Chapter 1 Wave Function

Problem 1.5: Consider the wave function

$$\psi(x,t) = Ae^{-\lambda|x|}e^{i\omega t}$$

Where A, λ and ω are positive real constants.

(a) Normalize ψ

(b) Determine the expectation values of x and x^2

(c) Find the standard deviation of x. Sketch the graph of $|\psi|^2$, as a function of x, and mark the points $(\langle x \rangle + \sigma)$ and , to illustrate the sense in which σ represents the "spread" in x. What is the probability that the particle would be found outside this range?

Solution: (a) $\psi(x,t) = Ae^{-\lambda|x|}e^{i\omega t}$

$$\int_{-\infty}^{+\infty} |\psi|^2 dx = 1 \Rightarrow |A|^2 \int_{-\infty}^{+\infty} e^{-2\lambda|x|} dx = |A|^2 \, 2\int_0^{\infty} e^{-2\lambda x} dx = 1$$

$$|A|^2 \cdot 2 \cdot \frac{e^{-2\lambda x}}{-2\lambda} \int_0^{\infty} = 1$$

$$|A|^2 \cdot \frac{1}{\lambda} = 1 \Rightarrow A = \sqrt{\lambda}$$

$$\int_0^{\infty} x^x e^{-\lambda x} = \frac{1}{\lambda(x+1)} \sqrt{(x+1)}$$

(b) $$\int_{-\infty}^{\infty} x|\psi|^2 dx = \lambda \int_{-\infty}^{\infty} xe^{-2\lambda|x|} dx = 0$$

$$2 \cdot \lambda \int_0^{\infty} x^2 e^{-2\lambda x} dx$$

$$2 \cdot \lambda \frac{1}{(2\lambda)^3} \sqrt{3} = \frac{2 \cdot \lambda x}{8 \cdot \lambda^2} x^{|x|} = \frac{1}{2\lambda^2}$$

(c) $$\sigma = \frac{1}{\sqrt{2}\lambda}$$

$$P\left(-\sqrt{2/\lambda} < x, x > \sqrt{2/\lambda}\right)$$

$$\lambda 2x \int_{\sqrt{2/\lambda}}^{\infty} e^{-2\lambda x} dx = 2\lambda x \frac{e^{-2\lambda x}}{2\lambda} = 2\lambda x \frac{e^{-2\sqrt{2}}}{2\lambda} = e^{-\sqrt{2}} = 0.2431$$