NET-JRF (June 2018)

PART- A

Q1. When a farmer was asked as to how many animals he had, he replied that all but two were cows, all but two were horses and all but two were pigs. How many animals did he have?

(a) 3 (b) 6 (c) 8 (d) 12

Ans. : (a)

Solution: When the farmer has just 3 animals, then all but two can be cows, all but two can be horses and all but two can be pigs. In this case the farmer has 1 cow, 1 horse and 1 pig. All other options violates the statement of the question.

Q2. Nine eleventh of the members of a parliamentary committee are men. Of the men, two-thirds are from the Rajya Sabha. Further, $7/11$ of the total committee members are from the Rajya Sabha. What fraction of the total number are women from the Lok Sabha?

(a) 1/11 (b) 6/11 (c) 2/11 (d) 3/11

Ans. : (a)

Solution: Let the total number of member in the committee be x.Then

Number of men in the committee $=\frac{9x}{11}$

and, number of women in the committee $=\frac{2x}{11}$

Number of men from Rajya Sabha $=\frac{9x}{11}\times\frac{2}{3}=\frac{6x}{11}$

Number of men from Lok Sabha $=\frac{9x}{11}-\frac{6x}{11}=\frac{3x}{11}$

From the question $\frac{7x}{11}$ members are from Rajya Sabha. Hence the number of committee members from Lok Shabha $=x-\frac{7x}{11}=\frac{4x}{11}$. In the Lok Sabha since $\frac{3x}{11}$ members are men, hence $\frac{4x}{11}-\frac{3x}{11}=\frac{x}{11}$ members are women. Hence the fraction of women from Lok Sabha out of the total members of committee $=\frac{x/11}{x}=\frac{1}{11}$

Q3. A librarian is arranging a thirteen-volume encyclopedia on the shelf from left to right in the following order of volume numbers: $8, 11, 5, 4, 9, 1, 7, 6, 10, 3, 12, 2$. In this pattern, where should the volume 13 be placed?

(a) Leftmost (b) Rightmost

(c) Between 10 and 3 (d) Between 9 and 1

Ans. : (c)

Q4. Pick the correct statement:

(a) The sky is blue because Sir C.V. Raman gave the correct explanation.

(b) Copernicus believed that the Sun, and not the Earth, was at the centre of the Solar system.

(c) The sky appears blue when seen from the Moon..

(d) No solar eclipse is visible for an astronaut standing on the Moon.

Ans. : (b)

Solution: In Copernician model of the solar system the solar system was considered Heleiocentric which means the Sun being at the centre of the solar system.

Q5. What is the last digit of $(2017)^{2017}$?

(a) 1 (b) 3 (c) 7 (d) 9

Ans. : (c)

Solution: The last digit of 7^4 is 1. Hence $\left[(2017)^4\right]^{504}.2017$ has its last digit $1\times 7 = 7$. Thus the last digit of $(2017)^{2017}$ is 7

Q6. What is the value of $(1+3+5+7+......+4033)+7983\times 2017$?

(a) 20170000 (b) 20172017 (c) 20171720 (d) 20172020

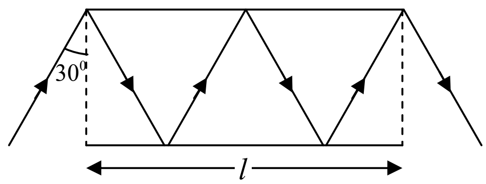
Ans. : (a)

Solution: The number of terms in $1+3+5+7+...4033$ is 2017. Hence

$$(1+3+5+7+...4033)+7983\times 2017$$

$$=\frac{1+4033}{2}\times 2017+7983\times 2017=2017(2017+7983)=20170000$$

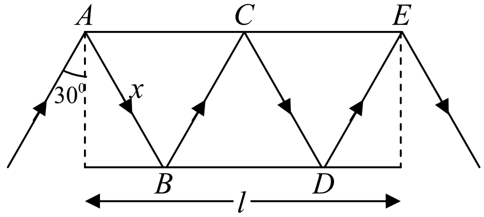
Q7. Path of a ray of light between two mirrors is shown in the diagram. If the length of each mirror is 'l', what is the total path length of the ray between the mirrors?



(a) $\frac{3}{4}l$ (b) $\frac{4}{3}l$ (c) $\frac{3}{2}l$ (d) $2l$

Ans. : (d)

Solution:



If we assume $AB = x$ then it can be proved that BC, CD and DE have the same length x.

From geometry $4x\sin 30^0 = l \Rightarrow 4x = \frac{l}{\sin 30} \Rightarrow 4x = \frac{l}{1/2} = 2l$

Q8. In a group of 11 persons, each shakes hand with every other once and only once. What is the total number of such handshakes?

(a) 110 (b) 121 (c) 55 (d) 66

Ans. : (c)

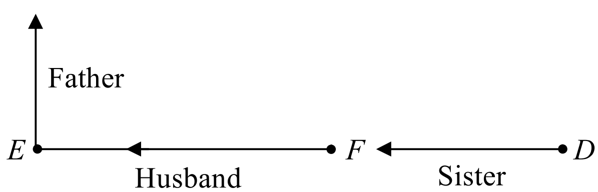
Solution: The total number handshakes is equal to the total number of combinations of 11 objects by taking 2 at a time. Thus total number of handshakes $= {}^{11}C_2 = \frac{11!}{2!9!} = \frac{10\times 11}{2} = 55$

Q9. Suppose (i) "$A * B$" means "A is the father of B", (ii) "$A \Delta B$" means "A is the husband of B", (iii) "$A \nabla B$" means "A is the wife of B", (iv) "$A \square B$" means "A is the sister of B". Which of the following represents "C is the father-in-law of the sister of D"

(a) $C \nabla E * F \square D$ (b) $C * E \nabla F \square D$ (c) $C \Delta E * F \square D$ (d) $C * E \Delta F \square D$

Ans. : (d)

Solution:



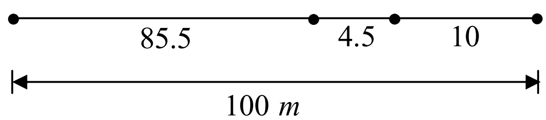
Using the code given in the question we have shown the diagram. Here we see that "C is the father-in-law of sister of D.

Q10. In a $100\ m$ race A beats B by $10m$. B beats C by $5m$. By how many meters does A beat C?

(a) $15.0m$ (b) $5.5\ m$ (c) $10.5\ m$ (d) $14.5\ m$

Ans.: (d)

Solution: When A runs $100m$, B runs $90m$. When B runs $100m$, C runs $95\ m$.



When B runs 90 m, C runs $\frac{95}{100}\times 90 = 85.5$

So in a $100m$ race A will be ahead of C by $100 - 85.5 = 14.5m$

Q11. If all the angles of a triangle are prime numbers, which of the following could be one such angle?

(a) 89^{o} (b) 79^{o} (c) 59^{o} (d) 29^{o}

Ans.: (a)

Solution: If one angle is 89^0, the other two possible angles are 2^0 and 89^0. No other angles are possible.

If we take possible angle as 79^0 then the sum of other two angles must be 101^0. But we can not find any two prime angles whose sum is 101^0.

Similarly, if we take the possible angle to be 59^0 then the sum of other two angles must be 121^0. But we can not find any two prime angles whose sum is 121^0.

Finally, if we take the possible angle to be 29^0, the sum of other two angles must be 151^0. Again we cannot find any two prime angles whose sum is 151^0.

Q12. A water tank that is 40% empty holds $40L$ more water than when it is 40% full. How much water does it hold when it is full?

(a) $100L$ (b) $75L$ (c) $120L$ (d) $200L$

Ans.: (d)

Solution: The water tank is 40% empty means it is 60% full. From the questions

60% of total volume - 40% of total volume $= 40L$

$\Rightarrow 100\%$ of total volume $= 200L$

Q13. How much gold and copper (in g), respectively, are required to make a $120\,g$ bar of 22 carat gold?

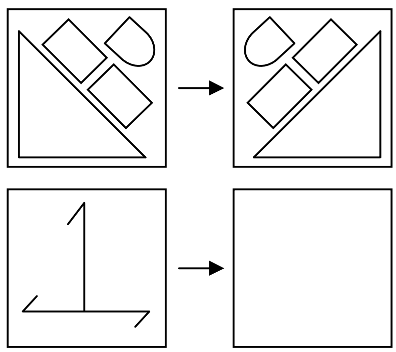
(a) 90 and 30 (b) 100 and 20

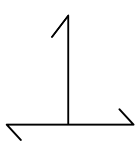
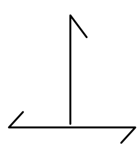
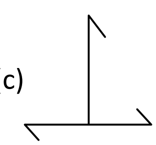
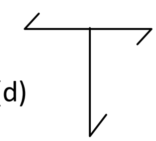
(c) 110 and 10 (d) 120 and 0

Ans.: (c)

Solution: In 22 carat gold, the ratio of the gold and copper is $22:2$, that is $11:1$. Hence in $120\,g$ bar of 22 craft gold, the amount of gold is $\frac{11}{12}\times 120 = 110g$ and $\frac{1}{12}\times 120 = 10g$ is the amount of copper.

Q14. Which of the following be the correct pattern in the empty square?



(a)  (b)  (c)  (d) 

Ans.: (c)

Q15. Areas of the three rectangles inside the full rectangle are given in the diagram

	8
12	4

What is the area of the full rectangle?

(a) 36 (b) 48 (c) 72 (d) 96

Ans.: (b)

Solution: Rectangle B has the same length as rectangle A but its area is twice. Hence width of rectangle B is twice the width of A. From the figure we also see that width of D is twice the width of C. Hence area of rectangle D is $2\times12=24$. Thus the total area of all the rectangles is $24+12+8+4=48$

D	B 8
C 12	A 4

Q16. The university needs to appoint a new Vice Chancellor which will be based on seniority. Ms. West is less senior to Mr. North but more senior to Ms. East. Mr. South is senior to Ms. West but junior to Mr. North. If the senior most declines the assignment, then who will be the new vice Chancellor of the University?

