IIT-JAM 2022 [Solution]

PHYSICS

Section A: Q.1 – Q.10 Carry ONE mark each

Q1. The equation $z^2+\bar{z}^2=4$ in the complex plane (where $\bar{z}$ is the complex conjugate of z) represents

(a) Ellipse
(b) Hyperbola
(c) Circle of radius 2
(d) Circle of radius 4

Topic: Mathematical Physics

Sub Topic: Complex Number

Ans. (b)

Solution: We know that $z=x+iy$, $\bar{z}=x-iy$

$$z^2+\bar{z}^2=4 \Rightarrow (x+iy)^2+(x-iy)^2=4$$

$$\Rightarrow x^2-y^2+2ixy+x^2-y^2-2ixy=4 \Rightarrow 2(x^2-y^2)=4 \Rightarrow x^2-y^2=2$$

This is the equation of Hyperbola

Q2. A rocket (S') moves at speed $\frac{c}{2}m/s$ along the positive x-axis, where c is the speed of light. When it crosses the origin, the clocks attached to the rocket and the one with a stationary observer (S) located at $x=0$ are both set to zero. If S observes an event at (x,t), the same event occurs in the S' frame at

(a) $x'=\frac{2}{\sqrt{3}}\left(x-\frac{ct}{2}\right)$ and $t'=\frac{2}{\sqrt{3}}\left(t-\frac{x}{2c}\right)$

(b) $x'=\frac{2}{\sqrt{3}}\left(x+\frac{ct}{2}\right)$ and $t'=\frac{2}{\sqrt{3}}\left(t-\frac{x}{2c}\right)$

(c) $x'=\frac{2}{\sqrt{3}}\left(x-\frac{ct}{2}\right)$ $t'=\frac{2}{\sqrt{3}}\left(t+\frac{x}{2c}\right)$

(d) $x'=\frac{2}{\sqrt{3}}\left(x+\frac{ct}{2}\right)$ $t'=\frac{2}{\sqrt{3}}\left(t+\frac{x}{2c}\right)$

Topic: Modern Physics

Sub Topic: STR: Lorentz Transformation

Ans. : (a)

Solution: The Lorentz transformation provides us

$$x' = \frac{x - vt}{\left(1-(v/c)^2\right)^{1/2}},\ t' = \frac{t - \frac{vx}{c^2}}{\left(1-(v/c)^2\right)^{1/2}}$$

Given that $v = c/2$

$$x' = \frac{2}{\sqrt{3}}\left(x - \frac{ct}{2}\right),\ t' = \frac{2}{\sqrt{3}}\left(t - \frac{x}{2c}\right)$$

Q3. Consider a classical ideal gas of N molecules in equilibrium at temperature T. Each molecule has two energy levels, $-\in$ and $\in$. The mean energy of the gas is

(a) 0

(b) $N \in \tanh\left(\frac{\in}{k_B T}\right)$

(c) $-N \in \tanh\left(\frac{\in}{k_B T}\right)$

(d) $\frac{\in}{2}$

Topic: Kinetic Theory of Gases and Thermodynamics

Sub Topic: Partition Function

Ans. (c)

Solution: The average energy of single molecule can be written in the following fashion

$$\langle E \rangle = \frac{\sum_i \varepsilon_i e^{-\beta\varepsilon_i}}{\sum_i e^{-\beta\varepsilon_i}} = \frac{-\varepsilon e^{+\beta\varepsilon} + \varepsilon e^{-\beta\varepsilon}}{e^{+\beta\varepsilon} + e^{-\beta\varepsilon}} = -\varepsilon \frac{e^{+\beta\varepsilon} - e^{-\beta\varepsilon}}{e^{+\beta\varepsilon} + e^{-\beta\varepsilon}} = -\varepsilon \tanh(\beta\varepsilon) = -\varepsilon \tanh(\frac{\varepsilon}{k_B T})$$

For N molecules, the average of the systems can be written as follows

$$N\langle E \rangle = -N\varepsilon \tanh\left(\frac{\varepsilon}{k_B T}\right)$$

Q4. At temperature T, let β and κ denote the volume expansivity and isothermal compressibility of gas, respectively. Then $\frac{\beta}{\kappa}$ is equal to

(a) $\left(\frac{\partial P}{\partial T}\right)_V$ (b) $\left(\frac{\partial P}{\partial V}\right)_T$ (c) $\left(\frac{\partial T}{\partial P}\right)_V$ (d) $\left(\frac{\partial T}{\partial V}\right)_P$

Topic: Kinetic Theory of Gases and Thermodynamics

Sub Topic: Thermodynamic Relation

Ans. : (a)

Solution: The expression for

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P,\ k = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

$$\beta=\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P,\ k=-\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \quad \frac{\beta}{k}=-\frac{\left(\frac{\partial V}{\partial T}\right)_P}{\left(\frac{\partial V}{\partial P}\right)_T}=-\left(\frac{\partial V}{\partial T}\right)_P\times\left(\frac{\partial P}{\partial V}\right)_T$$

We know that

$$\left(\frac{\partial P}{\partial V}\right)_T\times\left(\frac{\partial V}{\partial T}\right)_P\times\left(\frac{\partial T}{\partial P}\right)_V=-1\Rightarrow\left(\frac{\partial P}{\partial V}\right)_T=\frac{-1}{\left(\frac{\partial V}{\partial T}\right)_P\times\left(\frac{\partial T}{\partial P}\right)_V}$$

$$\frac{\beta}{k}=\frac{1}{\left(\frac{\partial T}{\partial P}\right)_V}=\left(\frac{\partial P}{\partial T}\right)_V$$

Q5. The resultant of the binary subtraction $1110101-0011110$ is

(a) 1001111 (b) 1010111 (c) 1010011 (d) 1010001

Topic: Solid State Devices and Electronics

Sub Topic: Digital Electronics

Ans. : (b)

Solution: $1\ 1\ 1\ 0 1 0\ 1 - 0\ 0\ 1\ 1 1 1\ 0 = 1\ 0\ 1 0 1 1\ 1$

Q6. Consider a particle trapped in a three-dimensional potential well such that $U(x,y,z)=0$ for $0\le x\le a, 0\le y\le a, 0\le z\le a$ and $U(x,y,z)=\infty$ everywhere else. The degeneracy of the 5th excited state is

(a) 1 (b) 3 (c) 6 (d) 9

Topic: Modern Physics

Sub Topic: Quantum Mechanics: 3 Dimension Box

Ans. (c)

Solution: For 5th Excited state

n_x	n_y	n_z
3	2	1
2	3	1
1	2	3
1	3	2
3	1	2
2	1	3

Therefore, the degeneracy is 6

Q7. A particle of mass m and angular momentum L moves in space where its potential energy is $U(r) = kr^2 \left(k > 0\right)$ and r is the radial coordinate.

If the particle moves in a circular orbit, then the radius of the orbit is

(a) $\left(\frac{L^2}{mk}\right)^{1/4}$ (b) $\left(\frac{L^2}{2mk}\right)^{1/4}$ (c) $\left(\frac{2L^2}{mk}\right)^{1/4}$ (d) $\left(\frac{4L^2}{mk}\right)^{1/4}$

Topic: Mechanics and General Properties of Matter

Sub Topic: Central Force Problem

Ans. (b)

Solution: Since, the effective potential can be written as follows

$$U_{eff} = \frac{l^2}{2mr^2} + kr^2$$

For circular orbit,

$$\left.\frac{dU_{eff}}{dr}\right|_{r=r_0} = \frac{-l^2}{mr_0^{\;3}} + 2kr_0 = 0 \Rightarrow r_0^{\;4} = \frac{l^2}{2mk} \Rightarrow r_0 = \left(\frac{l^2}{2mk}\right)^{1/4}$$

Q8. Consider a two-dimensional force field

$\vec{F}(x, y) = \left(5x^2 + ay^2 + bxy\right)\hat{x} + \left(4x^2 + 4xy + y^2\right)\hat{y}$

If the force field is conservative, then the values of a and b are

(a) $a = 2$ and $b = 4$ (b) $a = 2$ and $b = 8$

(c) $a = 4$ and $b = 2$ (d) $a = 8$ and $b = 2$

Topic: Mathematical Physics

Sub Topic: Vector Analysis

Ans. (b)

Solution: $\vec{F} = \left(5x^2 + ay^2 + bxy\right)\hat{x} + \left(4x^2 + 4y^2 + 4xy\right)\hat{y}$

For conservative force,

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \left(5x^2 + ay^2 + bxy\right) & \left(4x^2 + 4y^2 + 4xy\right) & 0 \end{vmatrix} = 0$$

$$\hat{x} \times 0 + \hat{y} \times 0 + \hat{z}\left(8x + 4y - 2ay - bx\right) = 0 \Rightarrow 8x - bx = 0, b = 8,\; 4y - 2ay = 0,\; a = 2$$

Q9. Consider an electrostatic field $\vec{E}$ in a region of space. Identify the INCORRECT statement.

