Previous Year's Solution
NET-JRF (JUNE 2020)

PART A

Q1. A couple lives in a house with their sons and daughters and no one else. The couple has four sons and each of the sons has exactly two sisters. How many persons live in that house?

(a) 8 (b) 10 (c) 12 (d) 14

Ans. : (a)

Solution: A couple has 2 persons. There are four sons and each son has two sisters. Hence total number of persons $=2+4+2=8$

Q2. A bank pays interest to its depositors compounded yearly. If a deposit becomes Rs. 54,000/- at the end of 3rd year and Rs. 64,800/- at the end of 6th year, what is the principal invested in the deposit?

(a) 40,000 (b) 42,500 (c) 45,000 (d) 48,000

Ans. : (c)

Solution: Let the principal invested be P. Then from the question

$$54000=P\left(1+\frac{r}{100}\right)^3 \quad \text{(i)}$$

$$64800=P\left(1+\frac{r}{100}\right)^6 \quad \text{(ii)}$$

Squaring equation (i) and dividing by equation (ii) gives

$$\frac{P^2}{P}=\frac{(54000)^2}{64800}\Rightarrow P=45000$$

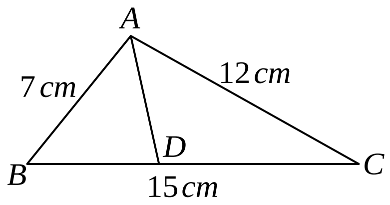
Q3. In the following ΔABC, $AB=7\,cm$, $BC=15\,cm$ and $AC=12\,cm$. D is a point on BC such that ΔADC and ΔABC are similar. Then AD (in cm) =

(a) 5.6

(b) 5.8

(c) 6.1

(d) 6.4



Ans. : (a)

Solution: The correct wording of the question should be ΔADC is similar to ΔBAC.

From the similarity condition we can write

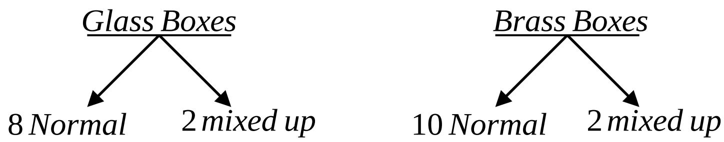
$$\frac{AD}{AC} = \frac{AB}{BC} \Rightarrow AD = \frac{AB}{BC} \times AC \Rightarrow AD = \frac{7}{15} \times 12 = 5.6$$

Q4. Ten glass vases were to be packed one each in 10 boxes marked "Glass". Twelve brass vases were to be packed one each in 12 boxes marked "Brass". Four vases and boxes got mixed up. A customer orders 1 glass and 1 brass vase and is sent appropriately marked boxes. The chance that the customer does not get the ordered vases in correctly marked boxes is

(a) $4/5$ (b) $5/6$ (c) $2/3$ (d) $1/3$

Ans. : (d)

Solution: According to wording of question, total four vases and boxes are mixed up. This is possible when 2 Glass boxes are mixed up and two Brass boxes are mixed up.



Total number of ways of not drawing 1 glass box and 1 brass box correctly

$$= 8\times 2 + 2\times 10 + 2\times 2 = 16 + 20 + 4 = 40$$

Total number of ways of drawing 1 glass box and 1 brass box correctly

$$= 10\times 12 = 120$$

Required probability $= \frac{40}{120} = \frac{1}{3}$

Q5. Anwara, Bharati, Colin and Tarun commute by different modes of transport namely, Cycle (C), Autorickshaw (A), Bus (B) and Train (T). The initials of the mode of transport and the name of the person match in exactly two cases. If Tarun travels by Train, and Colin rides neither an Autorickshaw nor a Bus, then

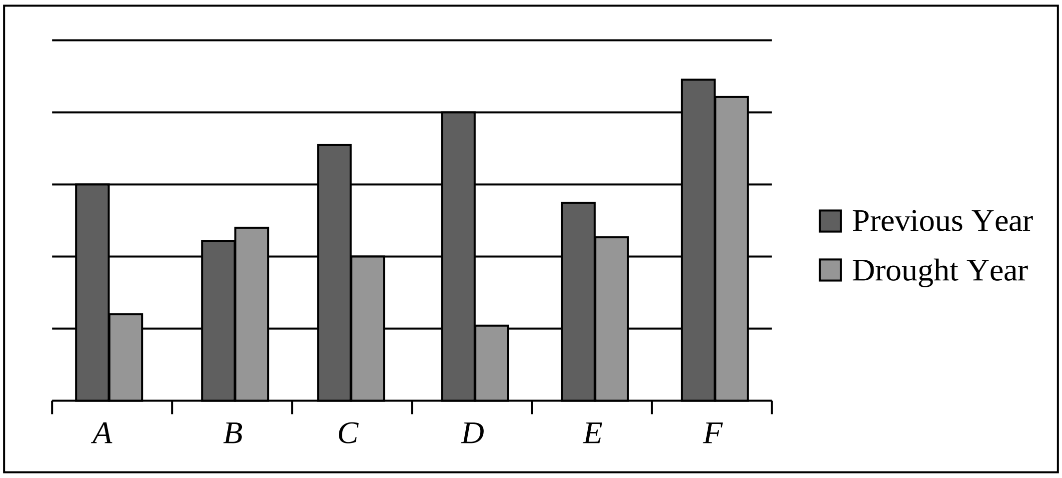
(a) Anwara rides an Autorikshaw (b) Anwara rides a Bus

(c) Bharati rides a Bus (d) Bharati rides a Cycle

Ans. : (b)

Solution: Tarun travels by Train. Colin rides neither an Autorickshaw nor a Bus hence Colin definitely travels by Cycle. Here we see that for both Tarun and Colin have their initials match with their mode of transports. Since, the initials of only two persons can match with their mode of transports, hence Anawara must ride a Bus and Bharati must ride an Autorickshaw.

Q6. Rice production in six states A, B, C, D, E and F in two consecutive years are shown in the diagram in linear scale



Among the states that saw a fall in production in the drought year, the maximum and minimum relative fall was, respectively, in states,

(a) D and F (b) C and B (c) C and E (d) D and A

Ans. : (a)

Solution: Relative fall $= \dfrac{\text{Fall}}{\text{Previous output}}$

Using this relation we see that maximum relative fall was in state D and minimum relative fall was in state F .

Q7. Based on the table, what is the maximum number of diamonds one can buy for Rs. 10 lakh?

Size (in carat)	Rate (Rs. Lakh per carat)	Number in stock
0.25	1	20
0.5	2	10
1	4	5
2	8	1

(a) 20 (b) 25 (c) 30 (d) 36

Ans. : (b)

Solution: In order to buy maximum number of diamonds, we must start with diamond size with cheapest rate then next cheapest and so on.

All 20 diamonds with carat size 0.25 can be purchased.

This is because $20 \times 0.25 \times 2 = 5$ lakh

Only 5 diamonds of 0.5 carat size can be purchased

This is because $5\times0.5\times2=5$ lakh

Now all money has been exhausted and no further diamonds can be purchased.

Hence required answer $=20+5=25$

Q8. For a disease, every infected person infects three others on the 5th day and recovers. On an average, men and women are infected in the proportion 4:1. The total number of women who were infected by the end of 35 days, is closest to

(a) 972 (b) 820 (c) 656 (d) 502

Ans. : (c)

Solution: The number of infected persons on the $5,10,15,.....,35$ days are shown below

Days	5	10	15	.....	30	35
Infected persons	3	3^2	3^3	.....	3^6	3^7
Infected women	$\frac{3}{5}$	$\frac{3^2}{5}$	$\frac{3^3}{5}$	.....	$\frac{3^6}{5}$	$\frac{3^7}{5}$

From the above table we see that the total number of infected women who were infected by the end of 35 days is

$$\frac{3}{5}+\frac{3^2}{5}+\frac{3^3}{5}+....+\frac{3^7}{5}=\frac{1}{5}\left(3+3^2+3^3+....+3^7\right)=\frac{3\left(3^7-1\right)}{3-1}\approx\frac{3^8}{2}=656.1\approx656$$

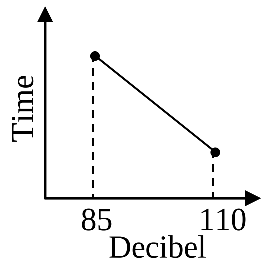
Q9. The maximum tolerable exposure time for noise is given to be about 8 hours at $85\,dB$ and 90 seconds at $110\,dB$. Assuming linear noise tolerance response of the ear, an increase of $3\,dB$ in noise level in this range would reduce the exposure time by roughly

(a) $45\,\text{min}$ (b) $60\,\text{min}$ (c) $90\,\text{min}$ (d) $120\,\text{min}$

Ans. : (b)

Solution: Assuming linearization exposure time per unit decibel is

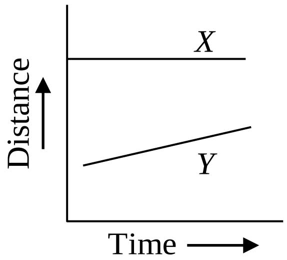
$$\frac{480-1.5}{110-85}=19.14$$

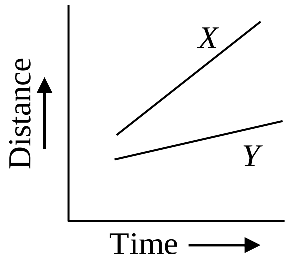


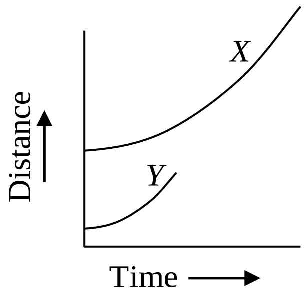
Hence an increase of 3 dB in noise level would reduce the exposure time by roughly

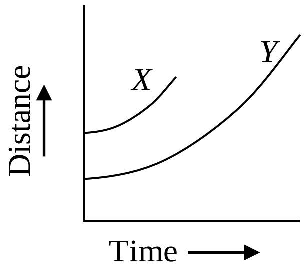
$$19.14\times3=57.42\approx60\text{ min}$$

Q10. Distance covered by cars, X and Y, with time is given below. Assuming constant acceleration for each car, which of the following graphs shows that X had higher acceleration than Y?

(a)


(b)


(c)


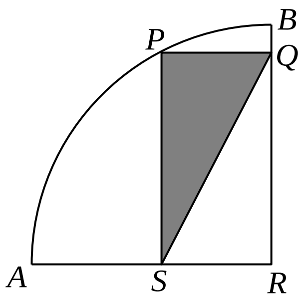
(d)


Ans. : (d)

Solution: The acceleration is given by $a = \frac{d^2x}{dt^2}$. Thus a is decided by the concavity of distance-time curve. Hence the last graph shows that X has higher acceleration than Y.

Q11. $PQRS$ is a rectangle inscribed in a quarter circle as shown. The area of shaded region is $24\,cm^2$ and $PQ = 6\,cm$.

The area of the quarter circle is



(a) 36π

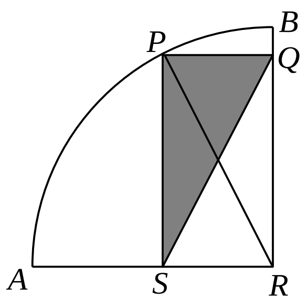
(b) 25π

(c) 13π

(d) 48π

Ans. : (b)

Solution: $PQRS$ is a rectangle, therefore $\angle QPS = 90^0$ and the triangle QPS is a right-angled triangle. This gives



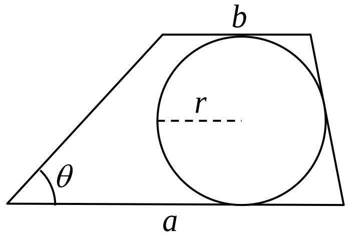
$$\frac{1}{2} \times PQ \times PS = 24 \Rightarrow \frac{1}{2} \times 6 \times PS = 24 \Rightarrow PS = 8$$

PR is a diagonal of the rectangle and it is also the radius of circle.

$$PR = \sqrt{(PQ)^2 + (PS)^2} = \sqrt{6^2 + 8^2} = 10\,cm$$

The area of quarter circle (in cm^2) is $= \frac{\pi(10)^2}{4} = 25\pi$

Q12. Area of the trapezium as shown in the figure, is



(a) $ab + r^2 \tan\theta$ (b) $r(a+b)\cos\theta$ (c) $2r(a+b)$ (d) $r(a+b)$

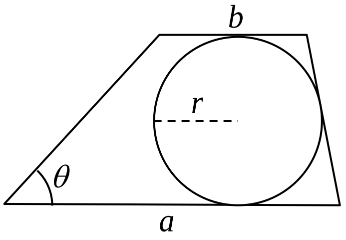
Ans. : (d)

Solution: The perpendicular distance between parallel sides

$= r + r = 2r$

Sum of parallel sides $= a+b$

Hence area of trapezium $= \frac{1}{2}\times(a+b)\times 2r = r(a+b)$



Q13. From an initially full bucket, water is dripping continuously from the bottom. The centre of mass of the bucket with water

(a) remains stationary (b) moves upward all the way

(c) moves downward all the way (d) moves downward first and then moves up

Ans. : (d)

Q14. Seven persons A, B, C, D, E, F and G are sitting in a row. E and B are sitting adjacent to each other. F is sitting between D and G. If C is sitting four places left of F, who among the following cannot be sitting at the centre?