(a) Mr. North (b) Mr. East

(c) Ms. West (d) Mr. South

Ans.: (d)

Solution: Using the seniority statements we can draw the seniority diagram

East $\rightarrow$ West $\rightarrow$ South $\rightarrow$ North

If the senior most declines thee assignment then the next person in thee seniority list will be Mr. south and will be the new vice chancellor

Q17. The prices of diamonds having a particular colour and clarity are tabulated below:

Weight of diamond (in carats)	**Price of diamond (in rupees / carat)**
0.25	1 lakh
0.5	2 lakh
1	4 lakh
2	8 lakh

How many 0.25 carat diamonds can be purchased for the price of a 2 carat diamond?

(a) 8 (b) 16 (c) 32 (d) 64

Ans.: (d)

Solution: The price of a 2 craft diamond is $0.25\times1\text{ lakh}=0.25\text{lakh}$

Hence the number of 0.25 carat diamond that can be purchased for the price of a 2 carat diamond $=\frac{16\text{lakh}}{0.25\text{lakh}}=64$

Q18. In a sequence of 24 positive integers, the product of any two consecutive integer is 24. If the 17^{th} member of the sequence is 6, the 7^{th} member is

(a) 24 (b) 4 (c) 6 (d) 17

Ans.: (c)

Solution: The only possible numbers of the sequence can be those integers which are divisors of 24. Thus $1,2,3,4,6,8,12$ and 24 can be members of the series. From the question the 17^{th} member will be 6 hence 16^{th} and 18^{th} member will be 4. Using the same reasoning we see that the sequence is

$$6,4,6,4,6,4,6,4,6,4,6,4,6,4,6,4,6,4,6,4,6,4,6,4$$

Hence, we conclude that the 7^{th} member of the series is 6.

Q19. Mohan lent Geeta as much money as she already had, she then spent Rs. 10. Next day, he again lent as much money as Geeta now had, and she spent Rs. 10 again. On the third day, Mohan again lent as much money as Geeta now had, and she again spent Rs. 10.If Geeta was left with no money at the end of third day, how much money did she have initially?

(a) Rs.11.25 (b) Rs.10 (c) Rs.7.75 (d) Rs.8.75

Ans.: (d)

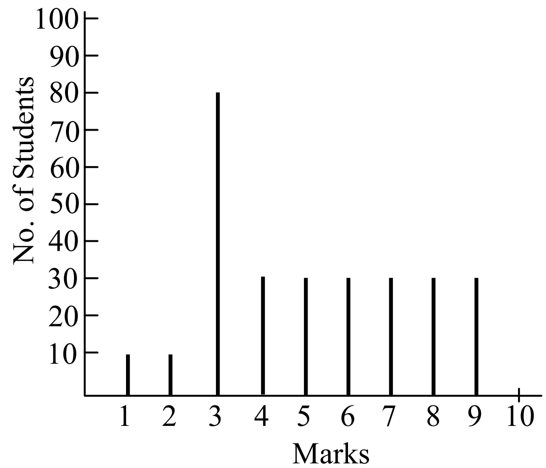
Solution: Let the amount with Geeta be x. When Mohan gives Geeta money for the first time Geeta has $2x$ rupees and after expenditure she has $2x-10$. When Mohan again lents Geeta, she has $4x-20$ and after expenditure she has $4x-30$.

On the third day the amount of money with Geeta is $2(4x-30)-10=8x-70$

From the question $8x-70=0 \Rightarrow x=8.75$

Thus initially Geeta had rupees 8.75

Q20. The distribution of marks of students in a class is given by the following chart:



If 3.30 marks is the passing score in a 10 mark question paper, which of the following is false?

(a) Majority of the students have scored above the pass mark

(b) mode of the distribution is 3

(c) Average marks of passing students is above 55%

(d) Average marks of students who have failed is below 20%

Ans.: (d)

Solution: From the diagram we see that 100 students have scored below the pass marks while 180 students have scored more than 3 marks, hence, majority of students have scored above pass marks.

There are 80 students who have scored 3 marks hence 3 is the mode of distribution.

The average marks obtained by passing students $= \dfrac{30(4+5+6+7+8+9)}{180} = 6.5$

Hence average marks obtained by passing students is above 55%.

Average marks obtained by failed students $= \dfrac{10\times 1 + 10\times 2 + 80\times 3}{100} = 2.7$

We see that average marks obtained by failed students is above 20%, hence (d) is incorrect.

PART B

Q21. Consider the following ordinary differential equation

$$\frac{d^2x}{dt^2}+\frac{1}{x}\left(\frac{dx}{dt}\right)^2-\frac{dx}{dt}=0$$

with the boundary conditions $x(t=0)=0$ and $x(t=1)=1$. The value of $x(t)$ at $t=2$ is

(a) $\sqrt{e-1}$ (b) $\sqrt{e^2+1}$ (c) $\sqrt{e+1}$ (d) $\sqrt{e^2-1}$

Topic: Mathematical Physics

Subtopic: Differential Equation

Ans.: (c)

Solution: The given equation can be written as

$$\frac{1}{x}\frac{d}{dt}\left(x\frac{dx}{dt}\right)-\frac{dx}{dt}=0\Rightarrow\frac{d}{dt}\left(x\frac{dx}{dt}\right)-x\frac{dx}{dt}=0$$

putting $y=x\frac{dx}{dt}$ gives

$$\frac{dy}{dt}-y=0\Rightarrow lny=t+lnc_1\Rightarrow y=c_1e^t$$

Since $x\frac{dx}{dt}=c_1e^t$ hence by integrating

$$\frac{x^2}{2}=c_1e^t+c_2 \qquad \text{(i)}$$

Using boundary conditions we obtain

$c_1+c_2=0$ and $c_1e+c_2=\frac{1}{2}$

Solving these equations we obtain $c_1=\frac{1}{2(e-1)}$ and $c_2=-\frac{1}{2(e-1)}$

Thus, $\frac{x^2}{2}=\frac{1}{2(e-1)}e^t-\frac{1}{2(e-1)}$

When $t=2$, we obtain, $x^2=\frac{e^2}{(e-1)}-\frac{1}{(e-1)}=\frac{(e^2-1)}{(e-1)}=e+1$

Therefore, $x(2)=\sqrt{e+1}$

Q22. What is the value of a for which $f(x,y)=2x+3(x^2-y^2)+2i(3xy+ay)$ is an analytic function of complex variable $z=x+iy$

(a) 1 (b) 0 (c) 3 (d) 2

Ans.: (a)

Solution: $f(x,y)=2x+3(x^2-y^2)+2i(3xy+\alpha y)$

$$u=2x+3(x^2-y^2)$$

$$v=2(3xy+\alpha y)$$

C-R conditions: $u_x=v_y, u_y=-v_x,$

$$2+3(2x)=2(3x+\alpha)\Rightarrow \alpha=1$$
$$-6y=-6y$$

Q23. Two particles A and B move with relativistic velocities of equal magnitude v, but in opposite directions, along the x-axis of an inertial frame of reference. The magnitude of the velocity of A, as seen from the rest frame of B, is

(a) $\dfrac{2v}{\left(1-\dfrac{v^2}{c^2}\right)}$ (b) $\dfrac{2v}{\left(1+\dfrac{v^2}{c^2}\right)}$ (c) $2v\sqrt{\dfrac{c-v}{c+v}}$ (d) $\dfrac{2v}{\sqrt{1-\dfrac{v^2}{c^2}}}$

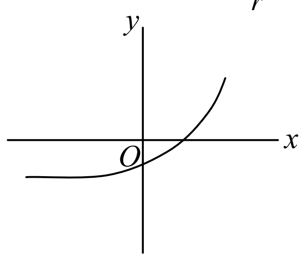
Ans.: (b)

Solution: $u_x'=v \qquad V=v$

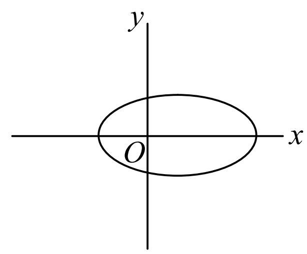
$$u_x=\frac{u_x'+V}{1+\frac{u_x'V}{c^2}} \qquad u_x=\frac{v+v}{1+\frac{v^2}{c^2}}=\frac{2v}{1+\frac{v^2}{c^2}}$$

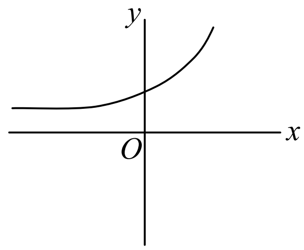
Q24. Which of the following figures best describes the trajectory of a particle moving in a repulsive central potential $V(r)=\dfrac{a}{r}$ ($a>0$ is a constant)?