(a) The work done in moving a charge in a closed path inside the region is zero

(b) The curl of $\vec{E}$ is zero

(c) The field can be expressed as the gradient of a scalar potential

(d) The potential difference between any two points in the region is always zero

Topic: Electricity and Magnetism

Sub Topic: Electrostats

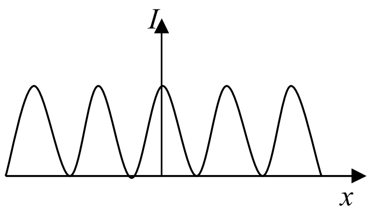
Ans. (d)

Solution: For electric field $\vec{E}$, the potential difference between point a and b can be written as follows

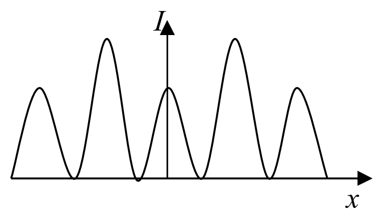
$V(b)-V(a)=-\int_a^b \vec{E}.\vec{dl}$, which can not be always zero. This is only zero for close path.

Q10. Which one of the following figures correctly depicts the intensity distribution for Fraunhofer diffraction due to a single slit? Here, x denotes the distance from the centre of the central fringe and I denotes the intensity.

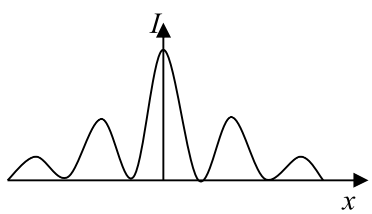
(a)



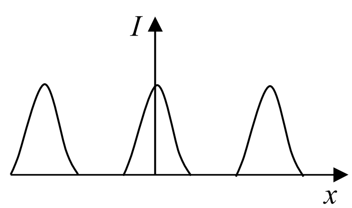
(b)



(c)



(d)



Topic: Waves and Optics

Sub Topic: Diffraction

Ans. : (c)

Solution: From Fraunhofer, the intensity distribution can be written as follows

$$I(x)=I_0\left(\frac{\sin(x)}{x}\right)^2$$

$$I(x)=I_0\left(\frac{\sin(x)}{x}\right)^2$$

The central maxima will be at $x=0$

Since, $\lim\limits_{x\to 0}\frac{\sin(x)}{x}=1$

Thus, plot c correctly describe the intensity plot.

Section A: Q11 – Q30 Carry TWO marks each.

Q11. The function $f(x)=e^{\sin x}$ is expanded as a Taylor series in x, around $x=0$, in the form $f(x)=\sum_{n=0}^{\infty}a_n x^n$. The value of $a_0+a_1+a_2$ is

(a) 0 (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) 5

Topic: Mathematical Physics

Sub Topic: Taylor Expansion

Ans. : (c)

Solution: $f(x)=e^{\sin(x)}\Rightarrow f(0)=e^{\sin(0)}=1$

$f'(x)=e^{\sin(x)}\cos(x)\Rightarrow f'(0)=e^{\sin(0)}\cos(0)=1$

$f''(x)=e^{\sin(x)}\cos^2(x)-\sin(x)e^{\sin(x)}\Rightarrow f''(0)=1$

Taylor's series

$$f(x)=f(0)+xf'(0)+\frac{x^2}{2}f''(0)..........=1+x+\frac{x^2}{2}..........=a_0x^0+a_1x^1+a_2x^2$$

$a_0=1, a_1=1, a_2=1/2,\ a_0+a_1+a_2=1+1+1/2=5/2$

Q12. Consider a unit circle C in the xy plane, centered at the origin. The value of the integral $\oint\left[(\sin x-y)dx-(\sin y-x)dy\right]$ over the circle C, traversed anticlockwise, is

(a) 0 (b) 2π (c) 3π (d) 4π

Topic: Mathematical Physics

Sub Topic: Vector Analysis

Ans. : (b)

Solution: $\oint(\sin x-y)dx-((\sin y-x))dy=\oint\left((\sin x-y)\hat{x}-((\sin y-x))\hat{y}\right).\vec{dl}$

$$=\oint\vec{F}.\vec{dl}=\oint\vec{F}.\vec{dl}=\iint\vec{\nabla}\times\vec{F}.\vec{ds}$$

$$\vec{\nabla}\times\vec{F}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ (\sin x-y) & -((\sin y-x)) & 0\end{vmatrix}=2$$

$$\iint \vec{\nabla}\times\vec{F}.\vec{ds} = 2\times\pi\times 1^2 = 2\pi$$

Q13. The current though a series RL circuit, subjected to a constant $emf\ \varepsilon$, obeys $L\frac{di}{dt}+iR=\varepsilon$. Let $L=1mH, R=1k\Omega$ and $\varepsilon=1V$. The initial condition is $i(0)=0$. At $t=1\,\mu s$, the current in mA is

(a) $1-2e^{-2}$ (b) $1-2e^{-1}$ (c) $1-e^{-1}$ (d) $2-2e^{-1}$

Topic: Electricity and Magnetism

Sub Topic: L-R Circuit

Ans. : (c)

Solution: $L\frac{di}{dt}+iR=\varepsilon$

$$\Rightarrow \frac{di}{dt}+\frac{i}{L}R=\frac{\varepsilon}{L}\Rightarrow i(t)e^{\frac{R}{L}t}=\frac{\varepsilon}{L}\int e^{\frac{R}{L}t}dt=\frac{\varepsilon}{L}\times\frac{L}{R}e^{\frac{R}{L}t}+c$$

$$i(t)=\frac{\varepsilon}{R}+ce^{-\frac{R}{L}t},\ i(0)=0=\frac{\varepsilon}{R}+c\Rightarrow c=-\frac{\varepsilon}{R}$$

For, $t=1$, $\varepsilon=1V, L=1mH, R=1k\Omega$

$$i(1)=\frac{\varepsilon}{R}-\frac{\varepsilon}{R}e^{-\frac{R}{L}t}=1-e^{-1}$$

Q14. An ideal gas in equilibrium at temperature T expands isothermally to twice its initial volume. If $\Delta S, \Delta U$ and ΔF denote the changes in its entropy, internal energy and Helmholtz free energy respectively, then

(a) $\Delta S<0, \Delta U>0, \Delta F<0$ (b) $\Delta S>0, \Delta U=0, \Delta F<0$

(c) $\Delta S<0, \Delta U=0, \Delta F>0$ (d) $\Delta S>0, \Delta U>0, \Delta F=0$

Topic: Kinetic Theory and Thermodynamics

Sub Topic: Law of Thermodynamics

Ans. : (b)

Solution: For isothermal expansion,

$\Delta U=0$

From 1st law of thermodynamics

$\Delta Q=\Delta U+\Delta W=\Delta W$

$$\Delta W=\int_{v_1}^{v_2}pdv=\int_{v_1}^{v_2}\frac{nRT}{v}dv=nRT\ln(\frac{v_2}{v_1})=nRT\ln(2)$$

$$\Delta S = \frac{\Delta Q}{T} = \frac{\Delta W}{T} = nR\ln(2) > 0$$

Now, Heltmotz free energy

$$F = U - TS \Rightarrow \Delta F = 0 - T\Delta S = -T\Delta S < 0$$

Q15. In a dilute gas, the number of molecules with free path length $\geq x$ is given by $N(x) = N_0 e^{-x/\lambda}$, where N_0 is the total number of molecules and λ is the mean free path. The fraction of molecules with free path lengths between λ and 2λ is

(a) $\frac{1}{e}$ (b) $\frac{e}{e-1}$ (c) $\frac{e^2}{e-1}$ (d) $\frac{e-1}{e^2}$

Topic: Kinetic Theory and Thermodynamics

Sub Topic: Mean Free Path

Ans. : (d)

