Problem 1.8: Suppose you add a constant V_0 to the potential energy (by "constant" I mean independent of x as well as t). In classical mechanics this doesn't change anything, but what about quantum mechanics? Show that the wave function picks up a time-dependent phase factor: $\exp\left(-iV_0/\hbar\right)$. What effect does this have on the expectation value of a dynamical variable?

Solution: $i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V\left(x\right)\psi$

$$i\hbar\frac{\partial\psi_0}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi_0}{\partial x^2}+\left(V+V_0\right)\psi_0$$

$$i\hbar\frac{\partial\psi_0}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi_0}{\partial x^2}+V\psi_0+V_0\psi_0$$

$$\psi_0=\psi e^{-iV_0t/\hbar}$$

$$i\hbar\frac{\partial\psi}{\partial t}e^{\frac{-iV_0t}{\hbar}}+i\hbar\psi e^{\frac{-iV_0t}{\hbar}}\left(-\frac{iV_0}{\hbar}\right)$$

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V\psi\right)e^{\frac{-iV_0t}{\hbar}}+V_0\psi e^{\frac{-iV_0t}{\hbar}}$$

$=-\frac{\hbar^2}{2m}\frac{\partial^2\psi_0}{\partial x^2}+\left(V+V_0\right)\psi_0$, where $\psi_0=\psi e^{\frac{-iV_0t}{\hbar}}$

$\langle\psi|Q|\psi\rangle=\langle\psi_0|Q|\psi_0\rangle$, where Q is dynamical variable