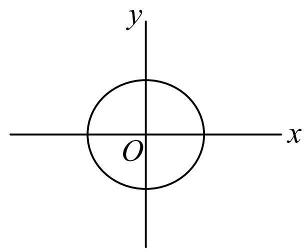
(a)



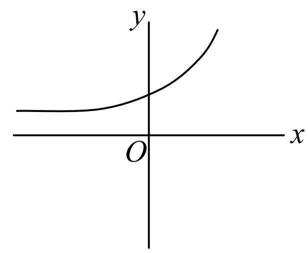
(b)



(c) 

(d) 

Ans.: (c)



Solution: The potential is $V(r)=\frac{a}{r}$ which is repulsive. So there is unbounded motion and mainly represent by scattering project

Q25. Consider the three vectors $\vec{v}_1 = 2\hat{i}+3\hat{k}, \vec{v}_2 = \hat{i}+2\hat{j}+2\hat{k}$ and $\vec{v}_3 = 5\hat{i}+\hat{j}+a\hat{k}$ where $\hat{i},\hat{j}$ and $\hat{k}$ are the standard unit vectors in a three-dimensional Euclidean space. These vectors will be linearly dependent if the value of a is

(a) $\frac{31}{4}$ (b) $\frac{23}{4}$ (c) $\frac{27}{4}$ (d) 0

Ans.: (a)

Solution: Given vector will be linearly dependent if the determinant of the matrix formed by taking these vectors as column is zero.

$$\begin{vmatrix} 2 & 1 & 5 \\ 0 & 2 & 1 \\ 3 & 2 & a \end{vmatrix} = 0 \Rightarrow 2(2a-2)-(-3)+5(-6)=0$$

$$\Rightarrow 4a-4+3-30=0 \Rightarrow 4a-31=0 \Rightarrow a=\frac{31}{4}$$

Q26. The Fourier transform $\int_{-\infty}^{\infty} dx f(x) e^{ikx}$ of the function $f(x)=e^{-|x|}$

(a) $-\frac{2}{1+k^2}$ (b) $-\frac{1}{2(1+k^2)}$ (c) $\frac{2}{1+k^2}$ (d) $\frac{2}{(2+k^2)}$

Ans.: (c)

Solution: $\int_{-\infty}^{+\infty} dx e^{-|x|} e^{ikx} = \int_{-\infty}^{+\infty} dx\, e^{-|x|} \cos kx dx$ odd function $\sin kx$ vanishes

$$\Rightarrow 2\int_0^\infty e^{-x}\cos kx\,dx = 2\frac{e^{-x}}{1+k^2}\left[-\cos kx + k\sin kx\right]_0^\infty$$

$$\because \int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx + b\sin bx\right]$$

$$\Rightarrow 2\int_0^\infty e^{-x}\cos kx dx = 2\frac{e^0}{1+k^2} = \frac{2}{1+k^2}$$

Q27. The value of the integral

$$\int_{-\pi/2}^{\pi/2} dx\int_{-1}^{+1} dy.\delta(\sin 2x)\delta(x-y) \text{ is}$$

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

Ans.: (b)

Solution: $I = \int\limits_{-\pi/2}^{\pi/2} dx\int\limits_{-1}^{1} dy\delta(\sin 2x)\delta(x-y) = \int\limits_{-\pi/2}^{\pi/2} dx\,\delta(\sin 2x)\int\limits_{-1}^{1}\delta(y-x)dy$

If we assume that x lies between -1 and $+1$ then the second integral is 1 and the given integral becomes

$$I = \int\limits_{-\pi/2}^{\pi/2}\delta(\sin 2x)dx$$

now $\delta(\sin 2x) = \sum\limits_{n=-\infty}^{\infty}\frac{\delta\left(x-\frac{n\pi}{2}\right)}{\left|2\cos 2\frac{n\pi}{2}\right|}$

Therefore, $I = \frac{1}{2}\int\limits_{-\pi/2}^{\pi/2}\delta(x)dx = \frac{1}{2}.1 = \frac{1}{2}$

Q28. A particle moves in the one-dimensional potential $V(x) = \alpha x^6$, where $a > 0$ is a constant. If the total energy of the particle is E, its time period in a periodic motion is proportional to

(a) $E^{-1/3}$ (b) $E^{-1/2}$ (c) $E^{1/3}$ (d) $E^{1/2}$

Ans.: (a)

Solution: $J = \oint Pdx$

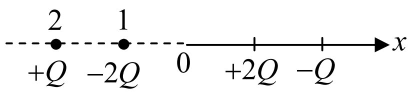
$$J \propto (2mE)^{1/2}\left(\frac{E}{a}\right)^{1/6} \quad \Rightarrow E \propto J^{3/2} \quad \nu = \frac{dE}{dJ} \propto J^{1/2} \Rightarrow \nu \propto E^{1/3} \quad \Rightarrow T \propto E^{-1/3}$$

Q29. Two point charges $+2Q$ and $-Q$ are kept at point with Cartesian coordinates $(1,0,0)$, respectively, in front of an infinite grounded conducting plate at $x=0$. The potential at $(x,0,0)$ for $x \gg 1$ depends on x as

(a) x^{-3} (b) x^{-5} (c) x^{-2} (d) x^{-4}

Ans.: (a)

Solution:

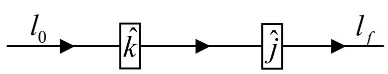


Monopole moment $2Q-Q-2Q+Q=0$

Dipole moment $\vec{p}=-Q(2\hat{x})+2Q(\hat{x})-2Q(-\hat{x})+Q(-2\hat{x}) \Rightarrow \vec{p}=0$

Thus $V \propto \frac{1}{x^3}$

Q30. Two Stern-Gerlach apparatus S_1 and S_2 are kept in a line (x-axis). The directions of their magnetic fields are along the positive z and y-axes, respectively. Each apparatus only transmits particles with spins aligned in thee direction of its magnetic field. If an initially unpolarized bearn of spin $\frac{1}{2}$ particles passes through this configuration, the ratio of intensities $l_0 : l_f$ of the initial and final beams is



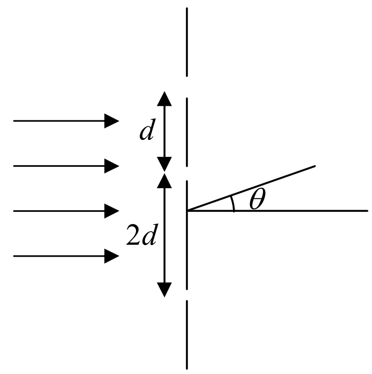
(a) $16:1$ (b) $2:1$ (c) $4:1$ (d) $1:0$

Ans.: (c)

Solution:

$I_f = \frac{I_0}{4}$ $\frac{I_0}{I_f}=\frac{4}{1}$

Q31. The following configuration of three identical narrow slits are illuminated by monochromatic light of wavelength λ (as shown in the figure below). The intensity is measured at an angle θ (where θ is the angle with the incident beam) at a large distance from the slits. If $\delta = \frac{2\pi d}{\lambda}\sin\theta$, the intensity is proportional to



(a) $2\cos\delta + 2\cos 2\delta$

(b) $3 + \frac{1}{\delta^2}\sin^2 3\delta$

(c) $3 + 2\cos\delta + 2\cos 2\delta + 2\cos 3\delta$

(d) $2 + \frac{1}{\delta^2}\sin^2 3\delta$

Ans.: (c)

Solution: $\vec{E}_1 = \vec{A}e^{i(\omega t)}$, $\vec{E}_2 = \vec{A}\, e^{i\delta}\, e^{i\omega t}$, $\vec{E}_3 = \vec{A}\, e^{i\delta_1}\, e^{i\omega t} = \vec{A}\, e^{3i\delta}\, e^{i\omega t}$

$$\because\ \delta = \frac{2\pi}{\lambda} d\sin\theta,\quad \delta_1 = \frac{2\pi}{\lambda}(3d\sin\theta) \approx 3\delta$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \vec{A}\left[1 + e^{i\delta} + e^{3i\delta}\right]e^{i\omega t}$$

$$\vec{E}^* = \vec{A}'\left[1 + e^{-i\delta} + e^{-3i\delta}\right]e^{-i\omega t}$$

$$I = \vec{E}.\vec{E}^* = A^2\left[1 + e^{i\delta} + e^{3i\delta}\right]\left[1 + e^{-i\delta} + e^{-3i\delta}\right]$$

$$I = A^2\left[3 + 2\frac{e^{i\delta} + e^{-i\delta}}{2} + 2\frac{e^{i2\delta} + e^{-i2\delta}}{2} + 2\frac{e^{i3\delta} + e^{-i3\delta}}{2}\right]$$

$$I = A^2\left[3 + 2\cos\delta + 2\cos 2\delta + 2\cos 3\delta\right]$$

Q32. A particle of mass m, kept in potential $V(x) = -\frac{1}{2}kx^2 + \frac{1}{4}\lambda x^4$ (where k and λ are positive constants), undergoes small oscillations about an equilibrium point. The frequency of oscillations is

(a) $\frac{1}{2\pi}\sqrt{\frac{2\lambda}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{\lambda}{m}}$

Ans.: (c)

Solution: $V = -\frac{1}{2}kx^2 + \frac{1}{4}\lambda x^4$

$$\frac{dV}{dx} = 0 \qquad -kx + \lambda x^3 = 0$$

$x=0,\quad x^2=\frac{k}{\lambda}\Rightarrow x=x_0=\sqrt{\frac{k}{\lambda}}$

$\frac{d^2V}{dx^2}=-k$ at $x=0$ so $x=0$ is unstable part

$\frac{d^2V}{dx^2}=2k$ at $x_0=\sqrt{\frac{k}{\lambda}}$ so $x_0=\sqrt{\frac{k}{x}}$ is stable equation point

$$\omega=\sqrt{\frac{\left.\frac{d^2V}{dx^2}\right|_{x=x_0}}{m}}=\sqrt{\frac{2k}{m}}\qquad f=\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$

Q33. The Hamiltonian of a spin $\frac{1}{2}$ particle in a magnetic field $\vec{B}$ is given by $H=-\mu.\vec{B}.\vec{\sigma}$, where μ is a real constant and $\vec{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ are the Pauli spin matrices. If $\vec{B}=(B_0,B_0,0)$ and the spin state at time $t=0$ is an eigenstate of σ_x, then of the expectation values $\langle\sigma_x\rangle,\langle\sigma_y\rangle$ and $\langle\sigma_z\rangle$

