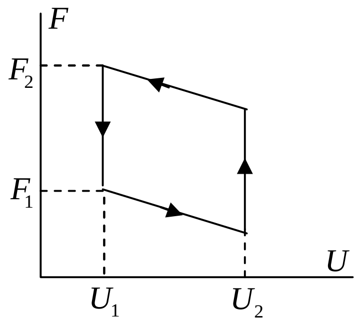
$$U = \frac{aS^5}{N^2V^2}$$

where U is the internal energy, V is the volume and N is the number of particles. Here a is a constant of the appropriate dimension. It follows that the equation of state of this gas relating its pressure P to its temperature T and its density $° = \frac{N}{V}$ is given by

(a) $\frac{P^5}{T^{5°2}} = \text{constant}$ (b) $\frac{P^5}{T^{4°3}} = \text{constant}$

(c) $\frac{P}{T°} = \text{constant}$ (d) $\frac{P^3}{T^{2°3}} = \text{constant}$

Q15. An ideal gas is passed through a cyclic process where the corresponding changes in the thermodynamic potentials are plotted on the adjoining graph. Here U is the internal energy and F is the Helmholtz free energy.



The efficiency of this cycle is given by

(a) $1-\frac{U_1}{U_2}$ (b) $1-\exp\left(-\frac{F_2}{F_1}\right)$

(c) $1-\frac{U_1}{U_2}\exp\left(-\frac{F_2}{F_1}\right)$ (d) $\exp\left(\frac{U_1}{U_2}\right)-\exp\left(-\frac{F_2}{F_1}\right)$

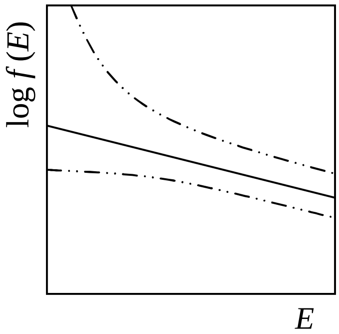
Q16. The mean free path a of molecules of a gas at room temperature is given approximately by

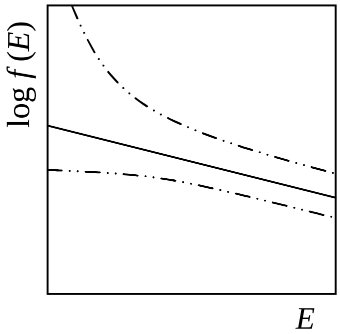
$$a = \frac{1}{n±}$$

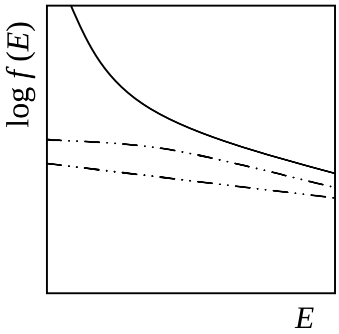
where n is the number density of the molecules and ± is the collision cross-section of two molecules. It follows that the mean free path of air molecules at normal temperature and pressure is of order

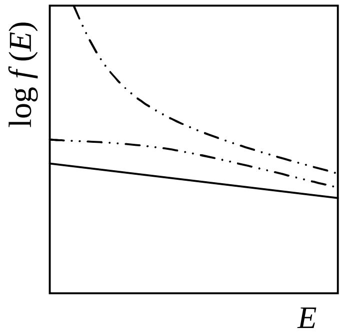
(a) $500\, «m$ (b) $50\ nm$ (c) $0.5\,nm$ (d) $500\,\text{fm}$

Q17. Four students are asked to draw on the same semi-logarithmic plot the energy distributions $f\left(E\right)$ of a classical gas (with a solid line), a boson gas (with a dashed line) and a fermion gas (with a dash-dot line) respectively, each as a function of energy E. Only one student's answer was correct. The graphs submitted by the four students are given below. The correct one is

(a)


(b)


(c)


(d)


Q18. The wave function of a particle subjected to a three-dimensional spherically-symmetric potential $V\left(r\right)$ is given by

$$\cdot\ \left(\vec{x}\right)=\left(x+y+3z\right)f\left(r\right)$$

the expectation value for the operator $\vec{L}^2$ for this state is

(a) $\hbar^2$ (b) $2\hbar^2$ (c) $5\hbar^2$ (d) $11\hbar^2$

Q19. A fermion of mass *m*, moving in two dimensions, is strictly confined inside a square box of side ℓ. The potential inside is zero. A measurement of the energy of the fermion yields the result

$$E=\frac{65\text{®}^2\hbar^2}{2m\ell^2}$$

The degeneracy of this energy state is

(a) 2 (b) 4 (c) 8 (d) 16

Q20. A sample of hydrogen gas was placed in a discharge tube and its spectrum was measured using a high-resolution spectrometer. The H_α line in the spectrum was found to be split into two lines, a high intensity line at $656.28\ nm$, and a low intensity line at $656.01\ nm$. This indicates that the hydrogen sample was contaminated with