(a) G (b) B (c) D (d) F

Ans. : (d)

Solution: For any person sitting at the centre, there are exactly three persons to his left and exactly three persons to his right. From the question, there are four or more persons to the left of F, hence F can not be sitting at the centre.

Q15. Starting from the same point at the same instant of time, three cyclists P, Q and R move on a circular path in the same direction with speeds 18, 27 and 36 km/h, respectively. The circumference of the circular path is $5.4\,km$. After a lapse of how much time would they all meet at the starting point again?

(a) $12\,\text{min}$ (b) $24\,\text{min}$ (c) $36\,\text{min}$ (d) $48\,\text{min}$

Ans. : (c)

Solution: The time taken by P, Q and R to complete the circle is $0.3hr$, $0.2hr$ and $0.15hr$ respectively. Hence they will again meet at the starting point after a time which is the LCM of $0.3hr$, $0.2hr$ and $0.15hr$.

LCM of $0.3hr$, $0.2hr$ and $0.15hr$ $= 0.6hr$

Now $0.6hr = 0.6\times 60 = 36$ minutes

Q16. Supply of food to a community is reducing at a constant rate, as a result of which the population is dying out. Ignoring other factors, which of these statements can be made about the long-term trend for the population?

(a) It will eventually die out completely

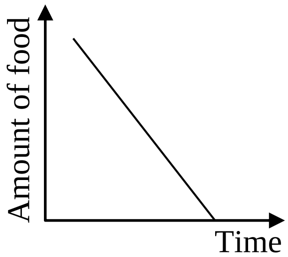
(b) It will stabilise at a non-zero number

(c) It will increase after reaching a minimum

(d) It will fall and rise repeatedly

Ans. : (a)

Solution: Amount of food is decreasing at a constant rate hence it will become zero after a certain interval of time however long it may be. Finally there will be no food available and all the population will die.



Q17. A marksman had four successes in six attempts. What is the probability that he had three consecutive successes?

(a) $9/15$ (b) $12/15$ (c) $13/15$ (d) $6/15$

Ans. : (a)

Solution: Number of ways in which four successes can be obtained in 6 attempts ${}^6C_4 = 15$. If there are four successes then there must be two failures. If the failures occurs on attempts 1 and 4 or 2 and 4 or 2 and 5 or 3 and 4 or 3 and 5 or 3 and 6 we can not get three consecutive successes. Hence there are $15-6=9$ ways of obtaining 3 consecutive successes.

Required probability $= \dfrac{9/64}{15/64} = \dfrac{9}{15}$

Q18. The scores of the six students of Group A in an examination are $38, 45, 42, 58, 62$ and 55. In the same examination, the scores of the six students of Group B of size 7 are $38, 41, 44, 46, 49$ and 52, where one score is missing. If the arithmetic means of the scores of the two groups are same, then what is the missing score?

(a) 80 (b) 65 (c) 63 (d) 62

Ans. : (a)

Solution: The mean of group A $=\dfrac{38+45+42+58+62+55}{6}=50$

From the question, the mean of group B is the same as the mean of group A. Therefore, if x is the missing score then

$$\frac{38+41+44+46+49+52+x}{7}=50 \qquad \Rightarrow x=350-270=80$$

Q19. A wire is bent into the shape of a square enclosing an area M. If the same wire is bent to form a circle, the area enclosed will be

(a) $\dfrac{4\sqrt{2}M}{\pi}$ (b) M (c) $\dfrac{4M}{\pi}$ (d) $\dfrac{\pi M}{2\sqrt{2}}$

Ans. : (c)

Solution: The perimeter of square $=4\sqrt{M}$

Now, circumference of circle = Perimeter of square

$$\Rightarrow 2\pi r=4\sqrt{M} \Rightarrow r=\frac{2\sqrt{M}}{\pi}$$

Area of circle $=\pi r^2=\pi\left(\dfrac{2\sqrt{M}}{\pi}\right)^2=\dfrac{4M}{\pi}$

Q20. In a flight of $600\,km$, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by $200\,km/h$ and the time of flight increased by 30 minutes. What was the scheduled duration of the flight?

(a) 1 hour (b) 1 hour 30 minutes

(c) 2 hours (d) 45 minutes

Ans. : (a)

Solution: Let the normal speed of aircraft be $v\ km/hr$ and the scheduled duration of the flight be t hours.

From the question

$vt = 600$ (i)

and $(v-200)\left(t+\frac{1}{2}\right) = 600$ (ii)

From equation (i) and (ii)

$$vt + \frac{v}{2} - 200t - 100 = vt$$

$\Rightarrow \frac{v}{2} - 200t - 100 = 0$ (iii)

Putting the value of v from equation (i) into equation (iii) gives

$$\frac{300}{t} - 200t - 100 = 0$$

$$\Rightarrow 200t^2 + 100t - 300 = 0 \Rightarrow 2t^2 + t - 3 = 0$$

$$\Rightarrow 2t^2 - 2t + 3t - 3 = 0 \Rightarrow 2t(t-1) + 3(t-1) = 0$$

$\Rightarrow (2t+3)(t-1) = 0 \Rightarrow t = -\frac{3}{2}$ or $t = 1$

Since negative value of t is unacceptable. Hence $t = 1$ hour.

PART B

Q21. A point mass m, is constrained to move on the inner surface of a paraboloid of revolution $x^2+y^2=az$ (where $a>0$ is a constant). When it spirals down the surface, under the influence of gravity (along $-z$ direction), the angular speed about the z - axis is proportional to

(a) 1 (independent of z) (b) z

(c) z^{-1} (d) z^{-2}

Topic: Classical Mechanics

Sub Topic: Lagrangian Formulation

Ans. : (c)

Solution: Using Lagrangian in cylindrical coordinate

$$L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+\dot{z}^2\right)-mgz \qquad \text{with constraint } x^2+y^2=az \Rightarrow r^2=az \Rightarrow \dot{z}=\frac{2r\dot{r}}{a}$$

$$L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+\left(\frac{2r\dot{r}}{a}\right)^2\right)-\frac{mgr^2}{a}$$

θ is cyclic coordinate so $\dfrac{\partial L}{\partial \theta}=0 \Rightarrow \dfrac{\partial L}{\partial \dot{\theta}}=J \Rightarrow mr^2\dot{\theta}=J \Rightarrow \dot{\theta}\propto \dfrac{1}{r^2}\propto \dfrac{1}{z}$

Q22. Two coupled oscillators in a potential $V(x,y)=\frac{1}{2}kx^2+2xy+\frac{1}{2}ky^2\ (k>2)$ can be decoupled into two independent harmonic oscillators (coordinates: x',y') by means of an appropriate transformation $\begin{pmatrix} x' \\ y' \end{pmatrix}=S\begin{pmatrix} x \\ y \end{pmatrix}$. The transformation matrix S is

(a) $\begin{pmatrix} \frac{1}{\sqrt{2}} & 1 \\ 1 & -\frac{1}{\sqrt{2}} \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ (c) $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Topic: Classical Mechanics

Sub Topic: Small Oscillations

Ans. : (b)

Solution: The normal mode of given potential is $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ and $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ in the basis of normal mode the potential can be diagonalise.

Q23. A heavy particle of rest mass M while moving along the positive z - direction, decays into two identical light particles with rest mass m (where $M > 2m$). The maximum value of the momentum that any one of the lighter particles can have in a direction perpendicular to the z - direction, is

(a) $\frac{1}{2}C\sqrt{M^2 - 4m^2}$ (b) $\frac{1}{2}C\sqrt{M^2 - 2m^2}$

(c) $C\sqrt{M^2 - 4m^2}$ (d) $\frac{1}{2}MC$

Topic: Classical Mechanics

Sub Topic: STR

Ans. : (a)

Solution:

Let P be the momentum of heavy mass M. And let P_1 be the momentum of the light particles of mass m in the direction perpendicular to z and P_2 be the momentum in z -direction.

According to conservation of momentum,

Momentum of mass M, $P = P_2 + P_2 = 2P_2 \Rightarrow P_2 = P/2$

Energy of mass M, $E = \sqrt{P^2c^2 + M^2c^4}$

Momentum of a mass m, $= \sqrt{P_1^2 + P_2^2} = \sqrt{P_1^2 + \frac{P^2}{4}}$

Energy of mass m, $E_1^2 = \left(P_1^2 + \frac{P^2}{4}\right)c^2 + m^2c^4$

As energy is conserved $E = E_1 + E_2 = 2E_1 \Rightarrow E_1 = \frac{E}{2}$ $\because E_1 = E_2$

Thus $E_1^2 = \frac{E^2}{4} = \left(P_1^2 + \frac{P^2}{4}\right)c^2 + m^2c^4 \quad \Rightarrow 4\left(P_1^2 + \frac{P^2}{4}\right)c^2 + 4m^2c^4 = P^2c^2 + M^2c^4$

$$4P_1^2c^2 + P^2c^2 + 4m^2c^4 = P^2c^2 + M^2c^4 \Rightarrow 4P_1^2c^2 + 4m^2c^4 = M^2c^4 \Rightarrow 4P_1^2 = M^2c^2 - 4m^2c^2$$

$$P_1^2 = \frac{c^2}{4}\left(M^2 - 4m^2\right) \Rightarrow P_1 = \frac{c}{2}\sqrt{M^2 - 4m^2}$$

Q24. A frictionless horizontal circular table is spinning with a uniform angular velocity ω about the vertical axis through its centre. If a ball of radius a is placed on it at a distance r from the centre of the table, its linear velocity will be

(a) $-r\omega\hat{r}+a\omega\hat{\theta}$ (b) $r\omega\hat{r}+a\omega\hat{\theta}$ (c) $a\omega\hat{r}+r\omega\hat{\theta}$ (d) 0 (zero)

Topic: Classical Mechanics

Sub Topic: Motion in a plane

Ans. : (d)

Solution: Since table is frictionless then there is not any tangential force, so ball will have zero speed .

Q25. An inductor L, a capacitor C and a resistor R are connected in series to an AC source, $V=V_0\sin\omega t$. If the net current is found to depend only on R, then

(a) $C=0$ (b) $L=0$ (c) $\omega=1/\sqrt{LC}$ (d) $\omega=\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$

Topic: Electronics

Sub Topic: Network Analysis

Ans. : (c)

Solution: The impedence value is LCR circuit is $Z=\sqrt{R^2+\left(X_C-X_L\right)^2}$

The net current is found to depend only on R, if $X_L=X_C$ the current and voltage will be in same phase and circuit will be behave as puerly resistive circuit . $\Rightarrow \omega L=\frac{1}{\omega C}\Rightarrow \omega=\frac{1}{\sqrt{LC}}$

Q26. Three point charges q are placed at the corners of an equilateral triangle. Another point charge $-Q$ is placed at the centroid of the triangle. If the force on each of the charges q vanishes, then the ratio Q/q is

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{3\sqrt{3}}$ (d) $\frac{1}{3}$

Topic: Electromagnetic Theory

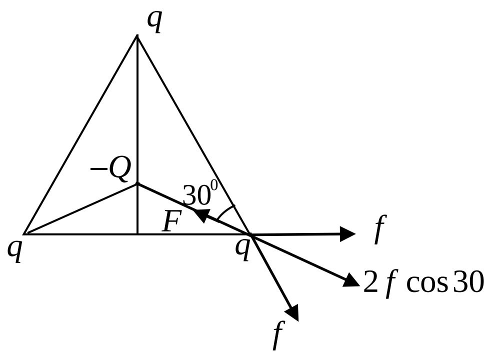
Sub Topic: Electrostatics

Ans. : (b)

Solution: $F=2f\cos 30$

$$\frac{kqQ}{\left(\frac{l}{\sqrt{3}}\right)^2}=2\frac{kq^2}{l^2}\frac{\sqrt{3}}{2}$$

$$\frac{Q}{q}=\frac{1}{\sqrt{3}}$$ (Note: sine component will cancel out).