Solution: Given that, $N(x) = N_0 e^{-x/\lambda}$

The number of molecule between free path $\lambda\, and\, 2\lambda$ is

$$\frac{\int_{\lambda}^{2\lambda} N_0 e^{-x/\lambda} dx}{\int_0^{\infty} N_0 e^{-x/\lambda} dx} = \frac{N_0(-\lambda)}{N_0(-\lambda)}\left[e^{-1} - e^{-2}\right] \Rightarrow \frac{e-1}{e^2}$$

Q16. Consider a quantum particle trapped in a one-dimensional potential well in the region $\left[\frac{-L}{2} < x < \frac{L}{2}\right]$, with infinitely high barriers at $x = \frac{-L}{2}$ and $x = \frac{L}{2}$. The stationary wave function for the ground state is $\psi(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$. The uncertainties in momentum and position satisfy

(a) $\Delta p = \frac{\pi\hbar}{L}$ and $\Delta x = 0$

(b) $\Delta p = \frac{2\pi\hbar}{L}$ and $0 < \Delta x < \frac{L}{2\sqrt{3}}$

(c) $\Delta p = \frac{\pi\hbar}{L}$ and $\Delta x > \frac{1}{2\sqrt{3}}$

(d) $\Delta p = 0$ and $\Delta x = \frac{L}{2}$

Topic: Modern Physics

Sub Topic: Quantum Mechanics: Particle in Box

Ans. : (Answer not matched)

Solution: $\langle x \rangle = 0 \Rightarrow \langle x^2 \rangle = \frac{\pi^2 - 6}{12\pi^2} \times L^2$. So $\Delta x = \sqrt{\frac{\pi^2 - 6}{12\pi^2}} \times L = 0.18L$

$$\langle p\rangle = 0 \Rightarrow \langle p^2\rangle = \frac{\pi^2\hbar^2}{L^2} \Rightarrow \Delta p = \frac{\pi\hbar}{L}$$

Q17. Consider a particle of mass m moving in a plane with a constant radial speed $\dot{r}$ and a constant angular speed $\dot{\theta}$. The acceleration of the particle in (r,θ) coordinates is

(a) $2r\dot{\theta}^2\hat{r} - \dot{r}\dot{\theta}\hat{\theta}$

(b) $-r\dot{\theta}^2\hat{r} + 2\dot{r}\dot{\theta}\hat{\theta}$

(c) $\ddot{r}\hat{r} + r\ddot{\theta}\hat{\theta}$

(d) $\ddot{r}\theta\hat{r} + r\ddot{\theta}\hat{\theta}$

Topic: Mechanics and General Properties of Matter

Sub Topic: Newton's Law in Polar Coordinate

Ans. (b)

Solution: Given that, $\dot{r} = c_1,\ \dot{\theta} = c_2$

$$\vec{a} -= (\ddot{r} - r\theta^2)\hat{r} + (r\dot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta},\ \ddot{r} = 0,\ \ddot{\theta} = 0$$

$$\vec{a} = (-r\theta^2)\hat{r} + \left(2\dot{r}\dot{\theta}\right)\hat{\theta}.$$

Q18. A planet of mass m moves in an elliptical orbit. Its maximum and minimum distances from the Sun are R and r, respectively. Let G denote the universal gravitational constant, and M the mass of the Sun. assuming $M >> m$, the angular momentum of the planet with respect to the center of the Sun is

(a) $m\sqrt{\frac{2GMRr}{(R+r)}}$

(b) $m\sqrt{\frac{GMRr}{2(R+r)}}$

(c) $m\sqrt{\frac{GMRr}{(R+r)}}$

(d) $2m\sqrt{\frac{2GMRr}{(R+r)}}$

Topic: Mechanics and General Properties of Matter

Sub Topic: Central Force Problem

Ans. : (a)

Solution: The energy expression can be written as follows

$$E = \frac{m}{2}\dot{r}^2 + \frac{J^2}{2mr^2} - \frac{GmM}{r}$$

$$E = \frac{m}{2}\dot{R}^2 + \frac{J^2}{2mR^2} - \frac{GmM}{R}$$

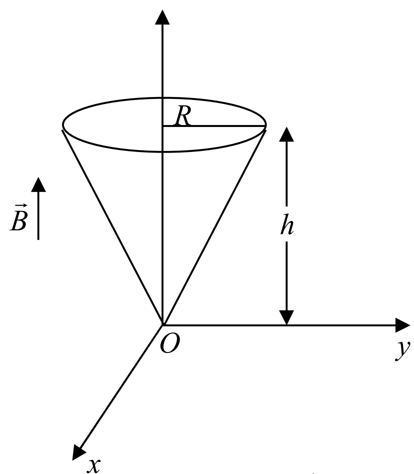
The energy can be written as follows,

$$E=-\frac{GmM}{r_{\max}+r_{\min}}=-\frac{GmM}{R+r}$$

Also, $\dot{r}=\dot{R}=0$

$$-\frac{GmM}{R+r}=0+\frac{J^2}{2mR^2}-\frac{GmM}{R}=\frac{J^2}{2mR^2}-\frac{GmM}{R}\Rightarrow J=m\sqrt{\frac{2GMRr}{R+r}}$$

Q19. Consider a conical region of height h and base radius R with its vertex at the origin. Let the outward normal to its base be along the positive z-axis, as shown in the figure. A uniform magnetic field, $\vec{B}=B_0\hat{z}$ exists everywhere. Then the magnetic flux through the base (Φ_b) and that through the curved surface of the cone (Φ_c) are



(a) $\Phi_b=B_0\pi R^2;\Phi_c=0$

(b) $\Phi_b=-\frac{1}{2}B_0\pi R^2;\Phi_c=\frac{1}{2}B_0\pi R^2$

(c) $\Phi_b=0;\Phi_c=-B_0\pi R^2$

(d) $\Phi_b=B_0\pi R^2;\Phi_c=-B_0\pi R^2$

Topic: Electricity and Magnetism

Sub Topic: Magnetostat

Ans. : (d)

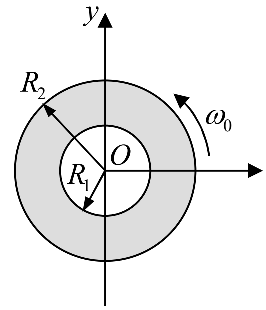
Solution: $\oiint B.ds=\iiint(\nabla.B)dv=0\ \left[\sin ce,(\nabla.B)=0\right]$

$\oiint B.ds=0$

$\phi_b+\phi_c=0,\ -\phi_b=\phi_c,\ \ Now,\phi_b=\oiint B.ds=B\pi R^2\quad [\sin ce, B\,\square\, ds]$

$\phi_c=-B\pi R^2$

Q20. Consider a thin angular sheet, lying on the xy-plane, with R_1 and R_2 as its inner and outer radii, respectively. If the sheet carries a uniform surface-charge density σ and spins about the origin O with a constant angular velocity $\vec{\omega} = \omega_0 \hat{z}$ then, the total current flow on the sheet is



(a) $\dfrac{2\pi\sigma\omega_0\left(R_2^3 - R_1^3\right)}{3}$ (b) $\sigma\omega_0\left(R_2^3 - R_1^3\right)$

(c) $\dfrac{\pi\sigma\omega_0\left(R_2^3 - R_1^3\right)}{3}$ (d) $\dfrac{2\pi\sigma\omega_0\left(R_2 - R_1\right)^3}{3}$

Topic: Electricity and Magnetism

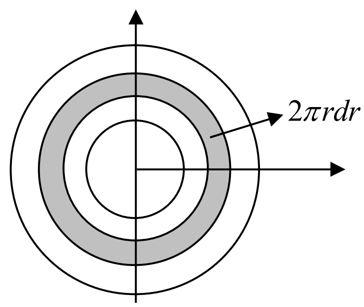
Sub Topic: Magnetostatics

Ans. : (Answer not matched)

Solution: $dq = 2\pi r dr \sigma$

$$dI = \frac{2\pi r dr \sigma}{2\pi / \omega} \Rightarrow dI = \sigma\omega r dr$$

$$I = \frac{\sigma\omega}{2}\left[R_2^2 - R_1^2\right]$$



Q21. A radioactive nucleus has a decay constant λ and its radioactive daughter nucleus has a decay constant 10λ. At time $t = 0, N_0$ is the number of parent nuclei and there are no daughter nuclei present. $N_1(t)$ and $N_2(t)$ are the number of parent and daughter nuclei present at time t, respectively. The ratio $\dfrac{N_2(t)}{N_1(t)}$ is