(a) deuterium (b) tritium

(c) helium (d) water vapour

Q21. The momentum operator

$$i\hbar\frac{d}{dx}$$

acts on a wavefunction $\psi(x)$. This operator is Hermitian

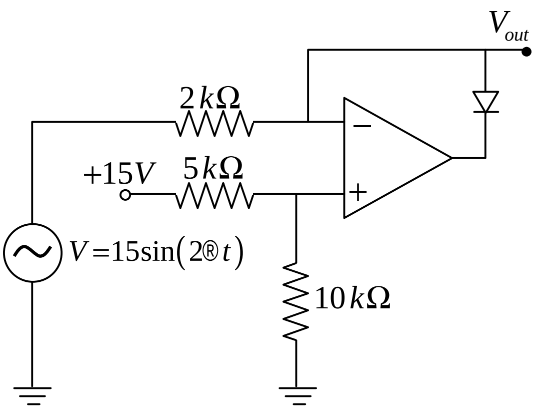
(a) provided the wavefunction $\psi(x)$ is normalized

(b) provided the wavefunction $\psi(x)$ and derivate $\psi'(x)$ are continuous everywhere

(c) provided the wavefunction $\psi(x)$ vanishes as $x \to \pm\infty$

(d) by its very definition

Q22.



In the above circuit, which of the following is the maximum value, in Volts, of voltage at V_{out} ?

(a) 10 (b) 15 (c) 0 (d) 5

Q23. A badly-designed voltmeter is modelled as an ideal voltmeter with a large resistor (R) and a large capacitor (C) connected in parallel to it. Given this information, which of the following statements describes what happens when this voltmeter is connected to a DC voltage source with voltage V and internal resistance $r(r \ll R)$?

(a) The reading on the voltameter rises slowly and becomes steady at a value slightly less than V

(b) The reading on the voltameter starts at a value slightly less than V and slowly falls to zero.

(c) The reading on the voltameter rises slowly to maximum value close to V and then slowly goes to zero.

(d) The reading on the voltameter reads zero even when connected to the voltage source.

Q24. An OR gate, a NOR gate and an XOR gate are to be constructed using only NAND gates.
If the minimum number of NAND gates needed to construct OR, NOR nd XOR gates is denoted $n(\mathrm{OR}), n(\mathrm{NOR})$ and $n(\mathrm{XOR})$ respectively, then

(a) $n(\mathrm{NOR})=n(\mathrm{XOR})>n(\mathrm{OR})$

(b) $n(\mathrm{NOR})=n(\mathrm{XOR})=n(\mathrm{OR})$

(c) $n(\mathrm{NOR})>n(\mathrm{XOR})>n(\mathrm{OR})$

(d) $n(\mathrm{NOR})<n(\mathrm{XOR})=n(\mathrm{OR})$

Q25. On passing electric current, a tungsten filament is emitting electrons by thermionic emission. In order to maintain the energy of the electron beam obtained from this source at a value approximately $100\ eV$, which of the following methods will work in practice?

(a) Float the filament at -100 Volts with a grounded aperture in front of it.

(b) Heat the filament so that the emitted electrons will have $100\ eV$ kinetic energy due to temperature.

(c) Apply a $+100$ Volts potential with respect to the filament potential to an aperture kept very close to the filament.

(d) Use an appropriate magnetic field to draw out the electron beam at the desired energy without applying any electric field.

SECTION B

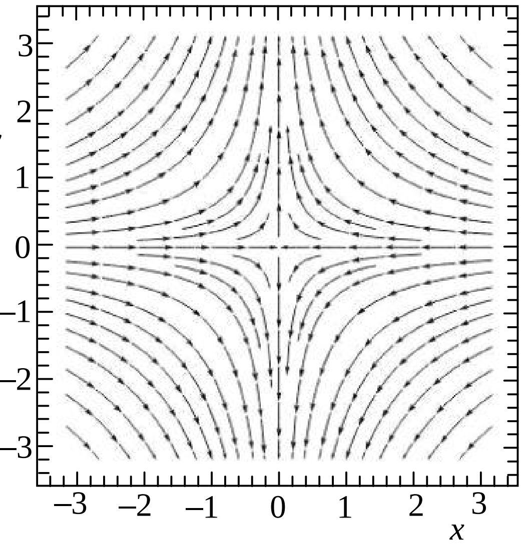
(only for Int.-Ph.D. candidates)

This Section consists of 15 Questions. All are of multiple-choice type. Mark only one option on the online interface provided to you. If more than one option is marked, it will be assumed that the question has not been attempted. A correct answer will get +5 marks, an incorrect answer will get 0 mark.