Q27. Three infinitely long wires, each carrying equal current are placed in the xy - plane along $x=0,+d$ and $-d$. On the xy - plane, the magnetic field vanishes at

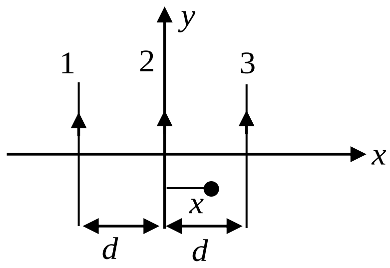
(a) $x=\pm\frac{d}{2}$ (b) $x=\pm d\left(1+\frac{1}{\sqrt{3}}\right)$ (c) $x=\pm d\left(1-\frac{1}{\sqrt{3}}\right)$ (d) $x=\pm\frac{d}{\sqrt{3}}$

Topic: Electromagnetic Theory

Sub Topic: Magnetostatics

Ans. : (d)

Solution: $B_1(-\hat{z})+B_2(-\hat{z})+B_3(\hat{z})=0$

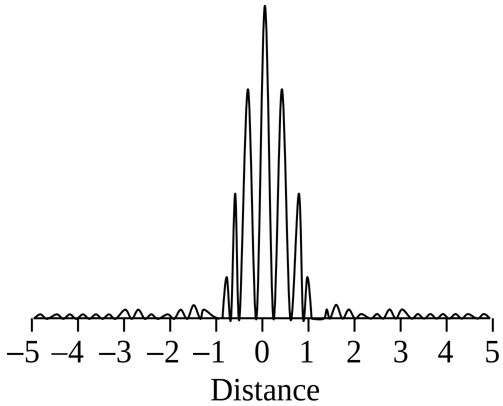


Using Ampere's Law on infinite wire,

$$-\frac{\mu_0 I}{2\pi(d+x)}-\frac{\mu_0 I}{2\pi x}+\frac{\mu_0 I}{2\pi(d-x)}=0$$

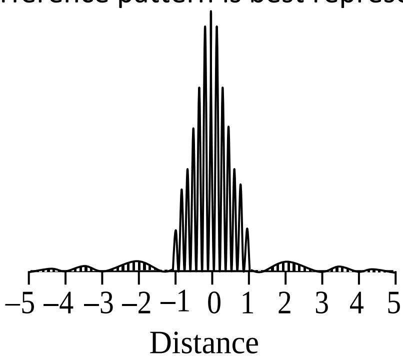
$$\frac{1}{d+x}+\frac{1}{x}=\frac{1}{d-x}\Rightarrow\frac{x+d+x}{x(d+x)}=\frac{1}{(d-x)}\Rightarrow x=\pm\frac{d}{\sqrt{3}}$$

Q28. The following figure shows the intensity of the interference pattern in the Young's double-slit experiment with two slits of equal width is observed on a distant screen

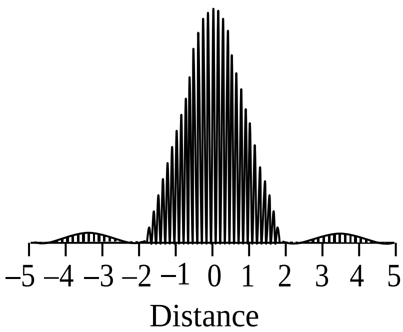


If the separation between the slits is doubled and the width of each of the slits is halved, then the new interference pattern is best represented by

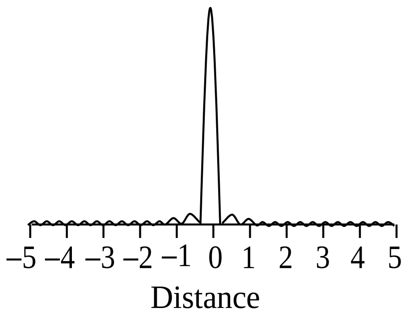
(a)



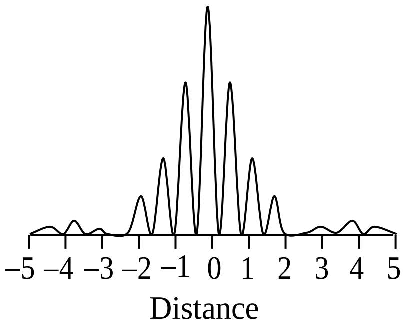
(b)



(c)



(d)



Topic: Electromagnetic Theory

Sub Topic: Interference

Ans. : (b)

Solution: (i) $\beta = \frac{D\lambda}{d}$

As d is increased to $2d$, so β will be halved and $\omega \to \frac{\omega}{2}$ so intensity will be twice

Q29. Let $\vec{E}(x,y,z,t) = \vec{E}_0 \cos(2x+3y-\omega t)$, where ω is a constant, be the electric field of an electromagnetic wave travelling in vacuum. Which of the following vectors is a valid choice for $\vec{E}_0$?

(a) $\hat{i} - \frac{3}{2}\hat{j}$ (b) $\hat{i} + \frac{3}{2}\hat{j}$ (c) $\hat{i} + \frac{2}{3}\hat{j}$ (d) $\hat{i} - \frac{2}{3}\hat{j}$

Topic: Electromagnetic Theory

Sub Topic: Electromagnetic Waves

Ans. : (d)

Solution: Let us check for $\vec{k}\cdot\vec{E} = 0$, $\vec{k} = 2\hat{i} + 3\hat{j}$, then Option (d) is correct.

Q30. Two time dependent non-zero vectors $\vec{u}(t)$ and $\vec{v}(t)$, which are not initially parallel to each other, satisfy $\vec{u}\times\frac{d\vec{v}}{dt} - \vec{v}\times\frac{d\vec{u}}{dt} = 0$ at all time t. If the area of the parallelogram formed by $\vec{u}(t)$ and $\vec{v}(t)$ be $A(t)$ and the unit normal vector to it be $\hat{n}(t)$, then

(a) $A(t)$ increases linearly with t, but $\hat{n}(t)$ is a constant

(b) $A(t)$ increases linearly with t, and $\hat{n}(t)$ rotates about $\vec{u}(t)\times\vec{v}(t)$

(c) $A(t)$ is a constant, but $\hat{n}(t)$ rotates about $\vec{u}(t)\times\vec{v}(t)$

(d) $A(t)$ and $\hat{n}(t)$ are constants

Topic: Mathematical Physics

Sub Topic: Vectors

Ans. : (d)

Solution: $\vec{A}(t) = \vec{u}\times\vec{v}$, $\quad \hat{n}(t) = \frac{\vec{A}(t)}{|A(t)|} = \frac{\vec{u}\times\vec{v}}{|\vec{u}\times\vec{v}|} = \frac{1}{A}\vec{u}\times\vec{v}$

$$\Rightarrow \frac{d\hat{n}}{dt} = \frac{1}{A}\frac{d\vec{u}}{dt}\times\vec{v} + \frac{1}{A}\vec{u}\times\frac{d\vec{v}}{dt} = \frac{1}{A}(-\vec{v})\times\frac{d\vec{u}}{dt} + \frac{1}{A}\vec{u}\times\frac{d\vec{v}}{dt} = 0 \Rightarrow \hat{n}(t) = const$$

$$\Rightarrow A(t) = |\vec{u}\times\vec{v}| = const$$

Q31. A basket consists of an infinite number of red and black balls in the proportion $p:(1-p)$. Three balls are drawn at random without replacement. The probability of their being two red and one black is a maximum for

(a) $p=\frac{3}{4}$ (b) $p=\frac{3}{5}$ (c) $p=\frac{1}{2}$ (d) $p=\frac{2}{3}$

Topic: Mathematical Physics

Sub Topic: Probability

Ans. : (d)

Solution: $P=p^2(1-p)\Rightarrow\frac{dP}{dp}=\frac{d}{dp}p^2(1-p)=0\Rightarrow p^2(-1)+(1-p)2p=0$

$$\Rightarrow -p^2+2p-2p^2=0\Rightarrow 3p^2=2p \quad\Rightarrow p=2/3$$

Q32. The eigenvalues of the 3×3 matrix $M=\begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix}$ are

(a) $a^2+b^2+c^2$, 0, 0 (b) b^2+c^2, a^2, 0

(c) a^2+b^2, c^2, 0 (d) a^2+c^2, b^2, 0

Topic: Mathematical Physics

Sub Topic: Matrices

Ans. : (a)

Solution: $M=\begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix}$. To make it simple, Let $a=1,b=1,c=1$ so $M=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{3\times3}$

$\Rightarrow \lambda=3,0,0$

So, option (a) is correct

Q33. A function of a complex variable z is defined by the integral $f(z)=\oint_\Gamma \frac{w^2-2}{w-z}dw$, where Γ is a circular contour of radius 3, centred at origin, running counter-clockwise in the w- plane. The value of the function at $z=(2-i)$ is

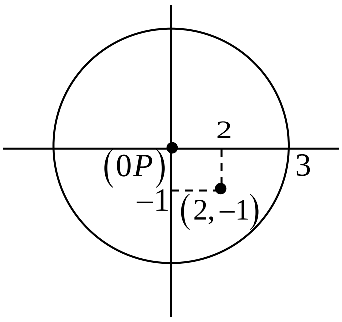
(a) 0 (b) $1-4i$ (c) $8\pi+2\pi i$ (d) $-\frac{2}{\pi}-\frac{i}{2\pi}$

Topic: Mathematical Physics

Sub Topic: Complex Variable

Ans. : (c)

Solution: $f(z) = \oint_{\Gamma} \frac{w^2 - 2}{w - z} dw$



$\omega = z$ is a simple pole.

Residue $\lim_{\omega \to z} (\omega - z) \frac{(\omega^2 - 2)}{(\omega - z)} = (2 - i)^2 - 2 = 4 - 1 - 4i - 2 = (1 - 4i)$

$f(z) = \oint_{\Gamma} \frac{w^2 - 2}{w - z} dw = 2\pi i (1 - 4i) = 2\pi i + 8\pi$

Q34. The temperatures of two perfect black bodies A and B are $400\,K$ and $200\,K$, respectively. If the surface area of A is twice that of B, the ratio of total power emitted by A to that by B is

(a) 4 (b) 2 (c) 32 (d) 16

Topic: Statistical Mechanics

Sub Topic: Black Body Radiation

Ans. : (c)

Solution: $A \to 400K$, $B \to 200K$

Energy densities, $\frac{u_A}{u_B} = \frac{400^4}{200^4} = \frac{4^4}{2^4} = \frac{4 \times 4 \times 4 \times 4}{2 \times 2 \times 2 \times 2} = 16$, $\frac{P_A}{P_B} = \frac{u_A 2A}{u_B A} = 2 \times 16 = 32$

Q35. Two ideal gases in a box are initially separated by a partition. Let N_1, V_1 and N_2, V_2 be the numbers of particles and volume occupied by the two systems. When the partition is removed, the pressure of the mixture at an equilibrium temperature T, is

(a) $k_B T \left(\frac{N_1 + N_2}{2(V_1 + V_2)} \right)$ (b) $k_B T \left(\frac{N_1 + N_2}{V_1 + V_2} \right)$

(c) $k_B T \left(\frac{N_1}{V_1} + \frac{N_2}{V_2} \right)$ (d) $\frac{1}{2} k_B T \left(\frac{N_1}{V_1} + \frac{N_2}{V_2} \right)$

Topic: Statistical Mechanics

Sub Topic: Kinetic Theory of Gases

Ans. : (b)

Solution:

N_1, V_1 P_1	N_2, V_2 P_2

Finally, At equilibrium $P(V_1 + V_2) = nRT$, n is number of moles = $\frac{N}{N_A}$

$$P=\frac{nRT}{V_1+V_2};\quad n=n_1+n_2;\quad n=\frac{N_1}{N_A}+\frac{N_2}{N_A}\ ,\ k_B=\frac{R}{N_A}$$

$$P=\frac{(N_1+N_2)k_BT}{V_1+V_2}=k_BT\left(\frac{N_1+N_2}{V_1+V_2}\right)$$

Q36. An idealised atom has a non-degenerate ground state at zero energy and a g-fold degenerate excited state of energy E. In a non-interacting system of N such atoms, the population of the excited state may exceed that of the ground state above a temperature $T>\frac{E}{2k_B\ln 2}$. The minimum value of g for which this is possible is

(a) 8 (b) 4 (c) 2 (d) 1

Topic: Statistical Mechanics

Sub Topic: Canonical Ensemble Theory

Ans. : (b)

Solution: $\varepsilon=E$ ———— $g=g \quad \to N_2$

$\varepsilon=0$ ———— $g=1 \quad \to N_1$

$$z=1+ge^{-\beta\varepsilon}$$

$$P_1=\frac{e^{-\beta 0}}{z},\qquad P_2=\frac{ge^{-\beta\varepsilon}}{z}$$

$$N_1=P_1N,\ N_2=P_2N$$

$$N_2=N_1\Rightarrow g\frac{e^{-\beta\varepsilon}}{z}=\frac{1}{z}\Rightarrow g\cdot e^{-E/k_BT}=1\Rightarrow g\cdot e^{-E/k_B(E/(2k_B\ln 2))}>1$$

$$\Rightarrow g=\exp\ln 2^2>4$$

Q37. The Hamiltonian of a system of N non-interacting particles, each of mass m, in one dimension is

$$H=\sum_{i=1}^{N}\left(\frac{p_i^2}{2m}+\frac{\lambda}{4}x_i^4\right)$$

where $\lambda>0$ is a constant and p_i and x_i are the momentum and position respectively of the i-th particle. The average internal energy of the system is

(a) $\frac{4}{3}k_BT$ (b) $\frac{3}{4}k_BT$ (c) $\frac{3}{2}k_BT$ (d) $\frac{1}{3}k_BT$

Topic: Statistical Mechanics

Sub Topic: Canonical Ensemble Theory

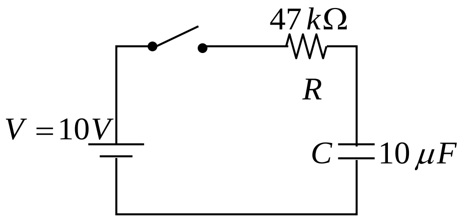
Ans. : (b)

Solution: $H=\sum_{i=1}^{N}\left(\frac{p_i^2}{2m}+\frac{\lambda}{4}x_i^4\right)$, $\left\langle\frac{p_i^2}{2m}\right\rangle=\frac{1}{2}k_BT$, $\left\langle\frac{\lambda}{4}x_i^4\right\rangle=\frac{\int_{-\infty}^{\infty}e^{-\beta\frac{\lambda}{4}x^4}\frac{\lambda}{4}x^4dx}{\int_{-\infty}^{\infty}e^{-\beta\frac{\lambda}{4}x^4}dx}$

$\int_0^{\infty}e^{-bx^4}dx=\frac{\overline{|5/4}}{b^{1/4}}$, $b\geq 0$ and $\int_0^{\infty}bx^4e^{-bx^4}dx=\frac{\overline{|5/4}}{4b^{1/4}}$

$\langle E\rangle=\frac{1}{2}k_BT+\frac{1}{4}k_BT=\frac{2+1}{4}k_BT=\frac{3}{4}k_BT$

Q38. A $10V$ battery is connected in series to a resistor R and a capacitor C, as shown the figure.