(a) $\dfrac{1}{9}\left[1 - e^{-9\lambda t}\right]$ (b) $\dfrac{1}{10}\left[1 - e^{-10\lambda t}\right]$ (c) $\left[1 - e^{-10\lambda t}\right]$ (d) $\left[1 - e^{-9\lambda t}\right]$

Topic: Modern Physics

Sub Topic: Nuclear Physics: Radioactivity

Ans. (a)

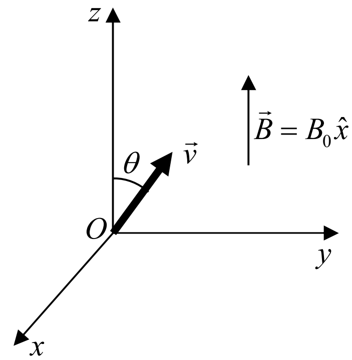
Solution: $N_1(t) = N_0 \exp^{(-\lambda t)}$

$$N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10}(\exp^{(-\lambda_1 t)} - \exp^{(-\lambda_2 t)})$$

$$= \frac{\lambda}{10\lambda - \lambda} N_0(\exp^{(-\lambda t)} - \exp^{(-10\lambda t)}) = \frac{1}{9} N_0(\exp^{(-\lambda t)} - \exp^{(-10\lambda t)})$$

$$\frac{N_2(t)}{N_1(t)} = \frac{1}{9}(1-\exp^{(-9\lambda t)})$$

Q22. A uniform magnetic field $B = B_0\hat{z}$, where $B_0 > 0$ exists as shown in the figure. A charged particle of mass m and charge $q(q>0)$ is released at the origin, in the yz-plane, with a velocity $\vec{v}$ directed at an angle $\theta = 45^0$ with respect to the positive z-axis. Ignoring gravity, which one of the following is TRUE.



(a) The initial acceleration $\vec{a} = \dfrac{qvB_0}{\sqrt{2}m}\hat{x}$

(b) The initial acceleration $\vec{a} = \dfrac{qvB_0}{\sqrt{2}m}\hat{y}$

(c) The particle moves in a circular path

(d) The particle continues in a straight line with constant sped

Topic: Electricity and Magnetism

Sub Topic: Charge Particle in Electromagnetic Field

Ans. : (a)

Solution: $\vec{V} = \dfrac{v}{\sqrt{2}}(\hat{j}+\hat{k}) \Rightarrow v_{\parallel} = \dfrac{v}{\sqrt{2}},\ v_{\perp} = \dfrac{v}{\sqrt{2}}$

Lorentz force, $\vec{F} = q(\vec{v}\times\vec{B}) = q(\dfrac{v}{\sqrt{2}}\hat{y}\times B_o\hat{z}) = q\dfrac{v}{\sqrt{2}}B_o\hat{x} \Rightarrow \vec{a} = \dfrac{q\dfrac{v}{\sqrt{2}}B_o\hat{x}}{m}$

Q23. For an ideal intrinsic semiconductor, the Fermi energy at $0\,K$

(a) lies at the top of the valence band

(b) lies at the bottom of the conduction band

(c) lies at the center of the bandgap

(d) lies midway between center of the bandgap and bottom of the conduction band

Topic: Solid State Devices and Electronics

Sub Topic: Semiconductor

Ans. : (c)

Solution: Location of Fermi level can be written as follows

$E_F = \dfrac{E_C+E_V}{2} + \dfrac{3}{4}kt\ln(\dfrac{m_h}{m_e})$. At $T=0$, $E_F = \dfrac{E_C+E_V}{2}$

It locates at the centre.

Q24. A circular loop of wire with radius R is centered at the origin of the xy-plane. The magnetic field at a point within the loop is, $\vec{B}=(\rho,\phi,z,t)=k\rho^3t^3\hat{z}$, where k is a positive constant of appropriate dimensions. Neglecting the effects of any current induced in the loop, the magnitude of the induced emf in the loop at time t is

(a) $\frac{6\pi kt^2R^5}{5}$ (b) $\frac{5\pi kt^2R^5}{6}$ (c) $\frac{3\pi kt^2R^5}{2}$ (d) $\frac{\pi kt^2R^5}{2}$

Topic: Electricity and Magnetism

Sub Topic: Faraday's Law

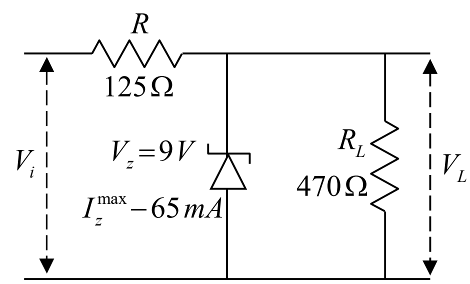
Ans. : (a)

Solution: Given that $B(\rho,\phi,z,t)=k\rho^3t^3\hat{z}$

Flux, $\phi=\int_0^R k\rho^3t^3\rho d\rho d\phi=\frac{2\pi kt^3R^5}{5}$

Induced emf, $\varepsilon=\left|\frac{d\phi}{dt}\right|=\frac{6\pi kt^2R^5}{5}$

Q25. For the given circuit, $R=125\Omega$, $R_L=470\Omega$, $V_z=9V$, and $I_z^{\max}=65\,mA$. The minimum and maximum values of the input voltage ($V_i^{\min}$ and $V_i^{\max}$) for which the Zener diode will be in the 'ON' state are



(a) $V_i^{\min}=9.0V$ and $V_i^{\max}=11.4V$ (b) $V_i^{\min}=9.0V$ and $V_i^{\max}=19.5V$

(c) $V_i^{\min}=11.4V$ and $V_i^{\max}=15.5V$ (d) $V_i^{\min}=11.4V$ and $V_i^{\max}=19.5V$

Topic: Electronics

Sub Topic: Zener Diode

Ans. : (d)

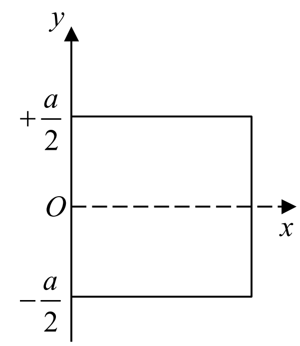
Solution: $v_A=\frac{470}{470+125}v_i$. The Zener diode to be in ON state $v_A\geq 9V$

$9=\frac{470}{470+125}v_{i,\min}\Rightarrow v_{i,\min}=11.39V$

$v_{i,\max}$ Zener is On (When maximum current pass through the Zener)

$$v_{i,\max}=v_z+(I_{z,\max}+I_L)R=9+(65\,mA+\frac{9}{470}A)125=19.51V$$

Q26. A square laminar sheet with side a and mass M, has mass per unit area given by $\sigma(x)=\sigma_0\left[1-\frac{x}{a}\right]$, (see figure). Moment of inertia of the sheet about y-axis is



(a) $\frac{Ma^2}{2}$ (b) $\frac{Ma^2}{4}$

(c) $\frac{Ma^2}{6}$ (d) $\frac{Ma^2}{12}$

Topic: Mechanics and General Properties of Matter

Sub Topic: Moment of Inertia

Ans. : (c)

Solution: $\sigma(x)=\sigma_0\left[1-\frac{x}{a}\right]$

$$I_{yy}=\int x^2dm\,,\ dm=\sigma(x)dxdy\,,\ I_{yy}=\int_0^a x^2\sigma(x)dx\int_{-a/2}^{a/2}dy$$

$$I_{yy}=a\sigma_0\int_{-a/2}^{a/2}\left(x^2-\frac{x^3}{a}\right)dx=a\sigma_0\left(\frac{a^3}{3}-\frac{a^3}{4}\right)=\frac{\sigma_0a^4}{12}$$

$$M=\iint\sigma(x)dxdy=\int_0^a\sigma(x)dx\int_{-a/2}^{a/2}dy=\int_0^a\sigma_0\left[1-\frac{x}{a}\right]dx\int_{-a/2}^{a/2}dy=\frac{\sigma_0a^2}{2}\Rightarrow\sigma_0=\frac{2M}{a^2}\Rightarrow I_{yy}=\frac{Ma^2}{6}$$

Q27. A particle is subjected to two simple harmonic motions along x and y axes, described by $x(t)=a\sin(2\omega t+\pi)$ and $y(t)=2a\sin(\omega t)$. The resultant motion is given by