(a) only $\langle\sigma_x\rangle$ changes with time

(b) only $\langle\sigma_y\rangle$ changes with time

(c) only $\langle\sigma_z\rangle$ changes with time

(d) all three change with time

Ans.: (d)

Solution: $\langle\sigma_x\rangle,\langle\sigma_y\rangle$ and $\langle\sigma_z\rangle$ will changes with time because Eigen state of σ_x ie $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ and $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$ and can be written in basis of eigen state of $H=-\mu.\vec{B}.\vec{\sigma}=-B_0\begin{pmatrix}0 & 1-i\\1+i & 0\end{pmatrix}$

Q34. A particle of mass m is constrained to move in a circular ring of radius R. When a perturbation $V'=\frac{a}{R^2}\cos^2\phi$ (where a is a real constant) is added , the shift in energy of the ground state, to first order in a, is

(a) $\frac{a}{R^2}$

(b) $\frac{2a}{R^2}$

(c) $\frac{a}{2R^2}$

(d) $\frac{a}{(\pi R^2)}$

Ans.: (c)

Solution: $V'=\frac{a}{R^2}\cos^2\phi$ where $|\phi_0\rangle=\frac{1}{\sqrt{2\pi}}$

$$\langle\phi_0|V'|\phi_0\rangle=\frac{a}{R^2}\int_0^{2\pi}\frac{1}{2\pi}\cos^2\phi$$

$$=\frac{a}{2\pi R^2}\int_0^{2\pi}\frac{1}{2}(1+\cos 2\phi)\,d\phi=\frac{a}{2\pi R^2}\frac{2\pi}{2}=\frac{a}{2R^2}$$

Q35. A particle of mass m is confined in a three-dimensional box by the potential

$$V(x,y,z)=\begin{cases}0, & 0\le x,y,z\le a\\ \infty & \text{otherwise}\end{cases}$$

The number of eigenstates of Hamiltonian with energy $\frac{9\hbar^2\pi^2}{2ma^2}$ is

(a) 1 (b) 6 (c) 3 (d) 4

Ans.: (c)

Solution: $E_{n_x,n_y,n_z}=\frac{9\pi^2\hbar^2}{2ma^2}$

$$\left.\begin{matrix} n_x & n_y & n_z\\ 1 & 2 & 2\\ 2 & 2 & 1\\ 2 & 1 & 2\end{matrix}\right\}$$

where $E_{xx,xy,xz}=\left(n_x^2+n_y^2+n_z^2\right)\frac{\pi^2\hbar^2}{2ma^2}$

Q36. The electric field $\vec{E}$ and the magnetic field $\vec{B}$ corresponding to the scalar and vector potentials,

$V(x,y,z,t)=0$ and $\vec{A}(x,y,z,t)=\frac{1}{2}\hat{k}\mu_0A_0(ct-x)$, where A_0 is a constant, are

(a) $\vec{E}=0$ and $\vec{B}=\frac{1}{2}\hat{j}\mu_0A_0$ (b) $\vec{E}=-\frac{1}{2}\hat{k}\mu_0A_0c$ and $\vec{B}=\frac{1}{2}\hat{j}\mu_0A_0$

(c) $\vec{E}=0$ and $\vec{B}=-\frac{1}{2}\hat{i}\mu_0A_0$ (d) $\vec{E}=\frac{1}{2}\hat{k}\mu_0A_0c$ and $\vec{B}=-\frac{1}{2}\hat{i}\mu_0A_0$

Ans.: (b)

Solution: $\vec{E}=\frac{-\partial\vec{A}}{\partial t}=-\left[\frac{1}{2}\mu_0A_0(c-0)\right]\hat{k}=-\frac{1}{2}\mu_0A_0c\hat{k}$

$$\vec{B}=\vec{\nabla}\times\vec{A}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & 0 & A_z\end{vmatrix}=\hat{x}\frac{\partial A_z}{\partial y}-\hat{y}\frac{\partial A_z}{\partial x}\Rightarrow\vec{B}=\frac{1}{2}\mu_0A_0\hat{j}$$

Q37. The electric field of a plane wave in a conducting medium is given by

$$\vec{E}(z,t) = \hat{i}E_0 e^{-z/3a} \cos\left(\frac{z}{\sqrt{3}a} - \omega t\right),$$

where ω is the angular frequency and $a > 0$ is a constant. The phase difference between the magnetic field $\vec{B}$ and the electric field $\vec{E}$ is

(a) 30^0 and $\vec{B}$ lags behind $\vec{B}$ (b) 30^0 and $\vec{B}$ lags behind $\vec{E}$

(c) 60^0 and $\vec{E}$ lags behind $\vec{B}$ (d) 60^0 and $\vec{B}$ lags behind $\vec{E}$

Ans.: (b)

Solution: $\vec{E}(z,t) = \hat{i}E_0 e^{-\kappa z}\cos(kz - \omega t + \delta_E)$ and $\vec{B}(z,t) = \hat{j}B_0 e^{-\kappa z}\cos(kz - \omega t + \delta_E + \phi)$

where $\phi = \tan^{-1}\left(\frac{\kappa}{k}\right)$.

$\because \vec{E}(z,t) = \hat{i}E_0 e^{-z/3a}\cos\left(\frac{z}{\sqrt{3}a} - \omega t\right) \Rightarrow \kappa = \frac{1}{3a}$ and $k = \frac{1}{\sqrt{3}a}$

$\Rightarrow \phi = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^0$

Q38. Which of the following statements concerning the coefficient of volume expansion α and the isothermal compressibility κ of a solid is true?

(a) α and κ are both intensive variables

(b) α is an intensive and κ is an extensive variable

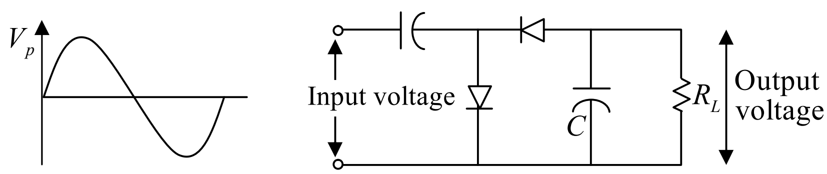
(c) α is an extensive and κ is an intensive variable

(d) α and κ are both extensive variables

Ans.: (a)

Solution: $\alpha = \frac{1}{V}\left(\frac{dV}{dT}\right)$, $\kappa = -\frac{1}{V}\left(\frac{\partial P}{\partial V}\right)_T$ both are intensive property

Q39. A sinusoidal signal with a peak voltage V_p and average value zero, is an input to the following circuit.



Assuming ideal diodes, the peak value of the output voltage across the load resistor R_L is

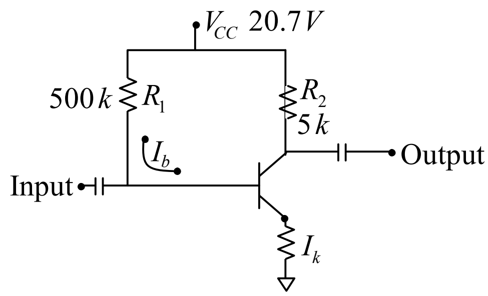
(a) V_p (b) $\frac{V_p}{2}$ (c) $2V_p$ (d) $\sqrt{2}V_p$

Ans.: (c)

Solution: It's a voltage doubler circuit

Peak value $= 2V_p$

Q40. In the following circuit, the value of the common-emitter forward current amplification factor β for the transistor is 100 and V_{BE} is $0.7V$.



The base current I_B is

(a) $40\ \mu A$ (b) $30\ \mu A$ (c) $44\ \mu A$ (d) $33\ \mu A$

Ans.: (d)

Solution: $I_B = \dfrac{V_{cc} - V_{BE}}{R_B + \beta R_E} = \dfrac{20.7 - 0.7}{500 + 100 \times 1} = \dfrac{20}{600K} = \dfrac{20 \times 1000}{600}\mu A = 33.3\mu A$

Q41. The number of ways of distributing 11 indistinguishable bosons in 3 different energy levels is

(a) 3^{11} (b) 11^3 (c) $\dfrac{(13)!}{2!(11)!}$ (d) $\dfrac{(11)!}{3!8!}$

Ans.: (c)

Solution: $n = 11 \quad g = 3$

$$w = \frac{\underline{|n+g-1}}{\underline{|n}\,\underline{|g-1}} = \frac{\underline{|11+3-1}}{\underline{|11}\,\underline{|2}} = \frac{\underline{|13}}{\underline{|11}\,\underline{|2}}$$

Q42. The van der Waals equation for one mole of a gas is $\left(p + \frac{a}{V^2}\right)(V - b) = RT$. The corresponding equation of state for n moles of this gas at pressure P, volume V and temperature T, is

(a) $\left(P + \dfrac{an^2}{V^2}\right)(V - nb) = nRT$ (b) $\left(P + \dfrac{a}{V^2}\right)(V - nb) = nRT$

(c) $\left(P + \dfrac{an^2}{V^2}\right)(V - nb) = nRT$ (d) $\left(P + \dfrac{a}{V^2}\right)(V - nb) = nRT$

Ans.: (a)

Solution: For n mole gas van der Waal equation is

$$\left(p+\frac{an^2}{V}\right)(V-nb)=nRT$$

Q43. In a system of N distinguishable particles, each particle can be in one of two states with energies 0 and $-E$, respectively. The mean energy of the system at temperature T is

(a) $-\frac{1}{2}N\left(1+e^{\varepsilon/k_BT}\right)$

(b) $-NE\left(1+e^{\varepsilon/k_BT}\right)$

(c) $-\frac{1}{2}NE$

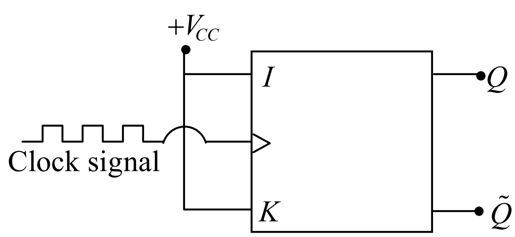
(d) $-NE\left(1+e^{-\varepsilon/k_BT}\right)$

Ans.: (d)

Solution: For one particle system

$$\langle E\rangle=\frac{0\times e^{\frac{-0}{k_BT}}+(-E)e^{+E/k_BT}}{e^{-0/k_BT}+e^{E/k_BT}}=\frac{-E\ e^{E/k_BT}}{1+e^{E/k_BT}}=\frac{-E}{e^{-E/k_BT}+1}\langle E\rangle=\frac{-NE}{1+e^{-E/k_BT}}$$

Q44. In the following JK flip-flop circuit, J and K inputs are tied together to $+V_{CC}$. If the input is a clock signal of frequency f, the frequency of the output Q is



(a) f (b) $2f$ (c) $4f$ (d) $\frac{f}{2}$

Ans.: (d)

Solution: It divides clock frequency by 2

Q45. Which of the following gates can be used as a parity checker?