The initial charge on the capacitor is zero. The switch is turned on and the capacitor is allowed to charge to its full capacity. The total work done by the battery in this process is

(a) $10^{-3}J$ (b) $2\times10^{-3}J$ (c) $5\times10^{-4}J$ (d) $47\times10^{-2}J$

Topic: Electronics
Sub Topic: Network Analysis

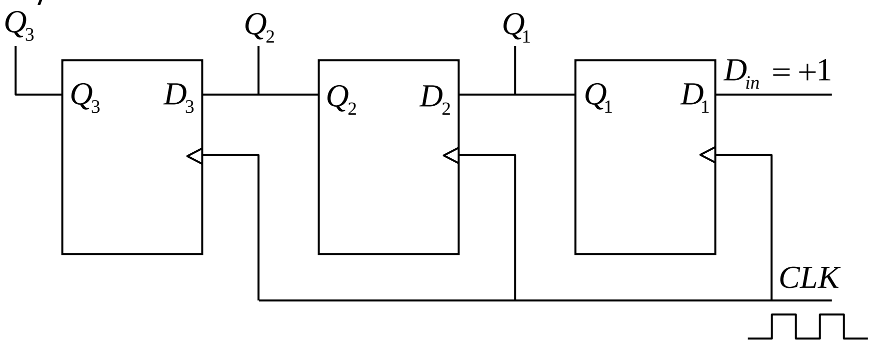
Ans. : (a)

Solution: Charge stored in the capacitance is $q=CV=10\times10^{-6}F\times10V=10^{-4}coulomb$

The total work done by the battery in this process is

$W=qV=(CV)V=10^{-4}\times10Joules=10^{-3}Jouls$

Q39. In the 3-bit register shown below, Q_1 and Q_3 are the least and the most significant bits of the output, respectively.



If Q_1, Q_2 and Q_3 are set to zero initially, then the output after the arrival of the second falling clock (CLK) edge is

(a) 001 (b) 100 (c) 011 (d) 110

Topic: Electronics
Sub Topic: Digital Electronics

Ans. : (c)

Solution:

Q_3	Q_3	Q_1	
0	0	1	(1)
0	1	1	(2)

Q40. The Boolean equation $Y = \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C$ is to be implemented using only two-input NAND gates. The minimum number of gates required is

(a) 3 (b) 4 (c) 5 (d) 6

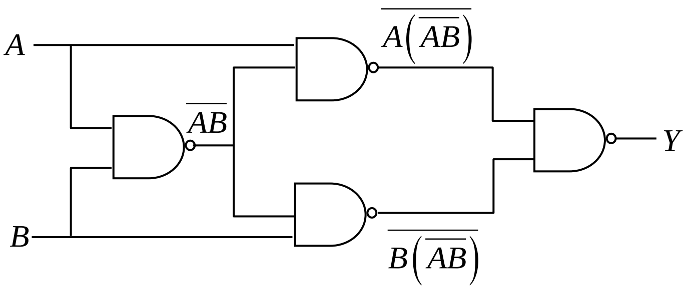
Topic: Electronics
Sub Topic: Digital Electronics

Ans. : (b)

Solution: $Y = \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C \quad \Rightarrow Y = \bar{A}B\left(C+\bar{C}\right) + A\bar{B}\left(\bar{C}+C\right)$

$\Rightarrow Y = \bar{A}B + A\bar{B} \qquad (Ex-OR)$

Implementing Ex-OR Gate



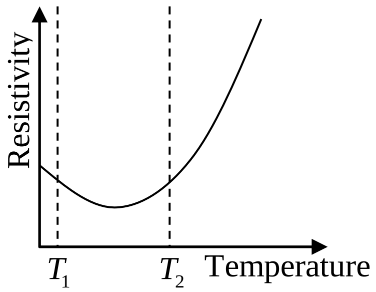
$\Rightarrow Y = \overline{\left(\overline{A\left(\overline{AB}\right)}\right)\left(\overline{B\left(\overline{AB}\right)}\right)} = A\left(\overline{AB}\right) + B\left(\overline{AB}\right)$

$\Rightarrow Y = A\left(\bar{A}+\bar{B}\right) + B\left(\bar{A}+\bar{B}\right) \Rightarrow Y = A\bar{B} + \bar{A}B$

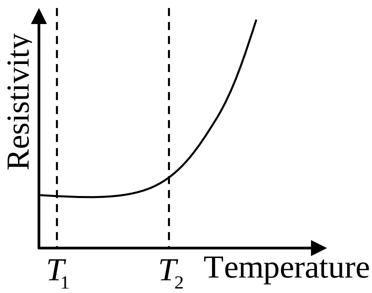
So minimum 4 number of gates are required.

Q41. The temperature variation of the resistivity of four materials are shown in the following graphs.

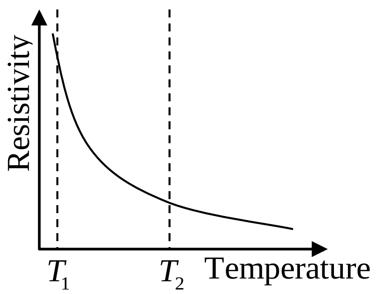
A



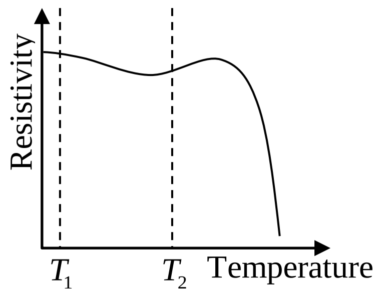
B



C



D



The material that would make the most sensitive temperature sensor, when used at temperatures between T_1 and T_2, is

(a) A (b) B (c) C (d) D

Topic: Electronics

Sub Topic: Experimental Method

Ans.: (c)

Solution: for good sensor In the range of T_1 and T_2 the variation of resistance with temperature must be single valued should have the property continuous, differentiable and monotonically changing . so only option (c) is satisfying the all property

Q42. Let $|n\rangle$ denote the energy eigenstates of a particle in a one-dimensional simple harmonic potential $V(x)=\frac{1}{2}m\omega^2x^2$. If the particle is initially prepared in the state $|\psi(t=0)\rangle=\sqrt{\frac{1}{2}}(|0\rangle+|1\rangle)$, the minimum time after which the oscillator will be found in the same state is

(a) $3\pi/(2\omega)$ (b) π/ω (c) $\pi/(2\omega)$ (d) $2\pi/\omega$

Topic: Quantum Mechanics

Sub Topic: Harmonic Oscillator

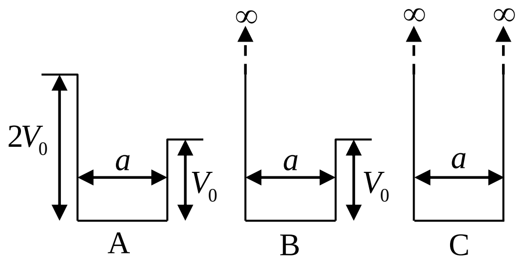
Ans. : (d)

Solution: $|\psi(t=0)\rangle=\sqrt{\frac{1}{2}}(|0\rangle+|1\rangle)$, $|\psi(t=t)\rangle=\sqrt{\frac{1}{2}}\left(|0\rangle e^{-\frac{i\omega t}{2}}+|1\rangle e^{-\frac{i3\omega t}{2}}\right)$

$$|\langle\psi(t)|\psi(0)\rangle|^2=1\Rightarrow\left|\frac{1}{2}\left(\exp-\frac{i\omega t}{2}+\exp-\frac{3i\omega t}{2}\right)\right|^2=1$$

$$|1+\exp(-i\omega t)|^2=4\Rightarrow t=\frac{2\pi}{\omega}$$

Q43. For the one dimensional potential wells A, B and C, as shown in the figure, let E_A, E_B and E_C denote the ground sate energies of a particle, respectively.



The correct ordering of the energies is

(a) $E_C>E_B>E_A$ (b) $E_A>E_B>E_C$ (c) $E_B>E_C>E_A$ (d) $E_B>E_A>E_C$

Topic: Quantum Mechanics

Sub Topic: Particle in Box

Ans. : (a) corresponding energy for infinite well is more than semi infinite well and energy of semi infite is more than finite well.

Q44. An angular momentum eigenstate $|j,0\rangle$ is rotated by an infinitesimally small angle ε about the positive y-axis in the counter clockwise direction. The rotated state, to order ε (upto a normalisation constant), is

(a) $|j,0\rangle-\frac{\varepsilon}{2}\sqrt{j(j+1)}(|j,1\rangle+|j,-1\rangle)$ (b) $|j,0\rangle-\frac{\varepsilon}{2}\sqrt{j(j+1)}(|j,1\rangle-|j,-1\rangle)$

(c) $|j,0\rangle-\frac{\varepsilon}{2}\sqrt{j(j-1)}(|j,1\rangle-|j,-1\rangle)$ (d) $|j,0\rangle-\frac{\varepsilon}{2}\sqrt{j(j+1)}|j,1\rangle-\frac{\varepsilon}{2}\sqrt{j(j-1)}|j,-1\rangle$

Topic: Quantum Mechanics

Sub Topic: Angular Momentum Algebra

Ans. : (b)

Solution: $U(R_y(\varepsilon)) \approx I-\frac{i}{\hbar}\varepsilon J_y = I-\frac{i\varepsilon}{\hbar}\left(\frac{J_+-J_-}{2i}\right)=I-\frac{\varepsilon}{2\hbar}J_+ +\frac{\varepsilon}{2\hbar}J_-$

$$U(R_y(\varepsilon))|j,0\rangle=\left(I-\frac{\varepsilon}{2\hbar}J_+ +\frac{\varepsilon}{2\hbar}J_-\right)|j,0\rangle=|j,0\rangle-\frac{\varepsilon}{2}\sqrt{j(j+1}(|j,+1\rangle-|j,-1\rangle)$$

Q45. The wavelength of the first Balmer line of hydrogen is $656\,nm$. The wavelength of the corresponding line for a hydrogenic atom with $Z=6$ and nuclear mass of $19.92\times10^{-27}\,kg$ is

(a) $18.2\,nm$ (b) $109.3\,nm$ (c) $143.5\,nm$ (d) $393.6\,nm$

Topic: Atomic and Molecular Physics

Sub Topic: Hydrogen Atom

Ans. : (a)

Solution: $R_H(1p)=R_\infty\frac{\mu}{m_e}=R_\infty\frac{m_p}{m_e+m_p}=R_\infty\frac{1836m_e}{m_e+1836m_e}=R_\infty\frac{1836}{1837}$

$$R_{H(6p)}=R_\infty\frac{\mu}{m_e}=R_\infty\frac{6m_p}{m_e+m_p}=R_\infty\frac{6\times1836}{11017} \quad \text{where } R_\infty=1.09\times10^7\,m^{-1}$$

First Balmer line corresponds to $n=3\rightarrow n=2$

$$\frac{1}{\lambda_H(1p)}=R_H(1p)\left(\frac{1}{2^2}-\frac{1}{3^2}\right) \quad \text{and} \quad \frac{1}{\lambda_H(6p)}=R_H(6p)\left(\frac{1}{2^2}-\frac{1}{3^2}\right)Z^2 \qquad Z=6$$

$$\lambda_H(1p)=\frac{R_H(1p)\lambda_H}{R_H(6p)Z^2}=\frac{1836}{1837}\times\frac{11017}{6\times1836}\times\frac{656}{6^2}nm \quad \Rightarrow \lambda_C=18.2nm$$

PART C

Q46. The state of an electron in a hydrogen atom is

$$|\psi\rangle = \frac{1}{\sqrt{6}}|1,0,0\rangle + \frac{1}{\sqrt{3}}|2,1,0\rangle + \frac{1}{\sqrt{2}}|3,1,-1\rangle$$

where $|n,l,m\rangle$ denotes common eigenstates of $\hat{H}$, $\hat{L}^2$ and $\hat{L}_z$ operators in the standard notation. In a measurement of $\hat{L}_z$ for the electron in this state, the result is recorded to be 0. Subsequently a measurement of energy is performed. The probability that the result is E_2 (the energy of the $n=2$ state) is

(a) 1 (b) $1/2$ (c) $2/3$ (d) $1/3$

Topic: Quantum Mechanics

Sub Topic: Hydrogen Atom

Ans. : (c)

Solution: We will use postulates 4 first then use postulate 2 and 3.