(a) $\frac{x^2}{a^2}+\frac{y^2}{4a^2}=1$ (b) $x^2+y^2=1$

(c) $y^2=x^2\left(1-\frac{x^2}{4a^2}\right)$ (d) $x^2=y^2\left(1-\frac{y^2}{4a^2}\right)$

Topic: Waves and Optics

Sub Topic: Superposition of Waves

Ans. : (d)

Solution: $x(t)=a\sin(2\omega t+\pi),\ y(t)=2a\sin(\omega t)$

$$\left(\frac{y(t)}{2a}\right)^2=\sin^2(\omega t)$$

$$x(t) = a\sin(2\omega t + \pi) = -a\sin(2\omega t) = -2a\sin(\omega t)\cos(\omega t) = -y\cos(\omega t) = -y\sqrt{1-\sin^2(\omega t)}$$

$$x(t) = -y\sqrt{1-\left(\frac{y(t)}{2a}\right)^2} \Rightarrow x^2(t) = y^2\left(1-\left(\frac{y(t)}{2a}\right)^2\right)$$

Q28. For a certain thermodynamic system, the internal energy $U = PV$ and P is proportional to T^2. The entropy of the system is proportional to

(a) UV (b) $\sqrt{\frac{U}{V}}$ (c) $\sqrt{\frac{V}{U}}$ (d) $\sqrt{UV}$

Topic: Kinetic Theory and Thermodynamics

Sub Topic: Thermodynamic Potential

Ans. : (d)

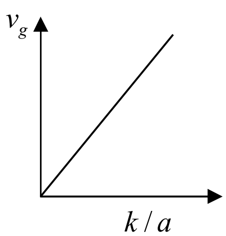
Solution: $U = PV$, $P\alpha T^2 \Rightarrow P = kT^2$

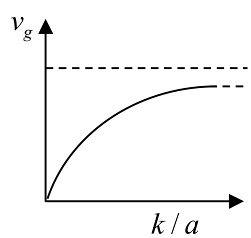
$$\Rightarrow U = VkT^2 \Rightarrow \frac{U}{kV} = T^2 \Rightarrow \sqrt{\frac{U}{kV}} = T$$

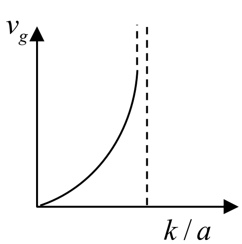
We know that $Tds = C_v dT + Pdv \Rightarrow ds = \frac{C_v dT}{T} \Rightarrow s = C_v \ln T \Rightarrow s\,\alpha\, C_v$

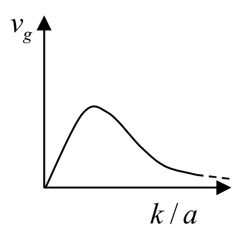
We know that, $C_v = \left(\frac{dU}{dT}\right)_v = VkT^2 = 2kTV = 2kV\sqrt{\frac{U}{kV}}\,\alpha\sqrt{UV} \Rightarrow S\alpha\sqrt{UV}$

Q29. The dispersion relation for certain type of waves is given by $\omega = \sqrt{k^2 + a^2}$, where k is the wave vector and a is a constant. Which one of the following sketches represents v_g, the group velocity?

(a)


(b)


(c)


(d)


Topic: Waves and Optics

Sub Topic: Group Velocity and Phase Velocity

Ans. : (b)

Solution: Given that

$$\omega=\sqrt{k^2+a^2}\Rightarrow\omega^2=k^2+a^2\Rightarrow 2\omega\frac{d\omega}{dk}=2k\Rightarrow\frac{d\omega}{dk}=\frac{k}{\omega}\Rightarrow v_g=\frac{k}{\omega}=\frac{k}{\sqrt{k^2+a^2}}$$

At $k\to 0\Rightarrow v_g=0$ At large k, $v_g=\frac{k}{k}=1\,\text{constant}$

Q30. Consider a binary number with m digits, where m is an even number. This binary number has alternating $1's$ and $0's$ with digit 1 in the highest place value. The decimal equivalent of this binary number is

(a) 2^m-1 (b) $\frac{(2^m-1)}{3}$ (c) $\frac{(2^{m+1}-1)}{3}$ (d) $\frac{2}{3}(2^m-1)$

Topic: Solid State Devices and Electronics

Sub Topic: Digital Electronics

Ans. : (d)

Solution: 101010101010............

The sum of this series will be

$$2^{m-1}+0+2^{m-3}+0+2^{m-5}+...............+2+0$$

$$=2^{m-1}+2^{m-3}+2^{m-5}+..............=2^{m-1}(1+1/4+1/16...............)$$

$$=2^{m-1}\left(\frac{(1/4)^{m/2}-1}{1/4-1}\right)=\frac{4}{3}2^{m-1}\left(1-2^{-m}\right)=\frac{2}{3}\left(2^m-1\right)$$

Section B: Q31 – Q40 Carry TWO marks each

Q31. Consider the 2×2 matrix $M=\begin{pmatrix}0 & a\\ a & b\end{pmatrix}$, where $a,b>0$. Then,

(a) M is a real symmetric matrix

(b) One of the eigenvalues of M is greater than b

(c) One of the eigenvalues of M is negative

(d) Product of eigenvalues of M is b

Topic: Mathematical Physics

Sub Topic: Matrices

Ans. : (a, b, c)

Solution: $M=\begin{pmatrix}0 & a\\ a & b\end{pmatrix}\Rightarrow M^T=\begin{pmatrix}0 & a\\ a & b\end{pmatrix}$

$$\begin{vmatrix} 0-\lambda & a \\ a & b-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - b\lambda - a^2 = 0, \lambda = \frac{b \pm \sqrt{b^2+4a^2}}{2}$$

$$\lambda_1 = \frac{b+\sqrt{b^2+4a^2}}{2} > b, \lambda_2 = \frac{b-\sqrt{b^2+4a^2}}{2} < 0 \quad \lambda_1\lambda_2 \neq b$$

Q32. In the Compton scattering of electrons, by photons incident with wavelength λ,

(a) $\frac{\Delta\lambda}{\lambda}$ is independent of λ

(b) $\frac{\Delta\lambda}{\lambda}$ increases with decreasing λ

(c) There is no change in photon's wavelength for all angles of deflection of the photon

(d) $\frac{\Delta\lambda}{\lambda}$ increases with increasing angle of deflection of the photon

Topic: Modern Physics

Sub Topic: Compton effect

Ans. : (b, d)

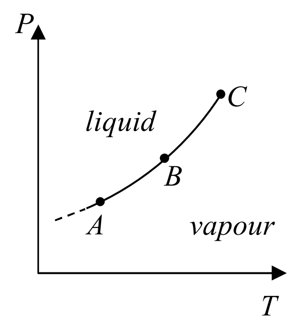
Solution: In Compton scattering, $\Delta\lambda = \lambda_c\left(1-\cos(\theta)\right)$

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda_c}{\lambda}\left(1-\cos(\theta)\right) \Rightarrow \frac{\Delta\lambda}{\lambda}\alpha\frac{1}{\lambda}, \ \frac{\Delta\lambda}{\lambda}\alpha\left(1-\cos(\theta)\right)$$

$\frac{\Delta\lambda}{\lambda}\alpha\frac{1}{\lambda}$, Which implies $\frac{\Delta\lambda}{\lambda}$ will increase with decreasing λ

$\frac{\Delta\lambda}{\lambda}\alpha\left(1-\cos(\theta)\right)$ Which implies $\frac{\Delta\lambda}{\lambda}$ will increase with increasing deflection angle.