(a) an OR gate

(b) a NOR gate

(c) an exclusive OR (XOR) gate

(d) an AND gate

Ans.: (c)

Q46. Which of the following statements is true for a 3×3 real orthogonal matrix with determinant $+1$?

(a) the modulus of each of its eigenvalues need not be 1, but their product must be 1

(b) at least one of its eigenvalues is $+1$

(c) all of its eigenvalues must be real

(d) none of its eigenvalues must be real

Ans.: (b)

Solution: The characteristic equation of any 3×3 matrix is of thee form $\lambda^3+a\lambda^2+b\lambda+c=0$ which implies that at least one of the eigenvalues must be real. It is a proven fact that modulus of each eigenvalues of an orthogonal matrix is 1.

If all eigenvalues of 3×3 orthogonal matrix are real then only possibilities for eigenvalues are

$\lambda_1=1,\lambda_2=1$ and $\lambda_3=1$ or $\lambda_1=-1,\lambda_2=-1,\lambda_3=1$ or $\lambda_1=-1,\lambda_2=1,\lambda_3=-1$

Thus we see that at least one eigenvalue is $+1$. Suppose one eigenvalues is real and other two eigenvalues are complex conjugates. Now

$$\lambda_1\lambda_2\lambda_3=1$$

$$\Rightarrow \lambda_1(a+ib)(a-ib)=1\Rightarrow \lambda_1\left(a^2+b^2\right)=1$$

Since a^2+b^2 is always positive hence $\lambda_1=1$.

In this case also we see that at least one eigenvalue must be $+1$

Q47. A particle of mass m moves in a central potential $V(r)=-\frac{k}{r}$ in an elliptic orbit $r(\theta)=\frac{a\left(1-e^2\right)}{1+e\cos\theta}$, where $0\le\theta<2\pi$ and a and e denote the semi-major axis and eccentricity, respectively. If its total energy is $E=-\frac{k}{2a}$, the maximum kinetic energy is

(a) $E\left(1-e^2\right)$ (b) $E\frac{(e+1)}{(e-1)}$ (c) $E/\left(1-e^2\right)$ (d) $E\frac{(e-1)}{(e+1)}$

Ans.: (b)

Solution: $E=T+V\quad T=E-V$

$$T=-\frac{k}{2a}+\frac{k}{r}\;T=-\frac{k}{2a}+\frac{k}{a\left(1-e^2\right)}(1+\cos\theta)$$

T. maximum $\cos\theta=1$

$$T_{\max} = -\frac{k}{2a} + \frac{k(1+e)}{a(1-e^2)} = -\frac{k}{2a} + \frac{k}{a}\frac{(1+e)}{(1-e)(1+e)}$$

$$= -\frac{k}{a}\left[\frac{1}{2} + \frac{1}{(1-e)}\right] = -\frac{k}{2a}\left(\frac{1+e}{1-e}\right) = E\left(\frac{1+e}{1-e}\right)$$

Q48. The Hamiltonian of a one-dimensional system is $H = \frac{xp^2}{2m} + \frac{1}{2}kx$, where m and k are positive constants. The corresponding Euler-Lagrange equation for the system is

(a) $m\ddot{x} + k = 0$ (b) $m\ddot{x} + 2\dot{x} + kx^2 = 0$

(c) $2mx\ddot{x} - m\dot{x}^2 + kx^2 = 0$ (d) $mx\ddot{x} + 2m\dot{x}^2 + kx^2 = 0$

Ans.: (c)

Solution: $H = \frac{xp^2}{2m} + \frac{1}{2}kx$

$$\frac{\partial H}{\partial p} = \dot{x} \Rightarrow \frac{xp}{m} = \dot{x} \qquad p = \frac{m\dot{x}}{x}$$

$$L = \dot{x}p - H \Rightarrow L = \dot{x}p - \frac{xp^2}{2m} - \frac{1}{2}kx = \frac{m\dot{x}^2}{x} - \frac{m\dot{x}^2}{2x} - \frac{1}{2}kx = \frac{m\dot{x}^2}{2x} - \frac{1}{2}kx$$

Eular Lagrangas equation is given by

$$\frac{d}{dr}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$\frac{m\ddot{x}}{x} - \frac{m\dot{x}\dot{x}}{2x^2} + \frac{1}{2}k = 0$$

$$2xm\ddot{x} - m\dot{x}^2 + kx^2 = 0$$

Q49. A hollow waveguide supports transverse electric (TE) modes with the dispersion relation $k = \frac{1}{c}\sqrt{\omega^2 - \omega_{mn}^2}$, where ω_{mn} is the mode frequency. The speed of flow of electromagnetic energy at the mode frequency is

(a) c (b) ω_{mn}/k (c) 0 (d) ∞

Ans.: (c)

Solution: Energy carried by the wave travels at the group velocity

$$v_g = \frac{d\omega}{dk} = c\sqrt{1 - \left(\frac{\omega_{mn}}{\omega}\right)^2} \qquad \text{at } \omega = \omega_{mn}, v_g = 0$$

Q50. The energy of a free relativistic particle is $E=\sqrt{|\vec{p}|^2 c^2+m^2c^4}$, where m is its rest mass, $\vec{p}$ is its momentum and c is the speed of light in vacuum. The ratio v_g/v_p of the group velocity v_g of a quantum mechanical wave packet (describing this particle) to the phase velocity v_p is

(a) $|\vec{p}|c/E$ (b) $|\vec{p}|mc^3/E^2$ (c) $|\vec{p}|^2 c^3/E^2$ (d) $|\vec{p}|c/2E$

Ans.: (c)

Solution: $E^2=p^2c^2+m^2c^4$ and $v_g=\frac{dE}{dp}, v_p=\frac{E}{p}$

$$2E\frac{dE}{dp}=2pc^2 \quad \Rightarrow \frac{E}{p}\frac{dE}{dp}=c^2$$

$$\frac{v_g}{v_p}=\frac{c^2}{v_p^2} \qquad \frac{v_g}{v_p}=\frac{c^3p^2}{E^2}$$

Q51. In the function $P_n(x)e^{-x^2}$ of a real variable x, $P_n(x)$ is polynomial of degree n. The maximum number of extrema that this function can have is

(a) $n+2$ (b) $n-1$ (c) $n+1$ (d) n

Ans.: (c)

Solution: $y=P_n(x)e^{-x^2} \Rightarrow P_n'(x)e^{-x^2}+P_n(x)e^{-x^2}(-2x)=0 \Rightarrow P_n'(x)-2xP_n(x)=0$

$P_0(x)=1, P_1(x)=2 \Rightarrow P_0'(x)-2xP_0(x)=0 \Rightarrow 0-2x.1=0$

$x=0,\ 1$ extrema

$P_1'(x)-2xP_1(x)=0$

$1-2x.x=0 \Rightarrow x=\pm\frac{1}{\sqrt{2}}$ i.e., 2 extrema.

Thus in general there are $(n+1)$ extrema.

Q52. The Green's function $G(x,x')$ for the equation $\frac{d^2y(x)}{dx^2}+y(x)=f(x)$, with the boundary values $y(0)=y\left(\frac{\pi}{2}\right)=0$, is

(a) $G(x,x')=\begin{cases} x\left(x'-\frac{\pi}{2}\right), & 0<x<x'<\frac{\pi}{2} \\ \left(x-\frac{\pi}{2}\right)x', & 0<x'<x<\frac{\pi}{2} \end{cases}$

(b) $G(x,x')=\begin{cases} -\cos x'\sin x, & 0<x<x'<\frac{\pi}{2} \\ -\sin x'\cos x, & 0<x'<x<\frac{\pi}{2} \end{cases}$

(c) $G(x,x')=\begin{cases} \cos x'\sin x, & 0<x<x'<\frac{\pi}{2} \\ \sin x'\cos x, & 0<x'<x<\frac{\pi}{2} \end{cases}$

(d) $G(x,x')=\begin{cases} x\left(\frac{\pi}{2}-x'\right), & 0<x<x'<\frac{\pi}{2} \\ x'\left(\frac{\pi}{2}-x\right), & 0<x'<x<\frac{\pi}{2} \end{cases}$

Ans.: (b)

Solution: $\frac{d^2y}{dx^2}+y=0\Rightarrow m^2+1=0\Rightarrow m=0\pm i$

$y_1(x)=\sin x(at), y_1'(x)=\cos x$

$y_2(x)=\cos x \quad \left(at\ \frac{\pi}{2}\right), y_2'(x)=-\sin x$

$A=\left\{P(x')\left[y_2'(x')y_1(x_1')-y_1'(x')\right]y_2(x')\right\}$

$\Rightarrow A=\{-\sin x'\sin x'-\cos'\cos x'\} \qquad \because P(x)=1\Rightarrow A=-1$