If L_z is measured and measurement is 0 then state is proportional to $\frac{1}{\sqrt{6}}|1,0,0\rangle + \frac{1}{\sqrt{3}}|2,1,0\rangle$

$$P(E=E_2) = \frac{\frac{1}{3}}{\frac{1}{6}+\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{1+2}{6}} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

Q47. A particle with incoming wave vector $\vec{k}$, after being scattered by the potential $V(r) = \frac{c}{r^2}$, goes out with wave vector $\vec{k}'$. The differential scattering cross-section, calculated in the first Born approximation, depends on $q = |\vec{k} - \vec{k}'|$, as

(a) $1/q^2$ (b) $1/q^4$ (c) $1/q$ (d) $1/q^{3/2}$

Topic: Quantum Mechanics

Sub Topic: Scattering

Ans. : (a)

Solution: Using Born Approximation for high energy

$$f(\theta) = -\frac{2m}{\hbar^2 q}\int_0^\infty rV(r)\sin qr\,dr \qquad \text{where } V(r) = \frac{c}{r^2}$$

$$f(\theta) = -\frac{2mc}{\hbar^2 q}\int_0^\infty \frac{\sin qr}{r}dr = -\frac{2mc}{\hbar^2 q}\frac{1}{2}\int_{-\infty}^{\infty}\frac{\sin qr}{r}dr \text{ solving from contour integration}$$

$$\int_{-\infty}^{\infty}\frac{\sin qr}{r}dr = \frac{\pi}{2} \quad \text{so } f(\theta) \propto \frac{1}{q} \Rightarrow D(\theta) = |f(\theta)|^2 \propto \frac{1}{q^2}$$

Q48. A quantum particle in a one-dimensional infinite potential well, with boundaries at 0 and a, is perturbed by adding $H' = \in \delta\left(x - \frac{a}{2}\right)$ to the initial Hamiltonian. The correction to the energies of the ground and the first excited states (to first order in $\in$) are respectively

(a) 0 and 0 (b) $2\in/a$ and 0 (c) 0 and $2\in/a$ (d) $2\in/a$ and $2\in/a$

Topic: Quantum Mechanics

Sub Topic: Perturbation Theory

Ans. : (b)

Solution: $E_n^1 = \frac{2}{a}\int_0^a \delta\left(x - \frac{a}{2}\right)\sin^2\frac{n\pi x}{a}dx = \frac{2}{a}\sin^2\frac{n\pi}{2}$ where $n = 1, 2, 3..$

For ground state $n = 1$, $E_1^1 = 2\in/a$

For first excited state $n = 2$, $E_2^1 = 0$

Q49. Spin $\frac{1}{2}$ fermions of mass m and $4m$ are in a harmonic potential $V(x) = \frac{1}{2}kx^2$. Which configuration of 4 such particles has the lowest value of the ground state energy?

(a) 4 particles of mass m

(b) 4 particles of mass $4m$

(c) 1 particle of mass m and 3 particles of mass $4m$

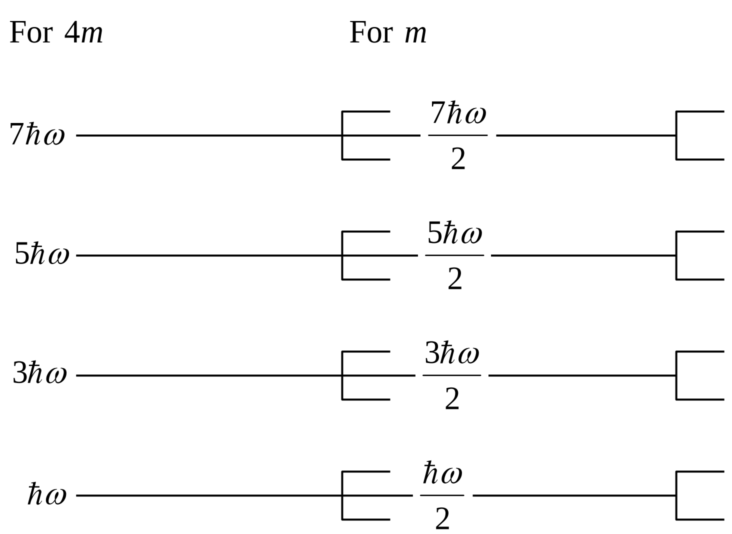
(d) 2 particles of mass m and 2 particles of mass $4m$

Topic: Statistical Mechanics

Sub Topic: Identical Particle

Ans. : (d)

Solution: $V(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$



For mass m: $V(x)=\frac{1}{2}m\omega^2x^2$ and $E_n=\left(n+\frac{1}{2}\right)\hbar\omega$

For mass $4m$: $V(x)=\frac{1}{2}(4m)\omega^2x^2=\frac{1}{2}m(2\omega)^2x^2$

$\omega_{eff}=2\omega \Rightarrow E_n=\left(n+\frac{1}{2}\right)\hbar(2\omega)$

(a) $2\left(\frac{\hbar\omega}{2}\right)+2\left(\frac{3\hbar\omega}{2}\right)=4\hbar\omega$ (b) $2(\hbar\omega)+2(3\hbar\omega)=8\hbar\omega$

(c) $\frac{\hbar\omega}{2}+2(\hbar\omega)+3\hbar\omega=\frac{11}{2}\hbar\omega=5.5\hbar\omega$ (d) $2\left(\frac{\hbar\omega}{2}\right)+2(\hbar\omega)=3\hbar\omega$

$3\hbar\omega$ is lowest among all so, (d) is correct.

Q50. Falling drops of rain break up and coalesce with each other and finally achieve an approximately spherical shape in the steady state. The radius of such a drop scales with the surface tension σ as

(a) $1/\sqrt{\sigma}$ (b) $\sqrt{\sigma}$ (c) σ (d) σ^2

Topic: Mechanics

Sub Topic: Surface Tension

Ans. : (a)

Solution: Work done while combining $W=\sigma\times$change in area $=\sigma\times\left(4\pi R^2-n4\pi r^2\right)$

Taking small r negligible: $W=\sigma\times 4\pi R^2 \Rightarrow R\propto\frac{1}{\sqrt{\sigma}}$

Q51. The velocity $v(x)$ of a particle moving in one dimension is given by $v(x)=v_0\sin\left(\frac{\pi x}{x_0}\right)$, where v_0 and x_0 are positive constants of appropriate dimensions. If the particle is initially at $x/x_0=\in$, where $|\in|\ll 1$, then, in the long time, it

(a) Executes an oscillatory motion around $x=0$

(b) Tends towards $x=0$

(c) Tends towards $x=x_0$

(d) Executes an oscillatory motion around $x=x_0$

Topic: Mechanics

Sub Topic: Small Oscillation

Ans. : (c)

Solution: $v(x)=v_0\sin\left(\frac{\pi x}{x_0}\right)\Rightarrow a(x)=v_0\cos\left(\frac{\pi x}{x_0}\right)\frac{\pi}{x_0}.\frac{dx}{dt}=v_0\cos\left(\frac{\pi x}{x_0}\right)\frac{\pi}{x_0}.v_0\sin\left(\frac{\pi x}{x_0}\right)$

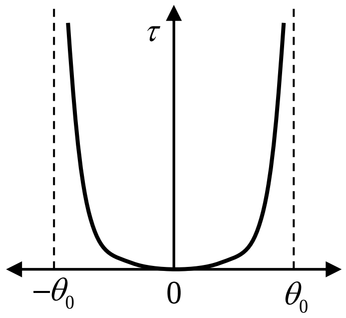
$$a(x)=\frac{\pi v_0^2}{2x_0}\sin\left(\frac{2\pi x}{x_0}\right)\Rightarrow a(x)\approx\frac{\pi v_0^2}{2x_0}\left(\frac{2\pi x}{x_0}\right)=\frac{\pi^2 v_0^2}{x_0^2}x.$$

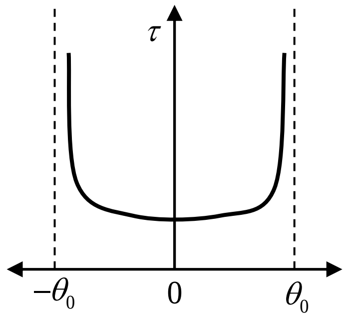
So motion is not oscillatory.

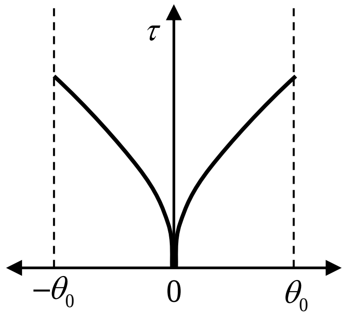
$$\frac{d^2x}{dt^2}-k^2x=0\Rightarrow x=Ae^{kt}+Be^{-kt}\text{ where }k=\frac{\pi v_0}{x_0}$$

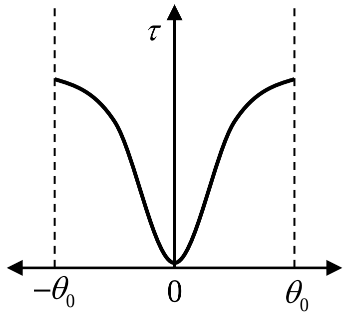
As $t\to\infty$, $x=Ae^{kt}$ if we assume k small and t is large we can assume x is some fixed quantity so (c) is the correct choice.

Q52. A pendulum executes small oscillations between angles $+\theta_0$ and $-\theta_0$. If $\tau(\theta)d\theta$ is the time spent between θ and $\theta+d\theta$, then $\tau(\theta)$ is best represented by

(a)


(b)


(c)


(d)


Topic: Classical Mechanics

Sub Topic: Small Oscillation

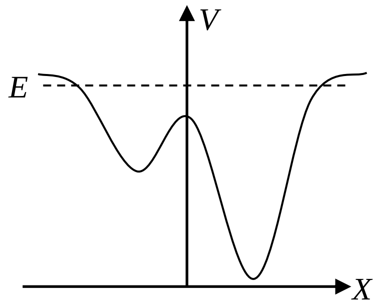
Ans. : (b)

Solution: $E=\frac{1}{2}ml^2\dot{\theta}^2-mgl\cos\theta$ for simplicity we can assume $\frac{1}{2}ml^2=1$ and $mgl=1$ and $E=1$

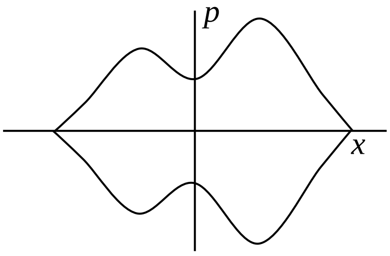
$$\Rightarrow\frac{d\theta}{dt}=\sqrt{1+\cos\theta}\Rightarrow dt=\sqrt{2}\sec\left(\frac{\theta}{2}\right)d\theta=\frac{dt}{d\theta}d\theta=\sqrt{2}\sec\left(\frac{\theta}{2}\right)d\theta$$

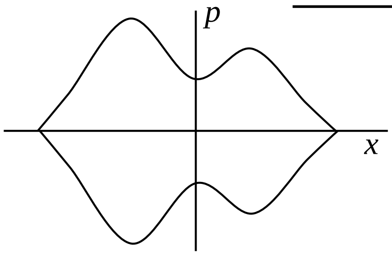
$$\Rightarrow\tau(\theta)d\theta=\sqrt{2}\sec\left(\frac{\theta}{2}\right)d\theta$$

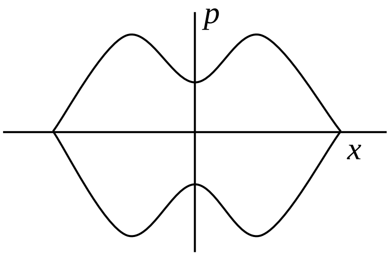
Q53. Consider a particle with total energy E moving in one dimension in a potential $V(x)$ as shown in the figure below.

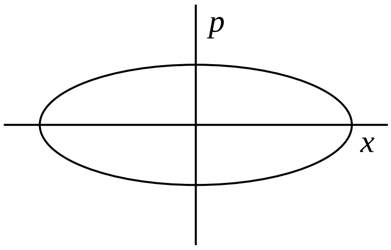


Which of the following figures best represents the orbit of the particle in the phase space?

(a)


(b)


(c)


(d)


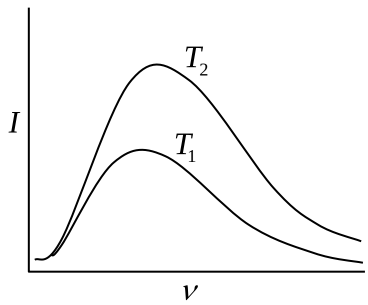
Topic: Classical Mechanics

Sub Topic: Phase Space

Ans. : (a)

Solution: Use concept $T = E - V$ where T is kinetic energy E is total energy and V is potential energy.