Q33. The figure shows a section of the phase boundary separating the vapour (1) and liquid (2) states of water in the $P-T$ plane. Here, C is the critical point. μ_1, v_1 and s_1 are the chemical potential, specific volume and specific entropy of the vapour phase respectively, while μ_2, v_2 and s_2 respectively denote the same for the liquid phase. Then



(a) $\mu_1 = \mu_2$ along AB

(b) $v_1 = v_2$ along AB

(c) $s_1 = s_2$ along AB

(d) $v_1 = v_2$ at the point C

Topic: Kinetic Theory and Thermodynamics

Subtopic: Phase Transition

Ans. : (a, d)

Q34. A particle is executing simple harmonic motion with time period T. Let x, v and a denote the displacement, velocity and acceleration of the particle, respectively, at time t. Then

(a) $\frac{aT}{x}$ does not change with time

(b) $(aT+2\pi v)$ does not change with time

(c) x and v are related by an equation of a straight line

(d) v and a are related by an equation of an ellipse

Ans. : (a, d)

Solution: $x = a\sin(\omega t+\phi) \quad T=\frac{2\pi}{\omega}$

$$v=\frac{dx}{dt}=a\omega\sqrt{\left(1-\frac{x^2}{a^2}\right)}=\omega\sqrt{\left(a^2-x^2\right)} \quad \text{and} \quad a=\frac{d^2x}{dt^2}=-\omega^2 x$$

$\frac{aT}{x}=\frac{2\pi}{\omega}\times-\frac{\omega^2 x}{x}=-2\pi\omega$ Independent on time

$$aT+2\pi v=-\omega^2 x\frac{2\pi}{\omega}+2\pi\omega\sqrt{\left(a^2-x^2\right)}=-2\pi\omega x+2\pi\omega\sqrt{\left(a^2-x^2\right)} \quad [x=f(t)]$$

$aT+2\pi v$ changes with time $v^2=\omega^2\left(a^2-x^2\right)$ They are related by straight line

Topic: wave and optics

Subtopic: simple harmonic motion

Q35. A linearly polarized light beam travels from origin to point $A(1,0,0)$. At the point A, the light is reflected by a mirror towards point $B(1,-1,0)$. A second mirror located at point B then reflects the light towards point $C(1,-1,1)$. Let $\hat{n}(x,y,z)$ represent the direction of polarization of light at (x,y,z).

(a) If $\hat{n}(0,0,0)=\hat{y}$, then $\hat{n}(1,-1,1)=\hat{x}$

(b) If $\hat{n}(0,0,0)=\hat{z}$, then $\hat{n}(1,-1,1)=\hat{y}$

(c) If $\hat{n}(0,0,0)=\hat{y}$, then $\hat{n}(1,-1,1)=\hat{y}$

(d) If $\hat{n}(0,0,0)=\hat{z}$, then $\hat{n}(1,-1,1)=\hat{x}$

Topic: Waves and Optics

Sub Topic: Polarisation

Ans. : (a, b)

Q36. Let (r,θ) denote the polar coordinates of a particle moving in a plane. If $\hat{r}$ and $\hat{\theta}$ represent the corresponding unit vectors, then

(a) $\frac{d\hat{r}}{d\theta}=\hat{\theta}$ (b) $\frac{d\hat{r}}{dr}=-\hat{\theta}$ (c) $\frac{d\hat{\theta}}{d\theta}=-\hat{r}$ (d) $\frac{d\hat{\theta}}{dr}=\hat{r}$

Topic: Mechanics and General Properties of Matter

Newton's Law in Polar Coordinate

Ans. : (a, c)

Solution: The representation of $\hat{r}$ and $\hat{\theta}$ in polar coordinate can be written as follows

$$\begin{pmatrix}\hat{r}\\ \hat{\theta}\end{pmatrix}=\begin{pmatrix}\cos(\theta) & \sin(\theta)\\ -\sin(\theta) & \cos(\theta)\end{pmatrix}\begin{pmatrix}\hat{x}\\ \hat{y}\end{pmatrix}$$

$$\hat{r}=\cos(\theta)\hat{x}+\sin(\theta)\hat{y}............(1) \quad \hat{\theta}=-\sin(\theta)\hat{x}+\cos(\theta)\hat{y}.........(2)$$

$$\frac{d\hat{r}}{d\theta}=-\sin(\theta)\hat{x}+\cos(\theta)\hat{y}=\hat{\theta}\ [from 1]\ \frac{d\hat{\theta}}{d\hat{r}}=0 \quad \frac{d\hat{r}}{dr}=0,\ \frac{d\hat{\theta}}{d\theta}=-\cos(\theta)\hat{x}-\cos(\theta)\hat{y}=-\hat{r}$$

Q37. The electric field associated with an electromagnetic radiation is given by $E=a(1+\cos\omega_1 t)\cos\omega_2 t$. Which of the following frequencies are present in the field?

(a) ω_1 (b) $\omega_1+\omega_2$ (c) $|\omega_1-\omega_2|$ (d) ω_2

Topic: Electricity and Magnetism

Sub Topic: Electromagnetic Wave

Ans. (b, c, d)

Solution: The given field is

$$E=a(1+\cos(\omega_1 t))\cos(\omega_2 t)=a\cos(\omega_2 t)+\frac{a}{2}\left[\cos(\omega_1+\omega_2)t+\cos(\omega_1-\omega_2)t\right]$$

Three frequencies are there ω_2, $\omega_1+\omega_2$, and $\omega_1-\omega_2$.

Q38. A string of length L is stretched between two points $x=0$ and $x=L$ and the endpoints are rigidly clamped. Which of the following can represent the displacement of the string from the equilibrium position?

(a) $x\cos\left(\frac{\pi x}{L}\right)$ (b) $x\sin\left(\frac{\pi x}{L}\right)$ (c) $x\left(\frac{x}{L}-1\right)$ (d) $x\left(\frac{x}{L}-1\right)^2$

Topic: Wave and Optics

Sub Topic: Standing Wave

Ans. (b, c, d)

Solution: From given condition it is clearly noticeable that, the displacement at equilibrium is zero at and $x = L$.

(a) $X = x\cos(\frac{\pi x}{L})$, at $x = 0$ X =0 but at $x = L$ $X = L\cos(\pi) = -L$, This is not possible

(b) $X = x\sin(\frac{\pi x}{L})$, at $x = 0$ X =0 . Also, at $x = L$ $X = 0$, **This is possible**

(c) $X = x\left(\frac{x}{L} - 1\right)$, at $x = 0$ X =0 . Also, at $x = L$ $X = 0$, **This is possible**

(d) $X = x\left(\frac{x}{L} - 1\right)^2$, at $x = 0$ X =0 . Also, at $x = L$ $X = 0$, **This is possible**

Q39. The Boolean expression $Y = \overline{PQ}R + Q\overline{R} + \overline{P}QR + PQR$ simplifies to

(a) $\overline{P}R + Q$ (b) $PR + \overline{Q}$ (c) $P + R$ (d) $Q + R$

Ans. : (d)

Solution: $Y = \overline{PQ}\,R + Q\overline{R} + \overline{P}QR + PQR$

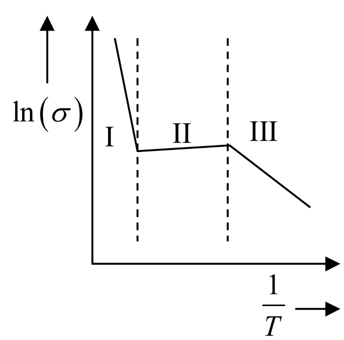
$$= R\left(\overline{PQ} + PQ\right) + Q\overline{R} + \overline{P}QR \quad \overline{PQ} + PQ = 1$$

$$= R + Q\overline{R} + \overline{P}QR = R + Q + \overline{P}QR = R\left(1 + \overline{P}Q\right) + Q = R + Q$$

Topic: Solid State Device and Electronics

Sub Topic: Digital Electronics

Q40. For an n-type silicon, an extrinsic semiconductor, the natural logarithm of normalized conductivity (σ) is plotted as a function of inverse temperature. Temperature interval-I corresponds to the intrinsic regime, interval-II corresponds to saturation regime and interval-III corresponds to the freeze-out regime, respectively. Then



(a) the magnitude of the slope of the curve in the temperature interval-I is proportional to the bandgap, E_g

(b) the magnitude of the slope of the curve in the temperature interval-III is proportional to the ionization energy of the donor, E_d

(c) in the temperature interval-II the carrier density in the conduction band is equal to the density of donors

(d) in the temperature interval-III, all the donor levels are ionized

Ans. : (a, b, c)

Solution: In region I

$$n_i = 2\times\left(\frac{2\pi kT}{h^2}\right)^{3/2}(m_h m_e)^{3/4}e^{-\frac{E_g}{2kT}}$$

$$\sigma = \mu n_i e = \mu e\times 2\times\left(\frac{2\pi kT}{h^2}\right)^{3/2}(m_h m_e)^{3/4}e^{-\frac{E_g}{2kT}}$$

$$\ln\sigma = \ln\left(\mu e\times 2\times\left(\frac{2\pi kT}{h^2}\right)^{3/2}\right)+3/4\ln(m_h m_e)-\frac{E_g}{2kT}$$

$$\ln\sigma = y \quad , \frac{1}{T}=x \quad y = mx,\ m = slope = \frac{E_g}{2k}\alpha E_g$$

In region III $n_i = N_d e^{-\frac{E_c-E_d}{kT}}$, $E_c - E_d =$ Donar Ionization energy

$$\sigma = \mu n_i e = \mu e N_d e^{-\frac{E_c-E_d}{kT}}$$

$$\Rightarrow \ln\sigma = \ln\left(\mu e N_d e\right) = \ln\left(\mu e N_d\right)-\frac{\left(E_c-E_d\right)}{kT}$$

Thus, slope in region in region III is proportional to ionization energy.