Thus $G(x,x')=\begin{cases} Ay_1(x)y_2(x'), & x<x' \\ Ay_2(x)y_1(x), & x>x' \end{cases}=\begin{cases} -\sin x\cos x', & 0<x<x'<\frac{\pi}{2} \\ -\cos x\sin x', & 0<x'<x<\frac{\pi}{2} \end{cases}$

Q53. The fractional error in estimating the integral $\int_0^1 x dx$ using Simpson's $\frac{1}{3}$ rule, using a step size 0.1, is nearest to

(a) 10^{-4} (b) 0 (c) 10^{-2} (d) 3×10^{-4}

Ans.: (b)

Solution: $I=\frac{h}{3}\left[y_0+2\left(y_2+y_4+....\right)+4\left(y_1+y_3+y_6+...\right)+y_n\right]$

$$=\frac{0.1}{3}\left[0+2\left(0.2+0.4+0.6+0.8\right)+4\left(0.1+0.3+0.5+0.7+0.9+1\right)\right]$$

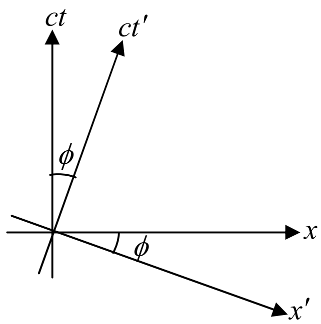
$$=\frac{1}{30}\left[4+10+1\right]=\frac{15}{30}=0.5$$

fractional error $=\frac{\Delta I}{I_{true}}=\frac{0.5-0.5}{0.5}\approx 0$

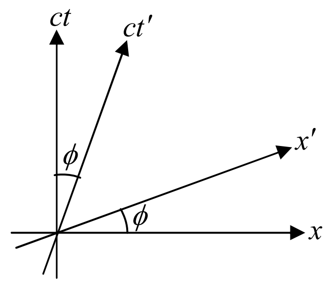
y_0	0
y_1	0.1
y_2	0.2
y_3	0.3
y_4	0.4
y_5	0.5
y_6	0.6
y_7	0.7
y_8	0.8
y_9	0.9
y_{10}	1.0

Q54. An inertial frame K' moves with a constant speed v with respect to another inertial frame K along their common x - direction. Let (x, ct) and (x', ct') denote the space-time coordinates in the frames K and K', respectively. Which of the following space-time diagrams correctly describes the t' - axis $(x' = 0 \text{ line})$ and the x' - axis $(t' = 0 \text{ line})$ in the x-ct plane? (In the following figures $\tan\phi = v/c$)

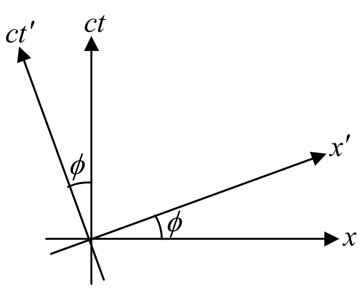
(a)



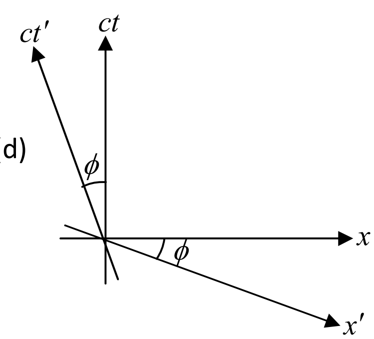
(c)



(c)

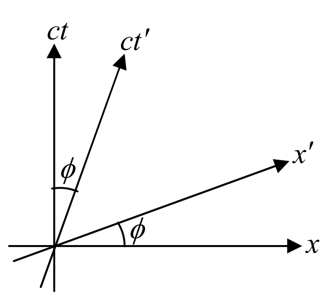


(d)



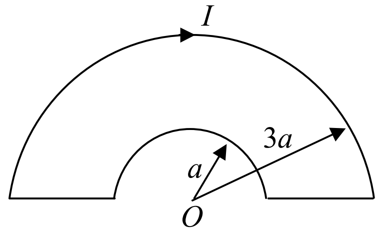
Ans. : (b)

Solution: $$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cosh\phi & -\sinh\phi & 0 & 0 \\ -\sinh\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$



Where $v = \cosh\phi, \beta v = \sinh\phi$ $\beta = \tanh\phi$

Q55. The loop shown in the figure below carries a steady current I .



The magnitude of the magnetic field at the point O is

(a) $\frac{\mu_0 I}{2a}$ (b) $\frac{\mu_0 I}{6a}$ (c) $\frac{\mu_0 I}{4a}$ (d) $\frac{\mu_0 I}{3a}$

Ans. : (b)

Solution: $B_a = \frac{1}{2}\frac{\mu_0 I}{2a}\odot,\quad B_{3a} = \frac{1}{2}\frac{\mu_0 I}{2(3a)}\otimes$

$$B = B_a - B_{3a} = \frac{\mu_0 I}{4a}\left(1-\frac{1}{3}\right) = \frac{\mu_0 I}{6a}$$

Q56. In the region far from a source, the time dependent electric field at a point (r,θ,ϕ) is

$$\vec{E}(r,\theta,\phi) = \hat{\phi}E_0\omega^2\left(\frac{\sin\theta}{r}\right)\cos\left[\omega\left(t-\frac{r}{c}\right)\right]$$

where ω is angular frequency of the source. The total power radiated (averaged over a cycle) is

(a) $\frac{2\pi}{3}\frac{E_0^2\omega^4}{\mu_0 c}$ (b) $\frac{4\pi}{3}\frac{E_0^2\omega^4}{\mu_0 c}$ (c) $\frac{4}{3\pi}\frac{E_0^2\omega^4}{\mu_0 c}$ (d) $\frac{2}{3}\frac{E_0^2\omega^4}{\mu_0 c}$

Ans. : (b)

Solution: $B = \frac{E}{c}$

$$|\vec{S}| = \frac{1}{\mu_0}E.B = \frac{E^2}{\mu_0 c} = \frac{E_0^2\omega^4}{\mu_0 c}\frac{Sm_\theta^2}{r^2}\cos^2\left[\omega\left(t-\frac{r}{c}\right)\right]$$

$$\langle|\vec{S}|\rangle = \frac{1}{2}\frac{E_0^2\omega^4}{\mu_0 c}\frac{\sin^2\theta}{r^2}$$

$$P = \oint_S \langle|\vec{S}|\rangle.d\vec{a} = \frac{E_0^2\omega^4}{2\mu_0 c}\int_0^\pi\int_0^{2\pi}\frac{\sin^2\theta}{r^2}r^2\sin\theta d\theta d\phi$$

$$P = \frac{E_0^2\omega^4}{2\mu_0 c}\times\frac{4}{3}\times 2\pi = \frac{4\pi}{3}\frac{E_0^2\omega^4}{\mu_0 c}$$

Q57. The pressure P of a system of N particles contained in a volume V at a temperature T is given by $P = nk_BT - \frac{1}{2}an^2 + \frac{1}{6}bn^3$, where n is the number density and a and b are temperature independent constants. If the system exhibits a gas-liquid transition, the critical temperature is

(a) $\frac{a}{bk_B}$ (b) $\frac{a}{2b^2k_B}$ (c) $\frac{a^2}{2bk_B}$ (d) $\frac{a^2}{b^2k_B}$

Ans. : (c)

Solution: $P = nk_BT - \frac{1}{2}an^2 + \frac{1}{6}bn^3 \qquad n = \frac{N}{V}$

For critical condition $\frac{\partial P}{\partial V} = 0$ and $\frac{\partial^2 P}{\partial V^2} = 0$

$$P = \frac{N}{V}k_BT - \frac{1}{2}a\frac{N^2}{V^2} + \frac{1}{6}b\frac{N^3}{V^3}$$

$$\frac{\partial P}{\partial V} = 0 \Rightarrow Nk_BT = \frac{aN^2}{V} - \frac{bN^3}{2V^2} \qquad \text{(i)}$$

$$\frac{\partial^2 P}{\partial V^2} = 0 \Rightarrow 2Nk_BT = \frac{3aN^2}{V} - \frac{2bN^3}{V^2} \qquad \text{(ii)}$$

From equation (i) and (ii)

$$V_C = \frac{bN}{a}$$

put the value of $V_C = \frac{b}{a}N$ in equation (i)

$$T = \frac{a^2}{2k_Bb}$$

Q58. Consider a particle diffusing in a liquid contained in a large box. The diffusion constant of the particle in the liquid is $1.0 \times 10^{-2} cm^2/s$. The minimum time after which the root-mean-squared displacement becomes more than $6\ cm$ is

(a) 10 min (b) 6 min (c) 30 min (d) $\sqrt{6}$ min

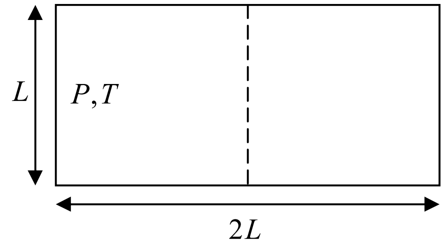
Ans. : (a)

Solution: $\langle r^2 \rangle = 6Dt$

$$\langle r^2 \rangle = (r.m.s)^2 = (6\,c.m)^2 \qquad D = 1 \times 10^{-2}\, c.m^2/\text{sec}$$

$$t=\frac{\langle r^2\rangle}{60}=\frac{(6)^2}{6\times1\times10^{-2}}=600\text{ sec}=10\text{ min}$$