Q26. A two-dimensional electrostatic field is defined as

$$\vec{E}(x,y) = -x\hat{i} + y\hat{j}$$

A correct diagram for the lines of force is

(a)


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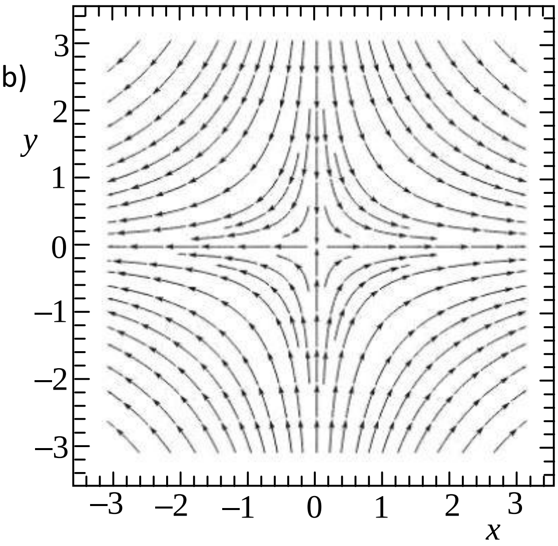
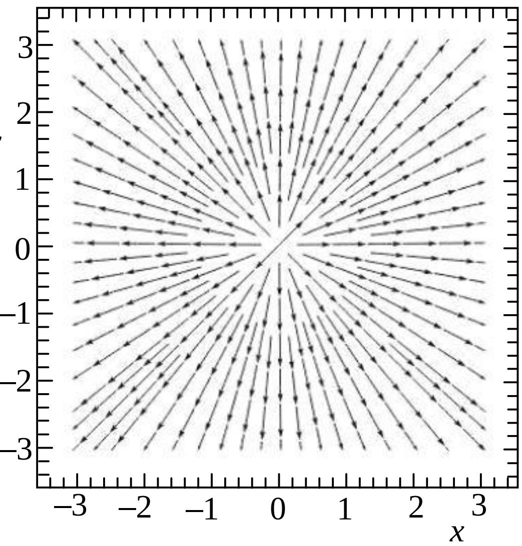
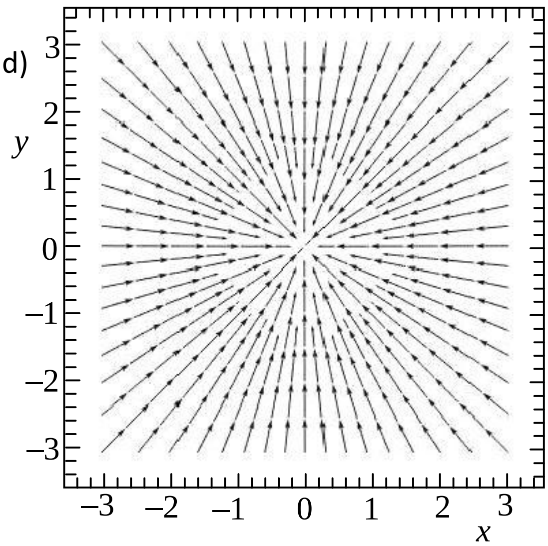


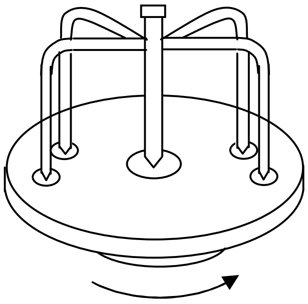
(c)


(d)


Q27. The sum of the infinite series $S=1+\frac{3}{5}+\frac{6}{25}+\frac{10}{125}+\frac{15}{625}+....$ is given by

(a) $S=\frac{125}{64}$ (b) $S=\frac{25}{16}$ (c) $S=\frac{25}{24}$ (d) $S=\frac{16}{25}$

Q28. A roundabout rotating base is a heavy uniform disc of radius $2\,m$ and mass $400\ kg$ has a central pillar and handles which are of negligible mass (see figure). The roundabout is set rotating at a steady rate of 20 r.p.m.

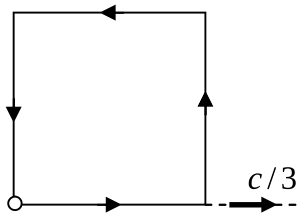


Four small children, of mass $10\,kg, 20\,kg, 30kg$ and $40kg$ respectively, step gently on to the edge of thee roundabout, each with velocity $7.2\,km/hr$ along a tangential direction and cling to the handles. After holding on for some time, the children step gently off the roundabout with the same velocity, but this time in a radial direction.

Neglecting all effects of friction and air drag the final rate of rotation of the roundabout will be about

(a) 28 r.p.m (b) 25 r.p.m (c) 36 r.p.m (d) 21 r.p.m

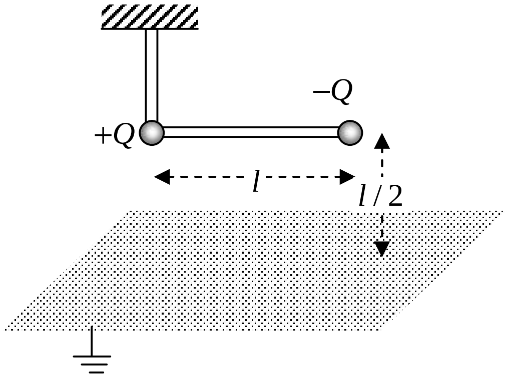
Q29. In the laboratory frame, a particle at rest starts moving with a speed $\frac{c}{2}$ from one corner of a square (see figure) and traverses the four sides of the square so that it returns to its original position. At each corner, it changes direction without any change in speed.



