

## Chapter 1 Wave Function

**Problem 1.7:** By using Ehrenfest theorem, calculate  $\frac{d\langle p \rangle}{dt}$ .

**Solution:** Ehrenfest theorem

Prove that  $\frac{d\langle P \rangle}{dt} = \left\langle -\frac{\partial V}{\partial x} \right\rangle$

$$\frac{d\langle A \rangle}{dt} = \frac{1}{i\hbar} \langle [A, H] \rangle + \left\langle \frac{\partial A}{\partial t} \right\rangle$$

$$\frac{d\langle P \rangle}{dt} = \frac{1}{i\hbar} \langle [P, H] \rangle + \left\langle \frac{\partial P}{\partial t} \right\rangle \Rightarrow \frac{1}{i\hbar} \left[ P, \frac{P^2}{2m} + V(x) \right] + 0$$

$$\frac{d\langle P \rangle}{dt} = \frac{1}{i\hbar} \left\langle \left[ P, \frac{P^2}{2m} \right] \right\rangle + \left\langle \frac{1}{i\hbar} [P, V(x)] \right\rangle$$

$$\frac{1}{i\hbar} [P, V(x)] \psi = \frac{-i\hbar}{i\hbar} \left[ \frac{\partial}{\partial x}, V(x) \right] \psi \Rightarrow \frac{\partial V(x)\psi}{\partial x} - V(x) \frac{\partial \psi}{\partial x} = - \left( \frac{\partial V}{\partial x} \psi + V \frac{\partial \psi}{\partial x} - V \frac{\partial \psi}{\partial x} \right)$$

So  $\frac{d\langle P \rangle}{dt} = \left\langle -\frac{\partial V}{\partial x} \right\rangle$