Q59. A thermally insulated chamber of dimensions $(L,L,2L)$ is partitioned in the middle. One side of the chamber is filled with n moles of an ideal gas at a pressure P and temperature T, while the other side is empty. At $t=0$, the partition is removed and the gas is allowed to expand freely. The time to reach equilibrium varies as



(a) $n^{1/3}L^{-1}T^{1/2}$ (b) $n^{2/3}LT^{-1/2}$ (c) $n^{0}LT^{-1/2}$ (d) $nL^{-1}T^{1/2}$

Ans. : (c)

from kinetic theory of gases $\langle F\rangle=\frac{m\langle v^2\rangle}{L}=MLt^{-2}=\frac{m}{L}\frac{3kT}{M}$ where $nM=m$ M is molecular mass

$t^{-2}=L^{-2}T\Rightarrow t=LT^{-1/2}$

Solution:

Q60. Two signals $A_1\sin(\omega t)$ and $A_2\cos(\omega t)$ are fed into the input and the reference channels, respectively, of a lock-in amplifier. The amplitude of each signal is $1\ V$. The time constant of the lock-in amplifier is such that any signal of frequency larger than ω is filtered out. The output of the lock-in amplifier is

(a) $2\ V$ (b) $1\ V$ (c) $0.5\ V$ (d) $0\ V$

Ans. : (d)

Solution: $v=A_1\sin\omega t.A_2\cos\omega t=\frac{A_1A_2}{2}\left[\sin(\omega t+\omega t)+\sin(\omega t-\omega t)\right]$

$$v=\frac{A_1A_2}{2}\sin2\omega t$$

This signal will be filtered out, so output is $0V$.

Q61. The maximum intensity of solar radiation is at the wavelength of $\lambda_{sun} \sim 5000\,\overset{0}{A}$ and corresponds to its surface temperature $T_{sun} \sim 10^4\,K$. If the wavelength of the maximum intensity of an X-ray star is $5\,\overset{0}{A}$, its surface temperature is of the order of

(a) $10^{16}\,K$ (b) $10^{14}\,K$ (c) $10^{10}\,K$ (d) $10^{7}\,K$

Ans. : (d)

Solution: From vein's law

$T_{\max}\lambda_{sun} =$ constant

$$5000 A_0^0 \times 10^4 = 5\,\overset{0}{A} \times T$$

$$T = \frac{5000 \times 10^4}{5} \qquad T = 10^7\,K$$

Q62. The full scale of a 3-bit digital-to-analog (DAC) converter is $7V$. Which of the following tables represents the output voltage of this 3-bit DAC for the given set of input bits?

(a)

Input bits	Output voltage
000	0
001	1
010	2
011	3

(b)

Input bits	Output voltage
000	0
001	1.25
010	2.5
011	3.75

(c)

Input bits	Output voltage
000	1.25
001	2.5
010	3.75
011	5

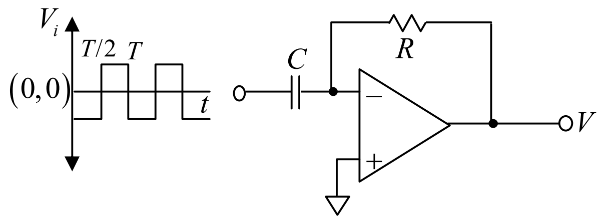
(d)

Input bits	Output voltage
000	1
001	2
010	3
011	4

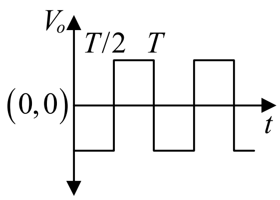
Ans. : (a)

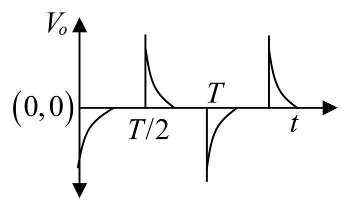
Solution: $(111) \rightarrow 7V$, $(001) \rightarrow 1V$, $(010) \rightarrow 2V$, $(011) \rightarrow 3V$, $(100) \rightarrow 4V$ ....

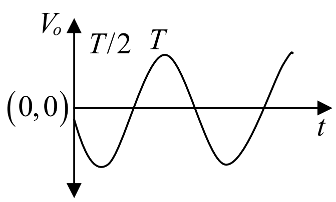
Q63. The input V_i to the following circuit is a square wave as shown in the following figure

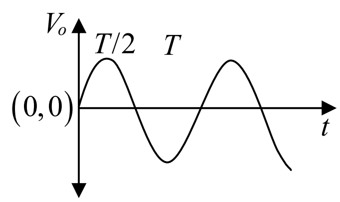


Which of the waveforms V_0 best describes the output?

(a)


(b)


(c)


(d)


Ans. : (b)

Solution: It's a differentiator circuit

Q64. The n^{th} energy eigenvalues E_n of a one-dimensional Hamiltonian $H = \frac{p^2}{2m} + \lambda x^4$ (where $\lambda > 0$ is a constant) in the WEB approximation, is proportional to

(a) $\left(n+\frac{1}{2}\right)^{4/3} \lambda^{1/3}$

(b) $\left(n+\frac{1}{2}\right)^{4/3} \lambda^{2/3}$

(c) $\left(n+\frac{1}{2}\right)^{5/3} \lambda^{1/3}$

(d) $\left(n+\frac{1}{2}\right)^{5/3} \lambda^{2/3}$

Ans. : (a)

Solution: From W.K.B approximation

$$4.\int_0^x P dx \propto \left(n+\frac{1}{2}\right) h$$

$$4\int_0^{\left(\frac{E}{\lambda}\right)^{1/4}} \sqrt{2m\left(E-\lambda x^4 dx\right)} \propto \left(n+\frac{1}{2}\right)h$$

making the integration dimensional

$$4\times(2mE)^{1/2}\left(\frac{E}{\lambda}\right)^{1/4}\int_0^1\sqrt{1-t^4}\,dt \propto \left(n+\frac{1}{2}\right) \Rightarrow E^{3/4} \propto \left(n+\frac{1}{2}\right)\lambda^{1/4} \Rightarrow E \propto \left(n+\frac{1}{2}\right)^{4/3}\lambda^{1/3}$$

Q65. The differential scattering cross-section $\frac{d\sigma}{d\Omega}$ for the central potential $V(r)=\frac{\beta}{r}e^{-\mu r}$, where β and μ are positive constants, is calculated in thee first Born approximation. Its dependence on the scattering angle θ is proportional to (A is a constant below)

(a) $\left(A^2+\sin^2\frac{\theta}{2}\right)$

(b) $\left(A^2+\sin^2\frac{\theta}{2}\right)^{-1}$

(c) $\left(A^2+\sin^2\frac{\theta}{2}\right)^{-2}$

(d) $\left(A^2+\sin^2\frac{\theta}{2}\right)^{2}$

Ans. : (c)

Solution: $f(\theta)\propto\int_0^\infty V(r)\sin kr dr \quad \Rightarrow D(\theta)\propto\left|f(\theta)\right|^2$

$$f(\theta)\propto\frac{1}{k}\int_0^\infty \beta\frac{e^{-\mu r}}{r}r\sin kr dr$$

$$f(\theta)\propto\frac{1}{k}\int_0^\infty \frac{e^{-\mu r}}{r}r\left(\frac{e^{ikr}-e^{-ikr}}{2i}\right)dr \Rightarrow \frac{1}{2ik}\int_0^\infty e^{-\mu r}e^{ikr}dr-\int_0^\infty e^{-\mu r}e^{-ikr}dr$$

$$\Rightarrow\frac{1}{2ik}\left(\int_0^\infty e^{-r(\mu-ik)}dr-\int e^{-r(\mu+ikr)}dr\right)\Rightarrow\frac{1}{2ik}\left[\frac{\mu+ik-\mu+ik}{\mu^2+k^2}\right]=\frac{2ik}{2ik}\left(\mu^2+k^2\right)^{-1}$$

$$f(\theta)\propto\frac{1}{\left(\mu^2+k^2\right)},\qquad D(\theta)=\left(\frac{1}{\mu^2+k^2}\right)^2$$

$D(\theta)=\left(\mu^2+k^2\right)^{-2}$, where $k\propto\sin\frac{\theta}{2}$

$D(\theta)\propto\left(\mu^2+\sin^2\frac{\theta}{2}\right)^{-2}$ or $D(\theta)=\left(A^2+\sin^2\frac{\theta}{2}\right)^{-2}$

Q66. At $t=0$, the wavefunction of an otherwise free particle confined between two infinite walls at $x=0$ and $x=L$ is $\psi(x,t=0)=\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L}-\sin\frac{3\pi x}{L}\right)$. Its wave function at a later time $t=\frac{mL^2}{4\pi h}$ is

(a) $\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L}-\sin\frac{3\pi x}{L}\right)e^{i\pi/6}$ (b) $\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L}+\sin\frac{3\pi x}{L}\right)e^{-i\pi/6}$

(c) $\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L}-\sin\frac{3\pi x}{L}\right)e^{-i\pi/8}$ (d) $\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L}+\sin\frac{3\pi x}{L}\right)e^{-i\pi/6}$

Ans. : (d)

Solution: $\psi(x,t=0)=\left(\sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}-\sqrt{\frac{2}{L}}\sin\frac{3\pi x}{L}\right)$

$$\psi(x,t=0)=|\varphi_1\rangle-|\varphi_3\rangle$$

$$\psi(x,t)=|\varphi_1\rangle e^{\frac{-iE_1t}{\hbar}}-|\varphi_3\rangle e^{\frac{-iE_3t}{\hbar}}$$

$$E_1=\frac{\pi^2\hbar^2}{2mL^2}E_3=\frac{9\pi^2\hbar^2}{2mL^2}t=\frac{mL^2}{4\pi\hbar}$$

$$\psi(x,t)=|\varphi_1\rangle e^{\frac{-i\pi}{8}}-|\varphi_3\rangle e^{\frac{=9i\pi}{8}} \qquad =e^{\frac{-i\pi}{8}}\left(|\varphi_1\rangle-|\varphi_3\rangle e^{-i\pi}\right)$$

$$=e^{\frac{-i\pi}{8}}\left(|\varphi_1\rangle+|\varphi_3\rangle\right) \qquad =e^{\frac{-i\pi}{8}}\left(\sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}+\sqrt{\frac{2}{L}}\sin\frac{3\pi x}{L}\right)$$

Q67. Sodium Chloride $(NaCl)$ crystal is a face-centered cubic lattice with a basis consisting of Na^+ and Cl^- ions separated by half the body diagonal of a unit cube. Which of the planes corresponding to the Miller indices given below will not give rise to Bragg reflection of X-rays?