If the entire square now moves with a speed $\frac{c}{3}$ in the laboratory frame, as indicated in the figure, the speed of the particle (in the laboratory frame) when it returns to its original position will be

(a) $\frac{2\sqrt{2}c}{15}$ (b) $\frac{c}{5}$ (c) $\frac{2\sqrt{2}c}{3}$ (d) $\frac{c}{5\sqrt{3}}$

Q30. A light rigid insulating rod of length ℓ is suspended horizontally from a rigid frictionless pivot at one of the ends (see figure). At a vertical distance h below the rod there is an infinite plane conducting plane, which is grounded.



If two small, light spherical conductors are attached at the ends of the rod and given charges $+Q$ and $-Q$ as indicated in the figure, the torque on the rod will be

(a) $\dfrac{Q^2}{4\pi \in_0 \ell}\hat{k}$ (b) $-\dfrac{Q^2}{4\pi \in_0 \ell}\hat{k}$

(c) $\dfrac{\left(4-\sqrt{2}\right)}{16\pi \in_0}\dfrac{Q^2}{\ell}\hat{k}$ (d) $-\dfrac{\left(4-\sqrt{2}\right)}{16\pi \in_0}\dfrac{Q^2}{\ell}\hat{k}$

Q31. The magnitude vector potential $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ is defined in a region R of space by

$A_x = 5\cos\pi y$ $A_y = 2+\sin\pi x$ $A_z = 0$

in an appropriate unit.

If L be a square loop of wire in the $x-y$ plane, with its end at

$(0,\ \ 0)$ $(0,\ \ 0.25)$ $(0.25,\ \ 0.25)$ $(0.25,\ \ 0)$

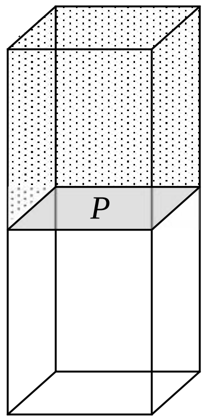
In appropriate unit and it lies entirely in the region R, the numerical value of the flux of the above magnetic field (in the same units) passing through L is

(a) 0.543 (b) 3.31 (c) -0.75 (d) $zero$

Q32. The volume V of a rectangular box is divided into two equal parts by a solid non permeable partition P. On one side of the partition P there is a vacuum, while the other side is filled with a real gas having equation of state

$$pVe^{a/RTV} = nRT$$

where a and b are constants, The gas was initially at a uniform temperature T_0. Then the partition P was removed instantaneously, and the gas was allowed to expand to fill the full volume of the box and come to equilibrium. The final temperature of the gas, in term of its specific heat C_v will be



(a) $T - \left(\frac{na}{C_V}\right)\ln 2$

(b) $T + \left(\frac{na}{C_V}\right)\ln 2$

(c) $T - 2n\left(\frac{RTa}{C_V}\right)^{3/2}$

(d) $T + 2n\left(\frac{RTa}{C_V}\right)^{3/2}$

Q33. A system is composed of a large number of non-interacting classical particles moving in two dimensions, which individually obey the Hamiltonian

$$\frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}m\mu^2\left(x^2 + y^2\right)$$

and the system is connected to a heat bath at a temperature T.

The probability of finding a particle within a radius R from the origin is given by

(a) $1 - \exp\left(-\frac{m\mu^2R^2}{2T}\right)$

(b) $\exp\left(-\frac{m\mu^2R^2}{2T}\right)$

(c) $erf\left(\sqrt{\frac{m}{2T}}\mu R\right)$

(d) $1 - \frac{m\mu^2R^2}{2T}$

Q34. A particle of mass m is confined inside a box with boundaries at $x = \pm L$. The ground state and the first excited state of this particle are E_1 and E_2 respectively.

Now a repulsive delta function potential ª¢(x) is introduced at the centre of the box where the constant ª satisfies

$$0 < ª \ll \frac{1}{32m}\left(\frac{h}{L}\right)^2$$

If the energies of the new ground state and the new first excited state be denoted as E_1' and E_2' respectively, it follows that

(a) $E_1' > E_1, E_2' > E_2$

(b) $E_1' = E_1, E_2' = E_2$

(c) $E_1' > E_1, E_2' = E_2$

(d) $E_1' = E_1, E_2' > E_2$

Q35. Three noninteracting particles whose masses are in the ratio $1:4:16$ are placed together in the same harmonic oscillator potential $V(x)$.

The degeneracies of the first three energy eigenstates (ordered by increasing energy) will be

(a) $1,1,1$ (b) $1,1,2$ (c) $1,2,1$ (d) $1,2,2$

Q36. The circuit shown represents a typical voltage-divider bias circuit for a transistor. Assume that resistance values and voltage values are typical for using the transistor as an amplifier.

Which of the following changes in the circuit would result in an increase in the collector voltage V_C?