Q54. The energy density I of a black body radiation at temperature T is given by the Planck's distribution function $I(\nu,T) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{\left(e^{\frac{h\nu}{k_BT}} - 1\right)}$, where ν is the frequency. The function $I(\nu,T)$ for two different temperatures T_1 and T_2 are shown in figure.



If the two curves coincide when $I(\nu,T)\nu^a$ is plotted against ν^b/T, then the values of a and b are, respectively,

(a) 2 and 1 (b) -2 and 2 (c) 3 and -1 (d) -3 and 1

Topic: Statistical Mechanics

Sub Topic: Black Body Radiation

Ans. : (d)

Solution: $I=\frac{8\pi\nu^3}{c^3}\frac{h}{\left(e^{\frac{h\nu}{k_BT}}-1\right)}\Rightarrow y=I\nu^a=\frac{8\pi\nu^{a+3}}{c^3}\frac{h}{\left(e^{\frac{h\nu}{k_BT}\nu^b\nu^{-b}}-1\right)}$

Let $x=\nu^b/T \quad \Rightarrow y=\frac{8\pi h}{c^3}\frac{\nu^{a+3}}{\left(e^{\frac{h\nu^{-b+1}}{k_B}x}-1\right)}$

For, $a=-3,\ b=1;\qquad y=\alpha\frac{1}{\left(e^{\beta x}-1\right)}$ Both graph are now same.

Q55. For an ideal gas consisting of N distinguishable particles in a volume V, the probability of finding exactly 2 particles in a volume $\delta V \ll V$, in the limit $N,V\to\infty$, is

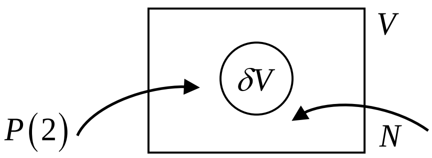
(a) $2N\delta V/V$ (b) $\left(N\delta V/V\right)^2$ (c) $\frac{\left(N\delta V\right)^2}{2V^2}e^{-N\delta V/V}$ (d) $\left(\frac{\delta V}{V}\right)^2 e^{-N\delta V/V}$

Topic: Statistical Mechanics

Sub Topic: Probability Distribution

Ans. : (c)

Solution:



We can use poisons here $f(x)=\frac{\mu^x}{x!}e^{-\mu}$ where $\mu=N\left(\frac{\delta V}{V}\right)$

$$f(2)=\frac{\left[N\left(\frac{\delta V}{V}\right)\right]^2}{2!}\cdot e^{-N\left(\frac{\delta V}{V}\right)}=\frac{\left(N\delta V\right)^2}{2V^2}e^{-N\left(\frac{\delta V}{V}\right)}$$

Q56. The Hamiltonian of a system of 3 spins is $H=J\left(S_1S_2+S_2S_3\right)$, where $S_i=\pm1$ for $i=1,2,3$. Its canonical partition function, at temperature T, is

(a) $2\left(2\sinh\frac{J}{k_BT}\right)^2$ (b) $2\left(2\cosh\frac{J}{k_BT}\right)^2$ (c) $2\left(2\cosh\frac{J}{k_BT}\right)$ (d) $2\left(2\cosh\frac{J}{k_BT}\right)^3$

Topic: Statistical Mechanics

Sub Topic: Canonical Ensemble Theory

Ans. : (b)

Solution:

S_1	S_2	S_3	H
1	1	1	2J
1	1	-1	0
1	-1	1	0
1	-1	-1	-2J
-1	1	1	0
-1	1	-1	-2J
-1	-1	1	0
-1	-1	-1	2J

Number of states $2^3 = 8$

$$H = J\left(S_1S_2 + S_2S_3\right)$$

$$Z = 2e^{-\beta 2J} + 2e^{\beta 2J} + 4 = 2\left[e^{\beta 2J} + e^{-\beta 2J}\right] + 4 = 2\left(\left[e^{\beta J} + e^{-\beta J}\right]^2 - 2\right) + 4$$

$$\Rightarrow Z = 2\left(\frac{2\left(e^{\beta J} + e^{-\beta J}\right)}{2}\right)^2 = 2\left(2\cosh\frac{J}{k_BT}\right)^2$$

Q57. A certain two-dimensional solid crystallises to a square monoatomic lattice with lattice constant a. Each atom can contribute an integer number of free conduction electrons. The minimum number of electrons each atom must contribute such that the free electron Fermi circle at zero temperature encloses the first Brillouin zone completely, is

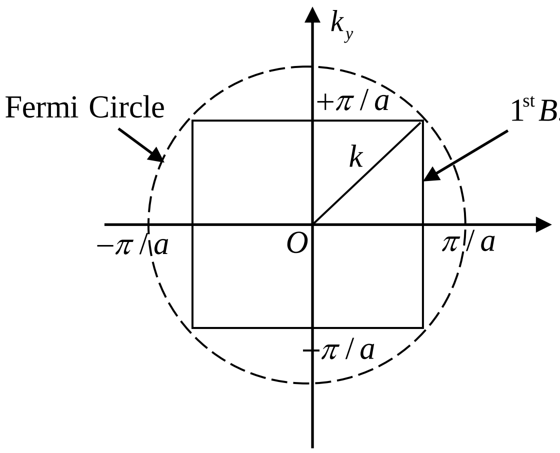
(a) 3 (b) 1 (c) 4 (d) 2

Topic: Condensed Matter Physics

Sub Topic: Free Electron Theory

Ans. : (c)

Solution: Brillouin zone of a square lattice of lattice constant 'a' is also a square as shown below



The radius of the Fermi circle in two-dimension is

$$k_F = (2\pi n)^{1/2} = \left(2\pi\frac{N}{a^2}\right)^{1/2}$$

$$k = \frac{\sqrt{2}\pi}{a} = \frac{4.443}{a}$$

The fermi circle will enclose the 1^{st} B.Z completely

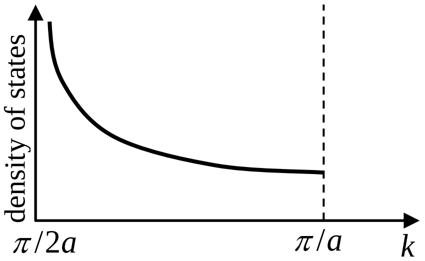
For $N=3, k_F=\dfrac{4.34}{a}$ which is less than $\dfrac{4.443}{a}$

For $N=4, k_F=\dfrac{5.01}{a}>\dfrac{4.443}{a}$ so it will encloses the first Brillouin zone completely

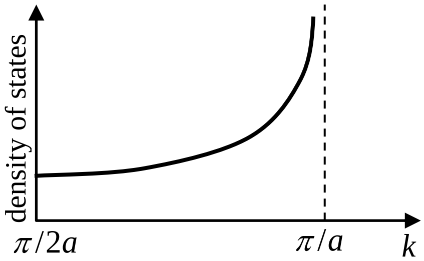
Thus $N=4$ is correct option.

Q58. A tight binding model of electrons in one dimension has the dispersion relation $\varepsilon(k)=-2t(1-\cos ka)$, where $t>0, a$ is the lattice constant and $-\dfrac{\pi}{a}<k<\dfrac{\pi}{a}$. Which of the following figures best represents the density of states over the range $\dfrac{\pi}{2a}\le k<\dfrac{\pi}{a}$?

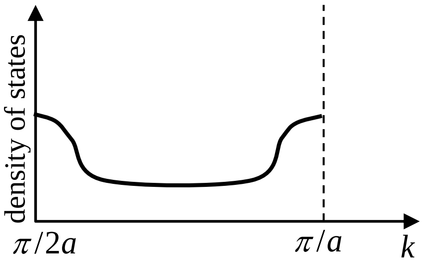
(a)



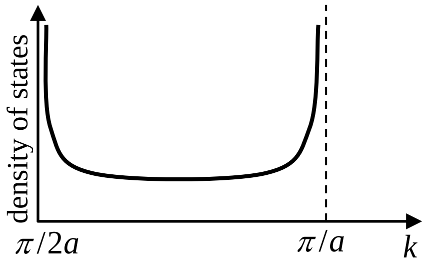
(b)



(c)



(d)



Topic: Condensed Matter Physics

Sub Topic: density of state

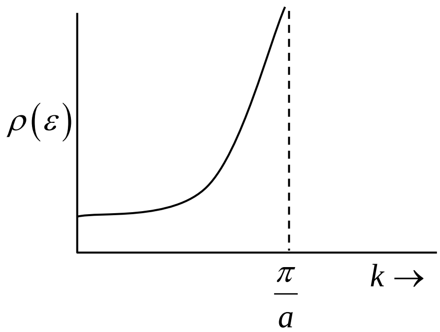
Ans. : (b)

Solution: $\varepsilon(k)=-2t(1-\cos ka)\Rightarrow d\varepsilon(k)=-2ta\sin(ka)dk$

Now, $g(k)dk=\dfrac{L}{\pi}dk \Rightarrow g(\varepsilon)d\varepsilon=\dfrac{L}{\pi}\cdot\dfrac{d\varepsilon(k)}{-2ta\sin(ka)}$

Density of state is $\rho(\varepsilon)=\dfrac{g(\varepsilon)d\varepsilon}{d\varepsilon}=\dfrac{L}{-2\pi ta}\cdot\dfrac{1}{\sin(ka)}$

at $k=\dfrac{\pi}{2a}$: $\quad \rho(\varepsilon)=\dfrac{2}{2\pi ta}\cdot\dfrac{1}{\sin\left(\dfrac{\pi}{2a}\times a\right)}=\dfrac{L}{2\pi ta}$



at $k=\frac{\pi}{a}$: $\rho(\varepsilon)=\frac{L}{2\pi ta}\cdot\frac{1}{\sin\left(\frac{\pi}{a}\times a\right)}=\infty$

Thus, variation of $\rho(\varepsilon)$ vs k is

Thus, correct option is (b)

Q59. A lattice is defined by the unit vectors $\vec{a}_1=a\hat{i},\vec{a}_2=-\frac{a}{2}\hat{i}+\frac{a\sqrt{3}}{2}\hat{j}$ and $\vec{a}_3=a\hat{k}$, where $a>0$ is a constant. The spacing between the (100) planes of the lattice is

(a) $\sqrt{3}a/2$ (b) $a/2$ (c) a (d) $\sqrt{2}a$

Topic: Condensed Matter Physics

Sub Topic: Crystallography

Ans. : (a)

Solution: Interplanar spacing for Hexagonal lattice is $\frac{1}{d^2}=\frac{4}{3}\left(\frac{h^2+hk+k^2}{a^2}\right)+\frac{l^2}{c^2}$

Here $|a|=|a_1|=a,\quad |b|=|a_2|=a$ and $|c|=|a_3|=a$

For (100) plane

$$\frac{1}{d^2}=\frac{4}{3}\left(\frac{1+0+0}{a^2}\right)+\frac{0}{c^2}\Rightarrow\frac{1}{d^2}=\frac{4}{3a}\Rightarrow d=\frac{\sqrt{3}}{2}a$$

Q60. A spacecraft of mass $m=1000\,kg$ has a fully reflecting sail that is oriented perpendicular to the direction of the sun. The sun radiates $10^{26}W$ and has a mass $M=10^{30}\,kg$. Ignoring the effect of the planets, for the gravitational pull of the sun to balance the radiation pressure on the sail, the area of the sail will be

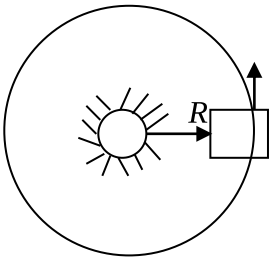
(a) $10^2\,m^2$ (b) $10^4\,m^2$ (c) $10^8\,m^2$ (d) $10^6\,m^2$

Topic: Electromagnetic Theory

Sub Topic: Radiation

Ans. : (d)

Solution: $m=10^3\,kg$, $M=10^{30}\,kg$, $P=10^{26}W$



Radiation pressure for fully reflecting Surface $=\frac{2I}{c}$

$I=$ Intensity $=\frac{\text{Power}}{\text{Area}}=\frac{P}{4\pi R^2}$

Radiation Pressure $=\frac{2P}{4\pi R^2 c}$

Gravitational pull $=\frac{GMm}{R^2}$

Force on sail $=$Radiation Press $\times$ Area of sail $=\dfrac{2P}{4\pi R^2 c}\times A$

$$\frac{P}{2\pi R^2 c}\times A=\frac{GMm}{R^2}\Rightarrow A=\frac{GMm\times 2\pi c}{P}\Rightarrow A=\frac{6.67\times10^{-11}\times10^{30}\times10^{3}\times2\pi\times3\times10^{8}}{10^{26}}$$

$$\Rightarrow A=6.67\times6\pi\times10^{4}=125.72\times10^{4}=1.25\times10^{6}$$

Option (d) is correct.