(a) (220) (b) (242) (c) (221) (d) (311)

Ans. : (c)

Solution: Mixed $(h\,k\,l)$ are absent in $NaCl$. (221) is mixed number of even and odd therefore this plane is absent.

Q68. The dispersion relation for the electrons in the conduction band of a semiconductor is given by $E = E_0 + \alpha k^2$ where α and E_0 are constants. If ω_c is the cyclotron resonance frequency of the conduction band electrons in a magnetic field B, the value of α is

(a) $\frac{\hbar\omega_c}{4eB}$ (b) $\frac{2\hbar^2\omega_c}{eB}$ (c) $\frac{\hbar^2\omega_c}{eB}$ (d) $\frac{\hbar^2\omega_c}{2eB}$

Ans. : (d)

Solution: $\omega_c = \frac{eB}{m^*}$ where $m^* = \frac{\hbar^2}{d^2E/dk^2}$

Since $E = E_0 + \alpha k^2 \Rightarrow \frac{d^2E}{dk^2} = 2\alpha$

$$\Rightarrow \omega_c = \frac{eB}{\hbar^2/2\alpha} = \frac{2\alpha}{\hbar^2}eB \Rightarrow \alpha = \frac{\hbar^2\omega_c}{2eB}$$

Q69. Hard disc of radius R are arranged in a two-dimensional triangular lattice. What is the fractional area occupied by the discs in the closest possible packing?

(a) $\frac{\pi\sqrt{3}}{6}$ (b) $\frac{\pi}{3\sqrt{2}}$ (c) $\frac{\pi\sqrt{2}}{5}$ (d) $\frac{2\pi}{7}$

Ans. : (a)

Solution: $P.F = \frac{n_{eff} \times \pi r^2}{A}$

where $n_{eff} = \frac{1}{3}\times 6 + 1 = 3$ and $A = 6\times\frac{\sqrt{3}}{4}a^2 = \frac{3\sqrt{3}}{2}a^2 = \frac{3\sqrt{3}}{2}(2r)^2 = 6\sqrt{3}r^2$

$$P.F. = \frac{3\times\pi r^2}{6\sqrt{3}r^2} = \frac{\pi}{2\sqrt{3}} = \frac{\pi\sqrt{3}}{6}$$

Q70. A photon of energy $115.62\ keV$ ionizes a K-shell electron of a Be atom. One L-shell electron jumps to the K-shell to fill this vacancy and emits a photon of energy $109.2\ keV$ in the process. If the ionization potential for the L-shell is $6.4\ keV$, the kinetic energy of the ionized electron is

(a) $6.42\,\text{keV}$ (b) $12.82\,\text{keV}$ (c) $20\,\text{eV}$ (d) $32\,\text{eV}$

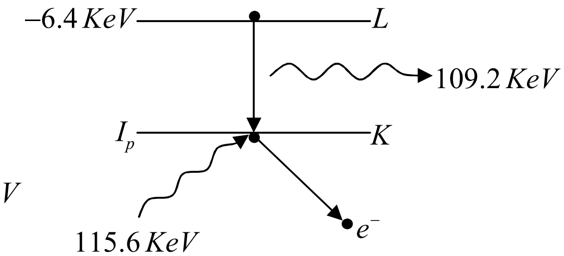
Ans. : (c)

Solution: Binding energy of K-shell electron

$I_p = 6.4\ KeV + 109.2\ KeV = 115.6\ KeV$

Thus, K.E. of ionized electron is

$= 115.62\ KeV - 115.6\ KeV = 0.02\ KeV = 20\ eV$



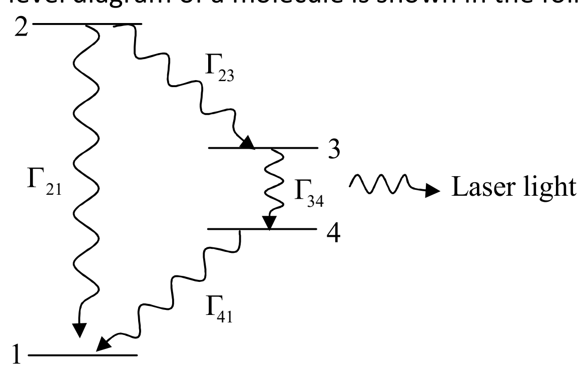
Q71. The value of the Lande g-factor for a fine-structure level defined by the quantum number $L=1, J=2$ and $S=1$, is

(a) $\frac{11}{6}$ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) $\frac{3}{2}$

Ans. : (d)

Solution: $g = 1 + \frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)} = 1 + \frac{2(2+1)+2-2}{2\times 2(2+1)} = 1 + \frac{1}{2} = \frac{3}{2}$

Q72. The electronic energy level diagram of a molecule is shown in the following figure,



Let Γ_{ij} denote the decay rate for a transition from the level i to j. The molecules are optically pumped from level 1 to 2. For the transition from level 3 to level 4 to be a lassing transition, the decay rates have to satisfy

(a) $\Gamma_{21} > \Gamma_{23} > \Gamma_{41} > \Gamma_{34}$ (b) $\Gamma_{21} > \Gamma_{41} > \Gamma_{23} > \Gamma_{34}$

(c) $\Gamma_{41} > \Gamma_{23} > \Gamma_{21} > \Gamma_{34}$ (d) $\Gamma_{41} > \Gamma_{21} > \Gamma_{34} > \Gamma_{23}$

Ans. : (c)

Solution: The state 3 is metastable state, therefore Γ_{34} would be lowest to enhance population inversion $\Gamma_{41} > \Gamma_{23}$. Thus correct sequence is $\Gamma_{41} > \Gamma_{23} > \Gamma_{21} > \Gamma_{34}$

Q73. The reaction $^{63}Cu_{29} + p \to {}^{63}Zn_{30} + n$ is followed by a prompt $\beta-$decay of zinc $^{63}Zn_{30} \to {}^{63}Cu_{29} + e^{+} + v_e$. If the maximum energy of the position is $2.4\ MeV$, the Q-value of the original reaction in MeV is nearest to

[Take the masses of electron, proton and neutron to be $0.5\,MeV/c^2, 938\,MeV/c^2$ and $939.5\,MeV/c^2$,respectively.]

(a) -4.4 (b) -2.4 (c) -4.8 (d) -3.4

Ans. : (a)

Solution: For $^{63}Zn_{30} \to {}^{63}Cu_{29} + e^{+} + v_e$

$Q_1 = (Zn - 30e) - [Cu - 29e + e] = Zn - Cu - 2e = 2.4\,MeV$

For $^{63}Cu_{29} + p \to {}^{63}Zn_{30} + n$

$Q_0 = [(Cu - 29e) + p] - [(Zn - 30e) + n]$

$= Cu - Zn + e + p - n = (-Q_1 - 2e) + e + p - n = -Q_1[e - p + n]$

$= -2.4 - (0.5 - 938 + 939.5) = -4.4.\,MeV$

Q74. A deuteron d captures a charged pion π^- in the $l = 1$ state, and subsequently decays into a pair of neutrons (n) via strong interaction. Given that the intrinsic parities of π^-, d and n are $-1, +1$ and $+1$ respectively, the spin wavefunction of the final state neutrons is

(a) linear combination of a singlet and a triplet

(b) singlet

(c) triplet

(d) doublet

Ans. : (b)

Solution: Parity must conserve intersections

$$\pi + d \to n + n$$

The parity of the initial state is

$$(-1)^l P_\pi P_d = (-1)^1 (-1)(+1) = +1$$

The parity of the final state is

$$(-1)^l P_n P_n = (-1)^l (+1)(+1) = (-1)^l = 1 \qquad \because l = 0, 2, ...$$

because the nucleons are identical fermions, the allowed states of two nucleons are ${}^1S_0,\ {}^3P_{0,1,2}$ corresponding to $l=0$ and $l=1$. Thus only $l=0$ (singlet) is allowed.

Q75. Which of the following elementary particle processes does not conserve strangeness?

(a) $\pi^0 + p \to k^+ + \wedge^0$

(b) $\pi^- + p \to k^0 + \wedge^0$

(c) $\Delta^0 \to \pi^0 + n$

(d) $K^0 \to \pi^+ + \pi^-$

Ans.: (d)

Solution: (a)

$$\begin{array}{lcccccc} & \pi^0 & +p & \to & k^+ & +\wedge^0 \\ S: & 0 & 0 & & +1 & -1 \end{array}$$ Conserved

(b)

$$\begin{array}{lcccccc} & \pi^- & +p & \to & k^0 & +\wedge^0 \\ S: & 0 & 0 & & +1 & -1 \end{array}$$ Conserved

(c)

$$\begin{array}{lccccc} & \Delta^0 & \to & \pi^0 & +n \\ S: & 0 & & 0 & 0 \end{array}$$ Conserved

(d)

$$\begin{array}{lccccc} & K^0 & \to & \pi^+ & +\pi^- \\ S: & +1 & & 0 & 0 \end{array}$$ Not conserved