(a) R_2 is decreased slightly (b) R_2 is increased slightly

(c) R_C is decreased slightly (d) R_C is increased slightly

Q37. A beam of X-rays is incident upon a powder sample of a material which forms simple cubic crystals of lattice constant $5.5\,\mathring{A}$. The maximum wavelength of the X-rays which can produce diffraction from the planes with Miller indices $(0,0,5)$ is

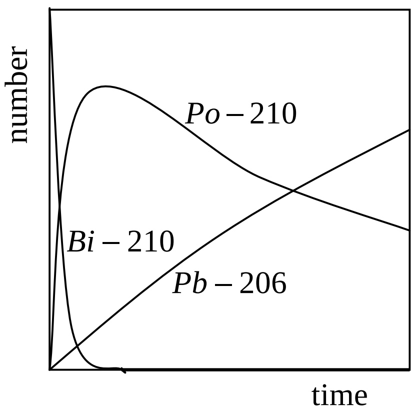
(a) $2.2\,\mathring{A}$ (b) $55.0\,\mathring{A}$ (c) $1.1\,\mathring{A}$ (d) $27.5\,\mathring{A}$

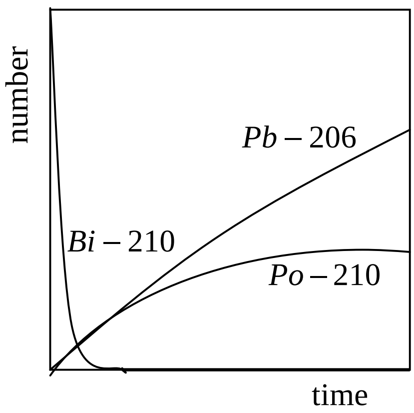
Q38. Consider the nuclear decay chain of radio-Bismuth to Polonium to Lead, i.e.

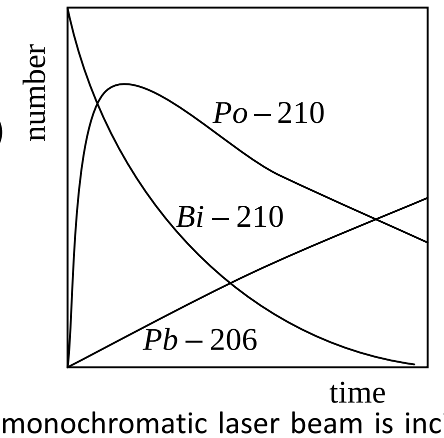
$$^{219}_{83}\mathrm{Bi} \rightarrow ^{210}_{84}\mathrm{Po} \rightarrow ^{206}_{82}\mathrm{Pb}$$

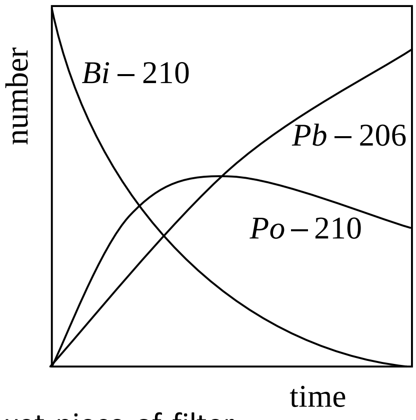
where Pb-206$\left(^{206}_{82}\mathrm{Pb}\right)$ is a stable nucleus, and Bi - 210$\left(^{219}_{82}\mathrm{Bi}\right)$ and Po - 206$\left(^{210}_{84}\mathrm{Po}\right)$ are radioactive nuclei with half lives of about 5 days and 138 days respectively.

If we start with a sample of pure Bi - 210$\left(^{219}_{82}\mathrm{Bi}\right)$, then a possible graph for the time evolution of the number of nuclei of these three species will be

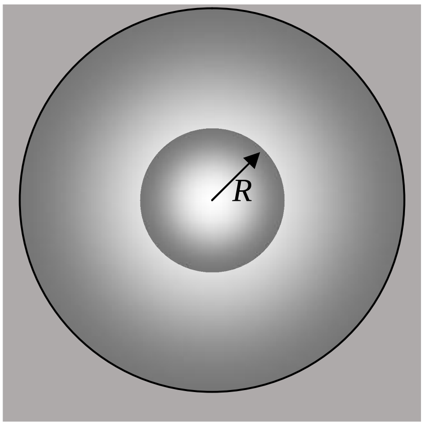
(a)


(b)


(c) 

(d) 

Q39. A monochromatic laser beam is incident on a wet piece of filter paper atop a sheet of glass of thickness d. The pattern observed on the paper is shown in figure. If the radius of the inner ring observed is R, the refractive index of the glass must be



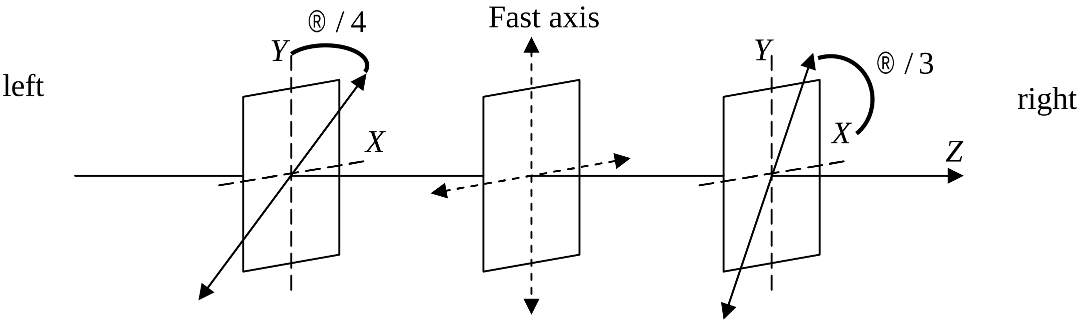
(a) $\sin\left\{\tan^{-1}\left(\frac{R}{2d}\right)\right\}$ (b) $\sin\left\{\tan^{-1}\left(\frac{R}{d}\right)\right\}$

(c) $\tan\left\{\sin^{-1}\left(\frac{R}{2d}\right)\right\}$ (d) $\tan\left\{\sin^{-1}\left(\frac{R}{d}\right)\right\}$

Q40. A plane polarised light wave with electric field expressed as

$$\vec{E}(z,t)=E_0\hat{j}\cos(kz-\mu t)$$

is incident from the left on the apparatus as sketched below.