Q61. The electric field due to a uniformly charged infinite line along the z - axis, as observed in the rest frame S of the line charge, is $\vec{E}(\vec{r})=\dfrac{\lambda}{2\pi\in_0}\dfrac{x\hat{i}+y\hat{j}}{(x^2+y^2)}$. In a frame M moving with a constant speed v with respect to S along the z - direction, the electric field $\vec{E}'$ is (in the following $\beta=v/c$ and $\gamma=1/\sqrt{1-\beta^2}$)

(a) $E_x'=E_x$ and $E_y'=E_y$

(b) $E_x'=\beta\gamma E_x$ and $E_y'=\beta\gamma E_y$

(c) $E_x'=E_x/\gamma$ and $E_y'=E_y/\gamma$

(d) $E_x'=\gamma E_x$ and $E_y'=\gamma E_y$

Topic: Electromagnetic Theory

Sub Topic: Relativistic Electrodynamics

And. : (d)

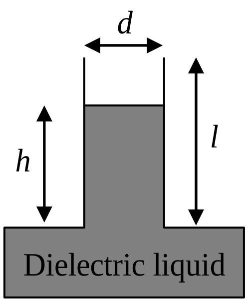
Solution: M is moving in z -direction

$E_z'=E_z$, $E_x'=\gamma(E_x-vB_y)$, $E_y^{'}=\gamma(E_y+vB_x)$ where $B_x=0, B_y=0$

Thus $E_x'=\gamma E_x$ and $E_y'=\gamma E_y$.

Q62. A parallel plate capacitor with rectangular plates of length l, breadth b and plate separation d, is held vertically on the surface of a dielectric liquid of dielectric constant κ and density ρ as shown in the figure. The length and breadth are large enough for edge effects to be neglected.

The plates of the capacitor are kept at a constant voltage difference V. Ignoring effects of surface tension, the height h upto which the liquid level rises inside the capacitor, is



(a) $\dfrac{V^2\varepsilon_0(\kappa-1)}{\rho gbd}$

(b) $\dfrac{V^2\varepsilon_0(\kappa-1)}{2\rho gb^2}$

(c) $\dfrac{V^2\varepsilon_0(\kappa-1)}{2\rho gd^2}$

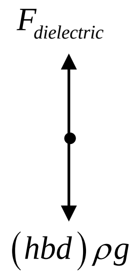
(d) $\dfrac{V^2\varepsilon_0(\kappa-1)}{\rho gd^2}$

Topic: Electromagnetic Theory

Sub Topic: Electrostatics

Ans. : (c)

Solution: It is basically a force balance problem. $F_{dielectric} = \frac{\varepsilon_0 b V^2}{2d}(\kappa - 1)$



If liquid rises to height h, then at equilibrium,

$$\frac{\varepsilon_0 b V^2 (\kappa - 1)}{2d} = (hbd)\rho g\,; \qquad h = \frac{\varepsilon_0 V^2 (\kappa - 1)}{2d^2 \rho g}$$

Q63. Using the following values of x and $f(x)$

x	0	0.5	1.0	1.5
$f(x)$	1	a	0	$-5/4$

the integral $I = \int_0^{1.5} f(x)\,dx$, evaluated by the Trapezoidal rule, is $5/16$. The value of a is

(a) $3/4$ (b) $3/2$ (c) $7/4$ (d) $19/24$

Topic: Mathematical Physics

Sub Topic: Numerical Technique

Ans. : (a)

Solution: $I = \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + \cdots)\right] = \frac{1}{4}\left[1 - \frac{5}{4} + 2(a + 0)\right] = \frac{5}{16}$

$$\Rightarrow 1 - \frac{5}{4} + 2a = \frac{5}{4}$$

$$\Rightarrow 2a = \frac{10}{4} - 1 \Rightarrow 2a = \frac{6}{4} \Rightarrow a = \frac{3}{4}$$

Q64. The Green's function for the differential equation $\frac{d^2x}{dt^2} + x = f(t)$, satisfying the initial conditions $x(0) = \frac{dx}{dt}(0) = 0$ is

$$G(t,\tau) = \begin{cases} 0 & \text{for} \quad 0 < t < \tau \\ \sin(t - \tau) & \text{for} \quad t > \tau \end{cases}$$

The solution of the differential equation when the source $f(t) = \theta(t)$ (the Heaviside step function) is

(a) $\sin t$ (b) $1 - \sin t$ (c) $1 - \cos t$ (d) $\cos^2 t - 1$

Topic: Mathematical Physics

Sub Topic: Green Function

Ans. : (c)

Solution: $\frac{d^2x}{dt^2}+x=f(t)$ and $x(0)=\dot{x}(0)=0$

$$G(t,\tau)=\begin{cases}0, & 0<t<\tau\\ \sin(t-\tau), & t>\tau\end{cases}$$

$$x(t)=\int_0^\infty G(t,\tau)f(\tau)d\tau$$

$$\Rightarrow x(t)=\int_0^t \sin(t-\tau)f(\tau)d\tau=\int_0^t \sin(t-\tau)d\tau=+\cos(t-\tau)\Big|_0^t=1-\cos t$$

Q65. The solution of the differential equation $\left(\frac{dy}{dx}\right)^2-\frac{d^2y}{dx^2}=e^y$, with the boundary conditions $y(0)=0$ and $y'(0)=-1$, is

(a) $-\ln\left(\frac{x^2}{2}+x+1\right)$ (b) $-x\ln(e+x)$ (c) $-xe^{-x^2}$ (d) $-x(x+1)e^{-x}$

Topic: Mathematical Physics

Sub Topic: Differential Equation

Ans. : (a)

Solution: $\left(\frac{dy}{dx}\right)^2-\frac{d^2y}{dx^2}=e^y$ put $y=\ln p$

$$\frac{dy}{dx}=\frac{1}{p}\frac{dp}{dx}\Rightarrow\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{1}{p}\frac{dp}{dx}\right)=\frac{1}{p}\frac{d^2p}{dx^2}-\frac{1}{p^2}\left(\frac{dp}{dx}\right)^2$$

Thus $\left(\frac{1}{p}\frac{dp}{dx}\right)^2-\frac{1}{p}\frac{d^2p}{dx^2}+\frac{1}{p^2}\left(\frac{dp}{dx}\right)^2=p$

$$\frac{2}{p^2}\left(\frac{dp}{dx}\right)^2-\frac{1}{p}\frac{d^2p}{dx^2}=p\Rightarrow\frac{2}{p^3}\left(\frac{dp}{dx}\right)^2-\frac{1}{p^2}\frac{d^2p}{dx^2}=1\Rightarrow\frac{1}{p^2}\frac{d^2p}{dx^2}-\frac{2}{p^3}\left(\frac{dp}{dx}\right)^2=-1$$

$$\Rightarrow\frac{d}{dx}\left(\frac{1}{p^2}\frac{dp}{dx}\right)=-1$$

Let $\frac{1}{p^2}\frac{dp}{dx}=z\Rightarrow\frac{dz}{dx}=-1\Rightarrow z=-x+c$

Thus $\frac{1}{p^2}\frac{dp}{dx}=-x+c\Rightarrow\int\frac{dp}{p^2}=\int(-x+c)dx$

$$-\frac{1}{p}=-\frac{x^2}{2}+cx+d\Rightarrow p=\frac{1}{\frac{x^2}{2}-cx-d}$$

$$y=\ln p=\ln\left(\frac{1}{\frac{x^2}{2}-cx-d}\right)=\ln\left(\frac{x^2}{2}-cx-d\right)$$

$$y(0)=0\Rightarrow y(0)=-\ln(-d)\Rightarrow d=-1$$

$$y=-\ln\left(\frac{x^2}{2}-cx+1\right)$$

$$y'(x)=-\frac{1}{\left(\frac{x^2}{2}-cx+1\right)}(x-c),\;\; y'(0)=-1\Rightarrow-\frac{(-c)}{1}=c=-1,\;\; y=-\ln\left(\frac{x^2}{2}+x+1\right)$$

Option (a) is correct.

Q66. If we take the nuclear spin I into account, the total angular momentum is $\vec{F}=\vec{L}+\vec{S}+\vec{I}$, where $\vec{L}$ and $\vec{S}$ are the orbital and spin angular momenta of the electron. The Hamiltonian of the hydrogen atom is corrected by the additional interaction $\lambda\vec{I}\cdot\left(\vec{L}+\vec{S}\right)$, where $\lambda>0$ is a constant. The total angular momentum quantum number F of the p - orbital state with the lowest energy is

(a) 0 (b) 1 (c) $1/2$ (d) $3/2$

Topic: Atomic & Molecular Physics

Sub Topic: Hyperfine Structure

Ans. : (b)

Solution: $\vec{F}=\vec{L}+\vec{S}+\vec{I}=\vec{J}+\vec{I}$; $H=\lambda\vec{I}\cdot\left(\vec{L}+\vec{S}\right)=\lambda\vec{I}\cdot\vec{J}$

$$\vec{F}=\vec{J}+\vec{I}\Rightarrow F^2=J^2+I^2+2\vec{I}\cdot\vec{J}\Rightarrow\vec{I}.\vec{J}=\frac{F^2-J^2-I^2}{2}\Rightarrow H=\lambda\frac{F^2-I^2-J^2}{2}$$

Now we can find all possible quantum number

For hydrogen atom, p -orbital electron $L=1, S=\frac{1}{2}\rightarrow J=\frac{1}{2},\frac{3}{2}$

Now for $J=\frac{1}{2}$ and $I=\frac{1}{2}$ the possible value of $F=0,1$

Now for $J=\frac{3}{2}$ and $I=\frac{1}{2}$ the possible value of $F=1,2$

$$\Delta E=\lambda\frac{F(F+1)-I(I+1)-J(J+1)}{2}\hbar^2$$ for minimum energy

The value of $I=\frac{1}{2}$ is fixed so for minimum value of ΔE J is maximum and F is minimum

$\Delta E_1\left(F=0, I=\frac{1}{2}, J=\frac{1}{2}\right)$; $\Delta E_1=\lambda\frac{0-\frac{3}{4}-\frac{3}{4}}{2}\hbar^2=-\frac{3}{4}\lambda\hbar^2$

$\Delta E_2\left(F=I, I=\frac{1}{2}, J=\frac{1}{2}\right)$; $\Delta E_2=\lambda\frac{2-\frac{3}{4}-\frac{3}{4}}{2}\hbar^2=\frac{1}{4}\lambda\hbar^2$

$\Delta E_3\left(F=1, I=\frac{1}{2}, J=\frac{3}{2}\right)$; $\Delta E_3=\lambda\frac{2-\frac{3}{4}-\frac{15}{4}}{2}\hbar^2=-\frac{5}{4}\lambda\hbar^2$

$\Delta E_4\left(F=2, I=\frac{1}{2}, J=\frac{3}{2}\right)$; $\Delta E_4=\lambda\frac{6-\frac{3}{4}-\frac{15}{4}}{2}=\frac{3}{4}\lambda\hbar^2$

Out of these four possibilities lowest state is corresponding to following quantum number

$$F=1, I=\frac{1}{2}, J=\frac{3}{2}, L=1, S=\frac{1}{2}$$

Q67. The absorption lines arising from pure rotational effects of HCl are observed at $83.03\,cm^{-1}$, $103.73\,cm^{-1}$, $124.30\,cm^{-1}$, $145.03\,cm^{-1}$ and $165.51\,cm^{-1}$. The moment of inertia of the HCl molecule is (take $\frac{\hbar}{2\pi c}=5.6\times10^{-44}\,kg\text{-}m$)

(a) $1.1\times10^{-48}\,kg\text{-}m^2$ (b) $2.8\times10^{-47}\,kg\text{-}m^2$

(c) $2.8\times10^{-48}\,kg\text{-}m^2$ (d) $1.1\times10^{-42}\,kg\text{-}m^2$

Topic: Atomic & Molecular Physics

Sub Topic: Rotational Spectra

Ans.: (b)

Solution: From the experimental data possible value of rotational parameter B

The distance between two absorption line is $2B$

$2B=103.73-83.03=20.70\,cm^{-1}$ $\Rightarrow 2B=124.30-103.73=20.57\,cm^{-1}$

$\Rightarrow 2B=145.03-124.30=20.73\,cm^{-1}$

Hence it is statistical data we can take

Average value $2B=\frac{20.7+20.57+20.73}{3}=\frac{62.00}{3}=20.67\,cm^{-1}\Rightarrow B=10.33\,cm^{-1}=1033\,m^{-1}$

$$B=\frac{h}{8\pi^2 Ic}\Rightarrow I=\frac{h}{8\pi^2 Bc}=\frac{27.99\times10^{-45}}{B}kg-m^2=\frac{27.99\times10^{-45}}{1033}=2.7\times10^{-47}\,kg-m^2$$

$\frac{h}{8\pi^2 c}=27.99\times10^{-45}$ in SI unit.

