Chapter 1 Wave Function

Problem 1.7: By using Ehrenfest theorem, calculate $\frac{d\langle p\rangle}{dt}$.

Solution: Ehrenfest theorem

Prove that $\frac{d\langle P\rangle}{dt}=\left\langle -\frac{\partial V}{\partial x}\right\rangle$

$$\frac{d\langle A\rangle}{dt}=\frac{1}{i\hbar}\langle[A,H]\rangle+\left\langle\frac{\partial A}{\partial t}\right\rangle$$

$$\frac{d\langle P\rangle}{dt}=\frac{1}{i\hbar}\langle[P,H]\rangle+\left\langle\frac{\partial P}{\partial t}\right\rangle\Rightarrow\frac{1}{i\hbar}\left[P,\frac{P^2}{2m}+V(x)\right]+0$$

$$\frac{d\langle P\rangle}{dt}=\frac{1}{i\hbar}\left\langle\left[P,\frac{P^2}{2m}\right]\right\rangle+\left\langle\frac{1}{i\hbar}\left[P,V(x)\right]\right\rangle$$

$$\frac{1}{i\hbar}\left[P,V(x)\right]\psi=\frac{-i\hbar}{i\hbar}\left[\frac{\partial}{\partial x},V(x)\right]\psi\Rightarrow\frac{\partial V(x)\psi}{\partial x}-V(x)\frac{\partial\psi}{\partial x}=-\left(\frac{\partial V}{\partial x}\psi+V\frac{\partial\psi}{\partial x}-V\frac{\partial\psi}{\partial x}\right)$$

So $\frac{d\langle P\rangle}{dt}=\left\langle -\frac{\partial V}{\partial x}\right\rangle$