The apparatus consists of (from left to right) a polariser with transmission axis at $\frac{®}{4}$ w.r.t. y-axis, followed by a quarter-wave plate with fast axis along the y-axis and finally, a polariser with transmission axis at $\frac{®}{3}$ about the x-axis.

If the incident intensity of the wave is I_0, what will be intensity of the light emerging out of the apparatus (on the right)?

(a) $\frac{I_0}{4}$ (b) $\frac{I_0}{8}$ (c) $\frac{3I_0}{8}$ (d) $\frac{I_0}{16}$

SECTION B

(Only for Ph.D. candidates)

This Section consists of 15 Questions. All are of multiple-choice type. Mark only one option on the online interface provided to you. If more than one option is marked, it will be assumed that question has not been attempted. A correct answer will get +5 marks, an incorrect answer will get 0 mark.

Q26. The solution of the differential equation

$$\frac{dy}{dx}=1+\frac{y}{x}-\frac{y^2}{x^2}$$

for $x>0$ with the boundary condition $y=0$ at $x=1$. is given by $y(x)=$

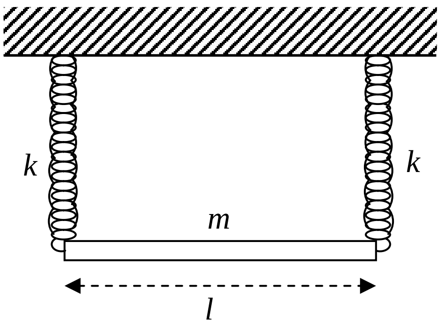
(a) $\frac{x(x^2-1)}{x^2+1}$ (b) $\frac{x(x-1)}{x+1}$ (c) $\frac{x-1}{x+1}$ (d) $\frac{x^2-1}{x^2+1}$

Q27. The value of the integral

$$\int_0^\infty \frac{dx}{x^4+4} \qquad \text{is}$$

(a) $\frac{®}{8}$ (b) $\frac{3®}{8}$ (c) $2®$ (d) $\frac{®}{4}$

Q28. A uniform rod of length ℓ and mass m is suspended horizontally from a rigid support by two identical massless springs, each with stiffness constant k, as sketched below.



If the springs can move only in the vertical direction, the frequency of small oscillations of the rod about equilibrium is given by

(a) $\sqrt{\frac{2k}{m}}$ and $\sqrt{\frac{6k}{m}}$ (b) $\sqrt{\frac{2k}{m}}$ and $\sqrt{\frac{2®k}{m}}$

(c) $\sqrt{\frac{®k}{2m}}$ and $\sqrt{\frac{6k}{m}}$ (d) $\sqrt{\frac{k}{m}}$ and $\sqrt{\frac{2®k}{m}}$

Q29. The Lagrangian of a system described by generalised coordinates q_1 and q_2 is given by

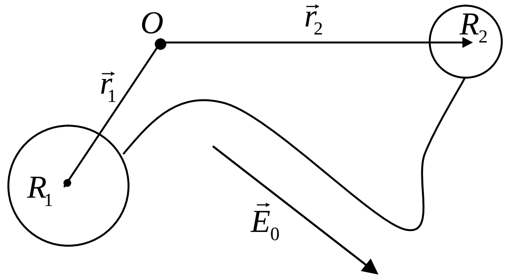
$$L = \frac{a}{2}\left(\dot{q}_1^2 + \dot{q}_2^2\right) - \frac{b^2}{®}\left(q_1^2 + q_2^2\right)$$

where a and b are constants. If follows that a conserved quantity in this system is

(a) $q_1\dot{q}_2 - q_2\dot{q}_1$ (b) $q_1\dot{q}_2 + q_2\dot{q}_1$

(c) $\frac{q_1\dot{q}_2 - q_2\dot{q}_1}{q_1^2 + q_2^2}$ (d) $2®\left(q_1^2\dot{q}_2 + q_2^2\dot{q}_1\right)$

Q30. Two conducting uncharged spheres of radii R_1 and R_2 are connected by an infinitesimally thin wire. The centres of the spheres are located at $\vec{r}_1$ and $\vec{r}_2$ respectively with respect to the origin O. The system is subjected to an uniform external electric field $\vec{E}_0$.