In region, the carrier density of conduction band is equal to the density of donors, because in this temperature range all donor get ionise

Topic: Solid State Device and Electronics

Sub Topic: Semiconductor

Section C: Q41 – Q50 Carry ONE mark each

Q41. The integral $\iint\left(x^2+y^2\right)dxdy$ over the area of a disk of radius 2 in the xy plane is ____ π.

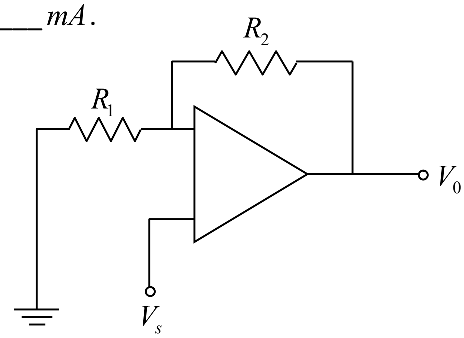
Ans. : 8 to 8

Solution: $I = \iint x^2+y^2 dxdy = \int r^2 r dr d\theta = \int_0^2 r^3 dr\int_0^{2\pi} d\theta = \frac{16}{4}2\pi = 8\pi$

Topic: Mathematical Physics

Sub Topic: Calculus of Multiple Variable

Q42. For the given operational amplifier circuit $R_1 = 120\Omega$, $R_2 = 1.5k\Omega$ and $V_s = 0.6V$, then the output current I_0 is ________ mA.



Ans. : 5 to 5

Solution: $\frac{V_o}{V_s} = (1+\frac{R_2}{R_1}) = (1+\frac{1500}{120})$

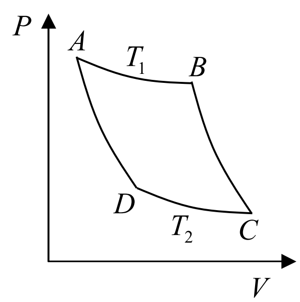
$$V_o = 0.6(1+\frac{1500}{120}) = 8.1V$$

$$V_o - V_s = IR_2\ I = \frac{V_o - V_s}{R_2} = \frac{7.5}{1.5}mA = 5\,mA$$

Topic: Solid State Device and Electronics

Sub Topic: Operational Amplifier

Q43. For an ideal gas AB and CD are two isothermals at temperatures T_1 and $T_2\left(T_1 > T_2\right)$, respectively. AD and BC represent two adiabatic paths as shown in figure. Let V_A, V_B, V_C and V_D be the volumes of the gas at A, B, C and D respectively. If $\frac{V_C}{V_B} = 2$, then $\frac{V_D}{V_A} =$ _______.



Ans. : 2 to 2

Solution: From given diagram

$B \to C$ adiabatic $TV^{\gamma-1}{}_B = TV^{\gamma-1}{}_C$ .....(1)

$D \to A$ adiabatic

$TV^{\gamma-1}{}_D = TV^{\gamma-1}{}_A$ .....(2)

Divide 1 by 2 $\frac{V^{\gamma-1}{}_B}{V^{\gamma-1}{}_D}=\frac{V^{\gamma-1}{}_C}{V^{\gamma-1}{}_A}\Rightarrow\frac{V_C}{V_B}=\frac{V_D}{V_A}=2\left[\because\frac{V_C}{V_B}=2\right]$

Topic: Kinetic Theory and Thermodynamics

Sub Topic: Law of Thermodynamics

Q44. A satellite is revolving around the Earth in a closed orbit. The height of the satellite above Earth's surface at perigee and apogee are $2500\,km$ and $4500\,km$, respectively. Consider the radius of the Earth to be $6500\,km$. The eccentricity of the satellite's orbit is ____ (Round off to 1 decimal place).

Ans. 0.1 to 0.1

Solution: $\frac{l}{r}=1+e\cos(\theta)\Rightarrow\frac{l}{r_{\max}}=1-e\ \ and\ \frac{l}{r_{\min}}=1+e$

$$\frac{r_{\max}}{r_{\min}}=\frac{1+e}{1-e}\Rightarrow e=\frac{r_{\max}-r_{\min}}{r_{\max}+r_{\min}}=\frac{1100-900}{1100+900}=\frac{200}{2000}=0.1$$

Topic: general properties of matter and mechanics

Subtopic: central force problem

Q45. Three masses $m_1=1, m_2=2$ and $m_3=3$ are located on the x-axis such that their center of mass is at $x=1$. Another mass $m_4=4$ is placed at x_0 and the new center of mass is at $x=3$. The value of x_0 is ________.

Ans. : 6 to 6

Solution: From given condition

$$x=\frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}=\frac{1x_1+2x_2+3x_3}{6}=1\Rightarrow 1x_1+2x_2+3x_3=6$$

Now, $3=\frac{1x_1+2x_2+3x+4x_0}{m_1+m_2+m_3+4}=\frac{6+4x_0}{10}\Rightarrow 6+4x_0=30\Rightarrow x_0=6$

Topic: Mechanics and General Properties of Matter

Sub Topic: Centre of Mass

Q46. A normal human eye can distinguish two objects separated by $0.35\,m$ when viewed from a distance of $1.0\,km$. The angular resolution of eye is _____ second (Round off to the nearest integer)

Ans. 71 to 73

Topic: Wave and Optics

Sub Topic: Resolution of Optical Instrument

Q47. A rod with a proper length of $3m$ moves along x-axis, making an angle of 30^0 with respect to the x-axis. If its speed is $\frac{c}{2}m/s$, where c is the speed of light, the change in length due to Lorentz contraction is _____ m. (Round off to 2 decimal places). [Use $c=3\times10^8\ m/s$]

Topic: Modern Physics

Sub Topic: STR length contraction

Ans. -0.31 to -0.29 or 0.29 to 0.31

Solution: $\Delta l = l_0 - l_0\sqrt{1-\left(\frac{v\cos\theta}{c}\right)^2} = 3-3\sqrt{1-\left(\frac{1\sqrt{3}}{4}\right)^2} = 3-3\sqrt{1-\frac{3}{16}} = 3-3\times\sqrt{\frac{13}{16}} = 0.3$

Q48. Consider the Bohr model of hydrogen atom. The speed of an electron in the second orbit $(n=2)$ is _______ $\times10^6\ m/s$ (Round off to 2 decimal places).

[Use $h=6.63\times10^{-34}Js,\ e=1.6\times10^{-19}C,\ \in_0=8.85\times10^{-12}C^2m^2/N$]

Topic: Modern Physics

Sub Topic: Hydrogen Atom

Ans. 1.08 to 1.10

Solution: The expression for velocity can be written as follows

$v_n = 2.18\times10^6\frac{z}{n}m/s$ For Hydrogen atom $z=1$ For $n=2$

$v_n = 2.18\times10^6\frac{1}{2}m/s = 1.09\times10^6\ m/s$

Q49. Consider a unit circle C in the xy plane with center at the origin. The line integral of the vector field, $\vec{F}(x,y,z) = -2y\hat{x} - 3z\hat{y} + x\hat{z}$, taken anticlockwise over C is ______ π.

Topic: Mathematical Physics

Sub Topic: Vector Analysis

Ans. 2 to 2

Solution: $\vec{F}(x,y,z) = -2y\hat{x} - 3z\hat{y} + x\hat{z}$

$\oint\vec{F}(x,y,z).\vec{dl} = \oint\vec{\nabla}\times\vec{F}(x,y,z).\vec{ds}$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -2y & -3z & x \end{vmatrix} = 3\hat{x} - \hat{y} + 2\hat{z}$$

$$\oiint \vec{\nabla}\times\vec{F}(x,y,z).\vec{ds} = \oiint 3\hat{x}-\hat{y}+2\hat{z}.ds\hat{z} = 2\left[For\,xy\;plane\,\vec{ds} = ds\hat{z}\right]$$

$$\oiint 2ds = 2\pi r^2 = 2\pi\left[For\,unit\,circle\,r=1\right]$$

Q50. Consider a $p-n$ junction at $T=300\,K$. The saturation current density at reverse bias is $-20\,\mu A/cm^2$. For this device a current density of magnitude $10\,\mu A/cm^2$ is realized with a forward bias voltage, V_F. The same magnitude of current density can also be realized with a reverse bias voltage, V_R. The value of $|V_F/V_R|$ is ______ (Round off to 2 decimal places).