Q68. The energies of the 3 lowest states of an atom are $E_0 = -14eV, E_1 = -9eV$ and $E_2 = -7eV$. The Einstein coefficients are $A_{10} = 3\times10^8 s^{-1}, A_{20} = 1.2\times10^8 s^{-1}$ and $A_{21} = 8\times10^7 s^{-1}$. If a large number of atoms are in the energy level E_2, the mean radiative lifetime of this excited state is

(a) $8.3\times10^{-9} s$ (b) $1\times10^{-8} s$ (c) $0.5\times10^{-8} s$ (d) $1.2\times10^{-8} s$

Topic: Atomic & Molecular Physics

Sub Topic: Laser

Ans. : (c)

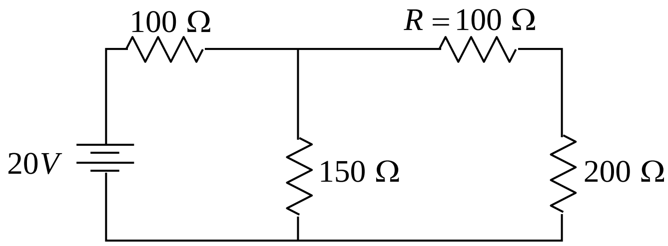
Solution: Rate of spontaneous decay from E_2 state $= (A_{20} + A_{21}) N_1 = A_2 N_1$

$$A_2 = A_{20} + A_{21} = (1.2\times10^8 + 0.8\times10^8) s^{-1} = 2.0\times10^8 s^{-}$$

$\therefore$ Mean radiative life time

$$\tau_2 = \frac{1}{A_2} = \frac{1}{2.0\times10^8} = 0.5\times10^{-8} s$$

Q69. Two voltmeters A and B with internal resistances $2M\Omega$ and $0.1k\Omega$ are used to measure the voltage drops V_A and V_B, respectively, across the resistor R in the circuit shown below.



The ratio V_A/V_B is

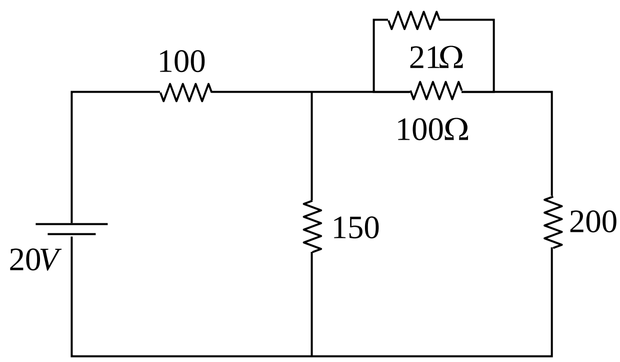
(a) 0.58 (b) 1.73 (c) 1 (d) 2

Topic: Electronics

Sub Topic: Network Analysis

Ans. : (b)

Solution:



Voltmeter A:

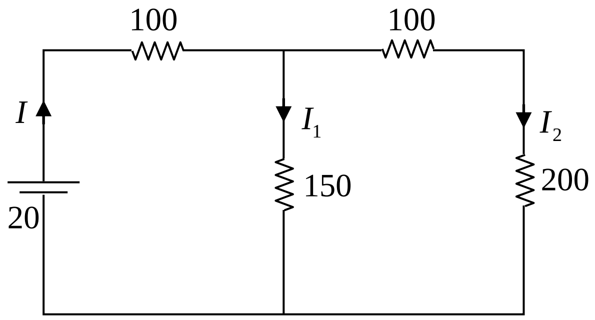
Equivalent resistance: $200\ \Omega$

Total Current $(I_T) = \frac{20}{200} = 0.1\, Ampere$

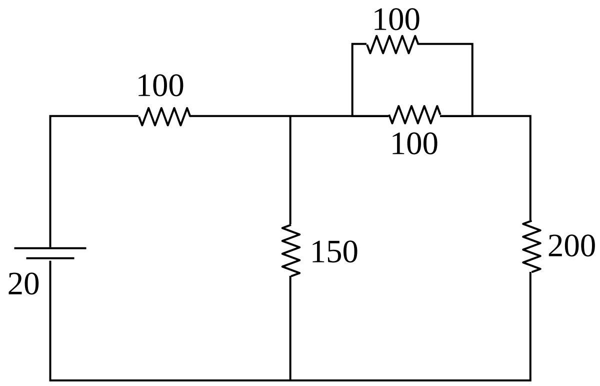
$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T$$

$$I_2 = \frac{150}{4500} = \frac{1}{30}\,Ampere$$

$$V_A = 100 \times \frac{1}{30} = \frac{10}{3}V$$



Voltmeter B:



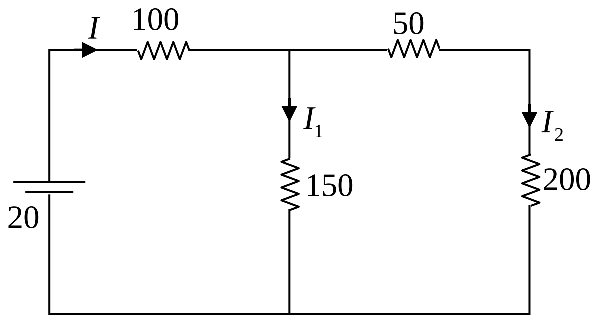
Equivalent resistance: $\frac{775}{4}\,\Omega$

Total Current $(I_T) = \frac{80}{775}\,Ampere$

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T = \frac{6}{155}\,Ampere$$

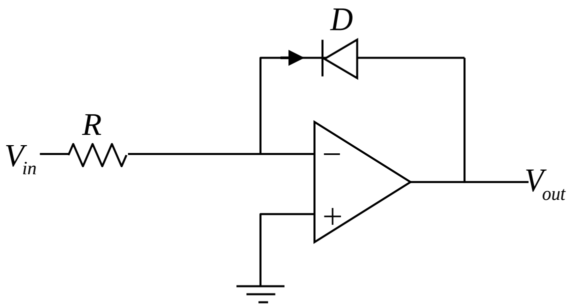
$$V_B = 50 \times \frac{6}{155} = \frac{60}{31}V$$

$$\frac{V_A}{V_B} = 1.73$$



Q70. The I-V characteristics of the diode D in the circuit below is given by

$$I = I_s\left(e^{\frac{qV}{k_B T}} - 1\right)$$



where I_s is the reverse saturation current, V is the voltage across the diode and T is the absolute temperature.

If the input voltage is V_{in}, then the output voltage V_{out} is

(a) $I_S R \ln\left(\frac{qV_{in}}{k_B T} + 1\right)$

(b) $\frac{1}{q} k_B T \ln\left(\frac{q(V_{in} + I_S R)}{k_B T}\right)$

(c) $\frac{1}{q} k_B T \ln\left(\frac{V_{in}}{I_S R} + 1\right)$

(d) $-\frac{1}{q} k_B T \ln\left(\frac{V_{in}}{I_S R} + 1\right)$

Topic: Electronics

Sub Topic: OPAMP

Ans. : (d)

Solution: $\because I = I_R \qquad \Rightarrow I_S\left(e^{-qV_O/k_BT}-1\right)=\frac{(V_{in})}{R}$

$$\Rightarrow e^{-qV_O/k_BT}-1=\frac{V_{in}}{I_SR}\Rightarrow e^{-qV_O/k_BT}=\frac{V_{in}}{I_SR}+1\Rightarrow V_O=-\frac{k_BT}{q}\ln\left(\frac{V_{in}}{I_SR}+1\right)$$

Q71. A rod pivoted at one end is rotating clockwise 25 times a second in a plane. A video camera which records at a rate of 30 frames per second is used to film the motion. To someone watching the video, the apparent motion of the rod will seem to be

(a) 10 rotations per second in the clockwise direction

(b) 10 rotations per second in the anticlockwise direction

(c) 5 rotations per second in the clockwise direction

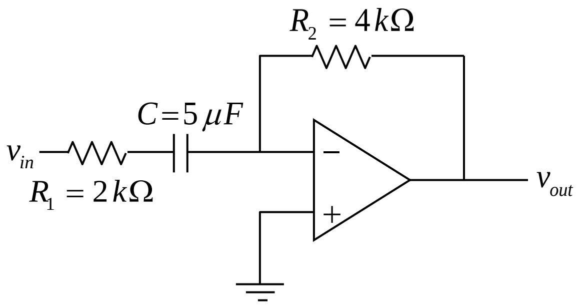
(d) 5 rotations per second in the anticlockwise direction

Topic: Classical Mechanics

Sub Topic: Rotation

Ans. : (d)

Q72. In the circuit shown below, the gain of the op-amp in the middle of its bandwidth is 10^5. A sinusoidal voltage with angular frequency $\omega=100$ rad/s is applied to the input of the op--amp.



The phase difference between the input and the output voltage is

(a) $5\pi/4$ (b) $3\pi/4$ (c) $\pi/2$ (d) π

Topic: Electronics

Sub Topic: OPAMP

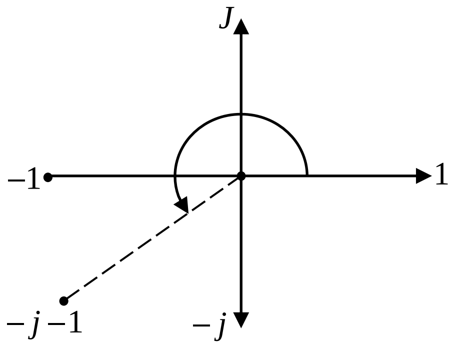
Ans. : (a)

Solution: $\frac{v_0}{v_m}=\frac{-R_2}{R_1+X_C}$

$$\Rightarrow \frac{v_0}{v_m}=\frac{-4\times10^3}{2\times10^3+\frac{1}{j\times100\times5\times10^{-6}}}=\frac{2}{j-1}$$

$$\Rightarrow \frac{v_0}{v_m}=\frac{2}{j-1}\times\frac{j+1}{j+1}=-j-1$$

So phase difference $\phi=\pi+\frac{\pi}{4}=\frac{5\pi}{4}$



Q73. Charged pions π^- decay to muons μ^- and anti-muon neutrinos $\bar{v}_\mu$; $\pi^- \to \mu^- + \bar{v}_\mu$. Take the rest masses of a muon and a pion to be $105\,MeV$ and $140\,MeV$, respectively. The probability that the measurement of the muon spin along the direction of its momentum is positive, is closest to

(a) 0.5 (b) 0.75 (c) 1 (d) 0

Topic: Nuclear & Particle Physics

Sub Topic: Particle Physics

Ans. : (c)

Q74. The binding energy B of a nucleus is approximated by the formula

$B = a_1 A - a_2 A^{2/3} - a_3 Z^2 A^{-1/3} - a_4 (A - 2Z)^2 A^{-1}$, where Z is the atomic number and A is the mass number of the nucleus. If $\frac{a_4}{a_3} \approx 30$. The atomic number Z for naturally stable isobars (constant value of A) is

(a) $\frac{30A}{60 + A^{2/3}}$ (b) $\frac{30A}{30 + A^{2/3}}$ (c) $\frac{60A}{120 + A^{2/3}}$ (d) $\frac{120A}{60 + A^{2/3}}$

Topic: Nuclear & Particle Physics

Sub Topic: Liquid Drop Model

Ans. : (c)

Solution: $B = a_1 A - a_2 A^{2/3} - a_3 Z^2 A^{-1/3} - a_4 (A - 2Z)^2 A^{-1}$

For most isobar $\frac{\partial B}{\partial Z} = 0 \Rightarrow -\frac{a_3 (2Z)}{A^{1/3}} - \frac{a_4 2(A - 2Z)(-2)}{A} = 0$

$$\Rightarrow a_3 \frac{Z}{A^{1/3}} = 2a_4 \frac{A}{A} - 4a_4 \frac{Z}{A} \Rightarrow \frac{Z}{A}\left(a_3 A^{2/3} + 4a_4\right) = 2a_4$$

$$\Rightarrow Z = \frac{2a_4 A}{a_3 A^{2/3} + 4a_4} = \frac{A}{2 + \frac{a_3}{2a_4} A^{2/3}} \Rightarrow Z = \frac{A}{2 + \frac{1}{60} A^{2/3}} = \frac{60A}{120 + A^{2/3}}$$

Q75. The magnetic moments of a proton and a neutron are $2.792\,\mu_N$ and $-1.913\,\mu_N$, where μ_N is the nucleon magnetic moment. The values of the magnetic moments of the mirror nuclei ${}^{19}_{9}F_{10}$ and ${}^{19}_{10}Ne_9$, respectively, in the Shell model, are closest to

(a) $23.652\,\mu_N$ and $-18.873\,\mu_N$ (b) $26.283\,\mu_N$ and $-16.983\,\mu_N$

(c) $-2.628\,\mu_N$ and $1.887\,\mu_N$ (d) $2.628\,\mu_N$ and $-1.887\,\mu_N$

Topic: Nuclear & Particle Physics

Sub Topic: Shell Model

Ans. : (d)

Solution: ${}^{19}_{9}F_{10} : p(9) : 1s^2_{1/2}1p^4_{3/2}1p^2_{1/2}1d^1_{5/2}$

$l = 2, s = \frac{1}{2} \quad j = \frac{5}{2} = 2 + \frac{1}{2}$

Hence there is unpaired proton in outer most orbit

so $\langle \mu_Z \rangle_{F^{19}} = \mu_N (j + 2.29) = \mu_N (2.5 + 2.29) = 4.79\mu_N$

Hence there is unpaired neutron ${}^{19}_{10}Ne_9 : N(9) : 1s^2_{1/2}1p^4_{3/2}1p^2_{1/2}1d^1_{5/2}$

$j = \frac{5}{2} = l + \frac{1}{2}$

$\langle \mu_z \rangle_{Ne^{19}} = -1.91\mu_N$ These values are closet to option (d)