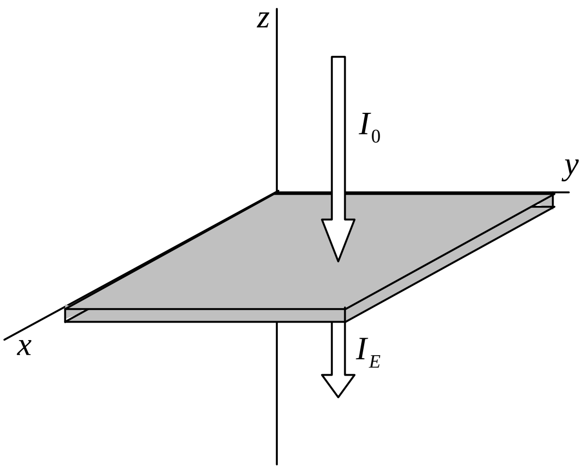
If the wire cannot support a net charge and two spheres are separated by distance much larger than the radii of each of them, the induced dipole moment in the system would be

(a) $4® \in_0 \frac{R_1R_2}{R_1+R_2}\left\{\vec{E}_0.(\vec{r}_2-\vec{r}_1)\right\}(\vec{r}_2-\vec{r}_1)$ (b) $\frac{1}{4® \in_0}\frac{R_1R_2}{(R_1+R_2)}\left\{\vec{E}_0.(\vec{r}_2-\vec{r}_1)\right\}(\vec{r}_2-\vec{r}_1)$

(c) $4® \in_0 \frac{R_1+R_2}{R_1R_2}\left\{\vec{E}_0.(\vec{r}_2-\vec{r}_1)\right\}(\vec{r}_2-\vec{r}_1)$ (d) zero

Q31. Consider the following situation.

An infinite plane metallic plate of thickness $1.8\ cm$ is placed along the $x-y$ plane, with z axis normal to the sheet (see figure).



A plane radio wave of intensity I_0 and frequency $29.5\ MHz$ propagates in vacuum along the negative z-axis and strikes the metal foil at normal incidence.

If the metal of the foil has conductivity $5.9\ \Omega^{-1}m^{-1}$ and magnetic permeability $« \simeq 1$, the intensity I_E of the emergent wave will be approximately

(a) $0.26\ I_0$ (b) $0.51 I_0$ (c) $0.29\times10^{-7}\ I_0$ (d) $2.08\times10^{-4}\ I_0$

Q32. In a certain atom, the ground state and first excited state of the valence electron are $-7.8\ eV$ and $-3.9\, eV$, while all the higher exited states have energies very close to zero. The ground state has a degeneracy of 2, while the first excited state has a degeneracy of 6.

It follows that if these atoms reside in the outer layers of a blue giant star at a temperature around $2.32\times10^{4}\ K$, the average per atom will be approximately

(a) $-5.1 eV$ (b) $-5.9 eV$ (c) $-6.8\ eV$ (d) $-4.4\ eV$

Q33. A square lattice consists $2N$ sites, of which alternate sites are labelled A and B. An example with $N=6$ is shown on the right. Now, N identical classical particles are distributed over these sites, such that each site can accommodate at most one particle.

A	B	A	B	A	B
B	A	B	A	B	A
A	B	A	B	A	B
B	A	B	A	B	A
A	B	A	B	A	B
B	A	B	A	B	A

The fraction of the total number N of particles occupying A sites is denoted ~ and the fraction occupying B sites is denoted ,so that $\sim + \ \ = 1$.

If ~, are fixed and $N \gg 1$, the entropy S of the system can be written

(a) $S = -2Nk_BT(\sim \ln\sim + \ \ \ln\ \ )$ (b) $S = 2Nk_BT(\sim \ln\sim + \ \ \ln\ \ )$

(c) $S = -2Nk_BT(\sim \ln\sim - \ \ \ln\ \ )$ (d) $S = 2Nk_BT(\sim \ln\sim - \ \ \ln\ \ )$

Q34. A particle of mass m is placed in one dimensional harmonic oscillator potential

$$V(x) = \frac{1}{2} m µ^2 x^2$$

At $t=0$, its wavefunction is $\cdot\ (x)$. At $t = 2® / µ$ its wavefunction will be

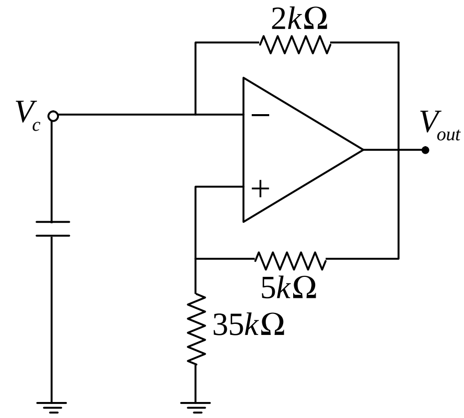
(a) $\cdot\ (x)$ (b) $-\cdot\ (x)$ (c) $-®\cdot\ (x)$ (d) $\frac{2®}{µ}\cdot\ (x)$

Q35. A spin-2 nucleus absorbs a spin-1/2 electron and is then observed to decay to a stable nucleus in two stages, recoiling against an emitted invisible particle in the first stage and against an emitted spin-1 photon in the second stage. If the stable nucleus is spinless, then the spin of the invisible particle is

(a) $\frac{3}{2}$ or $\frac{5}{2}$ (b) $\frac{3}{2}$

(c) $\frac{1}{2}$ or $\frac{3}{2}$ (d) $\frac{1}{2}$

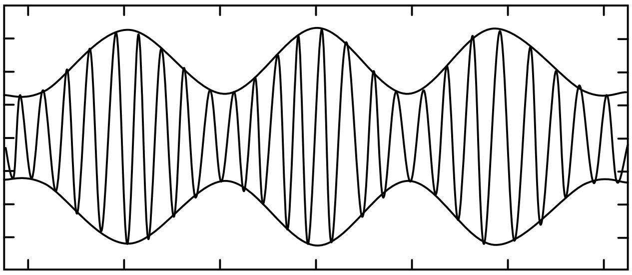
Q36. The circuit sketched below is called a relaxation oscillator.



For the parameters indicated in the figure, the ratio of the maximum voltage at V_{out} to the maximum voltage at V_C is

(a) $\frac{1}{8}$ (b) $\frac{1}{7}$ (c) $\frac{2}{7}$ (d) $\frac{1}{4}$

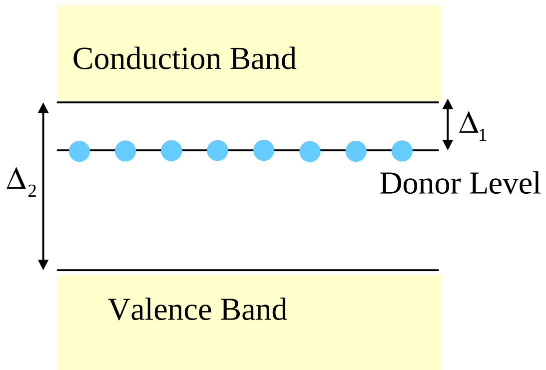
Q37. The figure below shows a carrier frequency $4kHz$ being amplitude-modulated by a sine wave-signal.



In order to transmit the signal (without distortion) the minimum bandwidth needed would be

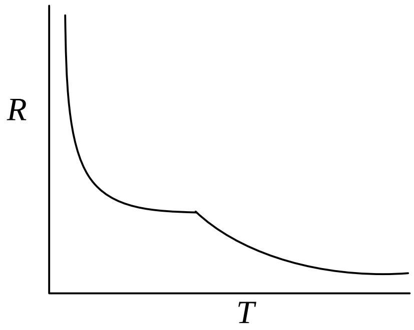
(a) 8kHz (b) 2kHz (c) 4kHz (d) 6kHz

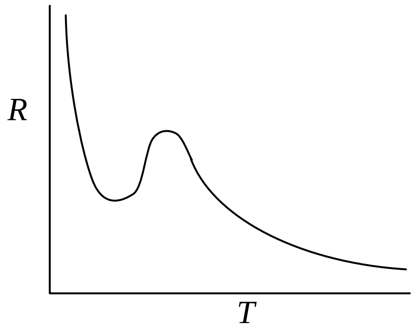
Q38. A semiconductor with donor impurities can be thought in terms of a filled valence band, a filled donor level and an empty valence band at $T = 0$, as shown in the figure below

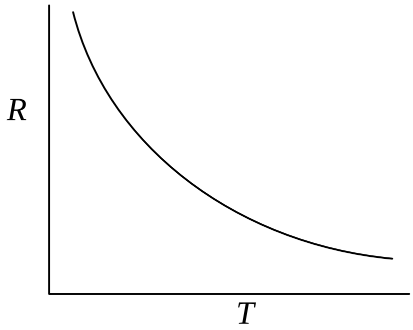


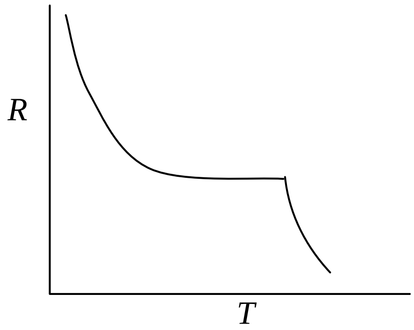
If the band gap between donor level and conduction band is Δ_1 and that between conduction and valence band is Δ_2 where $\Delta_2 \gg \Delta_1$, which of the following figures depict the qualitative features of the resistance (R) - vs-temperature (T) graph of the semi-conductor?

(Assume temperature-independent scattering rates and a flat density of states for the bands.)

(a)


(b)


(c)


(d)


Q39. Two atomic nuclei A and B have masses such that $m(B)=2m(A)$, in the laboratory frame, the nucleus B is kept stationary, while the nucleus A is given a kinetic energy $300\,MeV$ and a made to collide with B. It is found that the two nuclei fuse to form a compound nucleus C.

If the Q-value of the reaction is $-30\,MeV$, the excitation energy of the compound nucleus can be estimated as

(a) $81\ MeV$ (b) $170\,MeV$ (c) $330\,MeV$ (d) $270\ MeV$

Q40. Which of the following decays is forbidden?

(a) $® ^0 \to ¥ + ¥$ (b) $K^0 \to ®^+ + ®^- + ®^0$

(c) $«^- \to e^- + ¬_e + \overline{¬}_«$ (d) $n^0 \to p^+ + e^- + \overline{¬}_e$