Topic: Mathematical Physics

Sub Topic: Vector Analysis

Ans. : 0.57 to 0.61

Solution: We know that

$$J = J_0\left(e^{\frac{eV}{kT}}-1\right)$$

Under forward bias condition $J_F = J_0\left(e^{\frac{eV}{kT}}-1\right)$

$$10 = 20\left(e^{\frac{eV_F}{kT}}-1\right)\Rightarrow e^{\frac{eV_F}{kT}} = \ln(3/2)\Rightarrow V_F = \frac{kT}{e}\ln(3/2)$$

Under forward bias

$$J_R = J_0 e^{\frac{-eV_R}{kT}}\Rightarrow |V_R| = \frac{kT}{e}\ln(1/2)$$

$$\frac{|V_F|}{|V_R|} = \frac{\ln(3/2)}{\ln(1/2)} = 0.57$$

Section C: Q51. – Q60. Carry TWO marks each.

Q51. Consider the second order ordinary differential equation $y''+4y'+5y=0$. If $y(0)=0$ and $y'(0)=1$, then the value of $y\left(\frac{\pi}{2}\right)$ is _______ (Round off to 3 decimal places).

Topic: Mathematical Physics

Subtopic: Differential Equation

Ans. 0.041 to 0.045

Solution: $y''+4y'+5y=0 \Rightarrow m^2+4m+5=0$

$$m^2 = \frac{-4 \pm \sqrt{16-20}}{2} = -2 \pm i$$

$$y(x) = e^{-2x}\left[c_1 \cos x + c_2 \sin x\right],\ y(0) = c_1 = 0$$

$$y'(x) = -2e^{-2x} c_2 \sin x + e^{-2x} c_2 \cos x$$

$$y'(0) = c_2 = 1,\ y(x) = e^{-2x} \sin x,\ y(\frac{\pi}{2}) = e^{-\pi} = 0.044$$

Q52. A box contains a mixture of two different ideal monoatomic gases, 1 and 2, in equilibrium at temperature T. Both gases are present in equal proportions. The atomic mass for gas 1 is m, while the same for gas 2 is $2m$. If the rms speed of a gas molecule selected at random is $v_{rms} = x\sqrt{\frac{k_B T}{m}}$, then x is _______ (Round off to 2 decimal places).

Topic: Kinetic Theory and Thermodynamics

Sub Topic: Kinetic Theory of Gases

Ans. 1.50 to 1.50 or 1.57 to 1.59

Solution: $v_{rms1} = \sqrt{\frac{3k_B T}{m}}$, $v_{rms2} = \sqrt{\frac{3k_B T}{2m}}$

Resultant rms speed $v_{rms} = \frac{v_{rms1} + v_{rms2}}{2} = \frac{\sqrt{\frac{3k_B T}{m}} + \sqrt{\frac{3k_B T}{2m}}}{2} = 1.50\sqrt{\frac{k_B T}{m}}$

Q53. A hot body with constant heat capacity $800 J/k$ at temperature $925 K$ is dropped gently into a vessel containing $1 kg$ of water at temperature $300 K$ and the combined system is allowed to reach equilibrium. The change in the total entropy ΔS is _____ J/K (Round off to 1 decimal place).

[Take the specific heat capacity of water to be $4200 J/kg\,K$. Neglect any loss of heat to the vessel and air and change in the volume of water.]

Topic: Kinetic Theory and Thermodynamics

Sub Topic: Second Law of Thermodynamics

Ans. 537.5 to 537.7 or 549.8 to 550.2

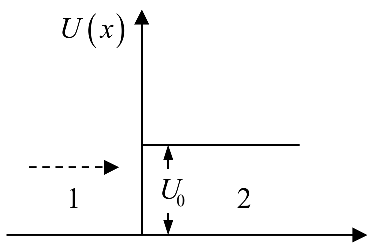
Solution: Heat lost = Heat gained

$800(925 - T_f) = 4200\left(T_f - 300\right)$ where $T_f = 400 K$

Now the change in entropy can be written as

$$\Delta s = 1\times 4200\ln\left(\frac{400}{300}\right) + 1\times 800\ln\left(\frac{400}{925}\right) = 537.5$$

Q54. Consider an electron with mass m and energy E moving along the x-axis towards a finite step potential of height U_0 as shown in the figure. In region $1(x<0)$, the momentum of the electron is $p_1 = \sqrt{2mE}$. The reflection coefficient at the barrier is given by $R = \left(\frac{p_1 - p_2}{p_1 + p_2}\right)^2$, where p_2 is the momentum in region 2. If, in the limit $E >> U_0, R \approx \frac{U_0^2}{nE^2}$, then the integer n is______



Topic: Modern Physics

Subtopic: Quantum Mechanics: Step Potential

Ans. 16 to 16

Solution: $R = \left(\frac{p_1 - p_2}{p_1 + p_2}\right)^2 = \left(\frac{\sqrt{2mE} - \sqrt{2m(E-U_0)}}{\sqrt{2mE} + \sqrt{2m(E-U_0)}}\right)^2 = \left(\frac{\sqrt{2mE} - \sqrt{2mE}\left(1 - \frac{U_0}{E}\right)^{1/2}}{\sqrt{2mE} + \sqrt{2m(E-U_0)}}\right)^2$

$$= \frac{2mE}{2mE}\left(\frac{1 - 1 - \frac{U_0}{2E}}{2}\right)^2 = \frac{1}{16}\left(\frac{U_0}{E}\right)^2 \qquad n = 16$$

Q55. A current density for a fluid flow is given by,

$$\vec{J}(x,y,z,t) = \frac{8e^t}{(1+x^2+y^2+z^2)}\hat{x}$$

At time $t=0$, the mass density $\rho(x,y,z,0)=1$.

Using the equation of continuity, $\rho(1,1,1,1)$ is found to be ______(Round off to 2 decimal places).

Topic: Electricity and Magnetism

Sub Topic: Continuity Equation

Ans. 2.70 to 2.74

Solution: From equation of Continuity

$$\nabla . j+\frac{d\rho}{dt}=0$$

$$\frac{-16xe^t}{\left(1+x^2+y^2+z^2\right)^2}+\frac{d\rho}{dt}=0$$

$$\rho\left(x,y,z,t\right)=\frac{16xe^t}{\left(1+x^2+y^2+z^2\right)^2}+c_1$$

$$\rho\left(x,y,z,0\right)=\frac{16x}{\left(1+x^2+y^2+z^2\right)^2}+c_1=1\Rightarrow c_1=1-\frac{16x}{\left(1+x^2+y^2+z^2\right)^2}$$

$$\rho\left(x,y,z,t\right)=\frac{16xe^t}{\left(1+x^2+y^2+z^2\right)^2}+1-\frac{16x}{\left(1+x^2+y^2+z^2\right)^2}$$

$$\rho\left(1,1,1,1\right)=\frac{16e^1}{16}+1-\frac{16}{16}=e=2.72$$

Q56. The work done in moving $-5\mu C$ charge in an electric field $\vec{E}=\left(8r\sin\theta\hat{r}+4r\cos\theta\hat{\theta}\right)V/m$, from a point $A\left(r,\theta\right)=\left(10,\frac{\pi}{6}\right)$ to a point $B\left(r,\theta\right)=\left(10,\frac{\pi}{2}\right)$, is _______ mJ.

Topic: Electricity and Magnetism

Sub Topic: Electrostatics

Ans. -1 or 1

Solution: The work done can be written as follows

$$W=q\Delta V=-q\int_{10,\pi/6}^{10,\pi/2}\vec{E}.\vec{dl}=q\int_{10,\pi/6}^{10,\pi/2}\left(8r\sin(\theta)\hat{r}+4r\cos(\theta)\hat{\theta}\right).\left(dr\hat{r}+rd\theta\hat{\theta}\right)$$

$$=5\times10^{-6}\int_{10,\pi/6}^{10,\pi/2}\left(8r\sin(\theta)dr+4r\cos(\theta)rd\theta\right)=5\times10^{-6}\left[\left(8r^2\sin(\theta)\right)\right]\Big|_{10,\pi/6}^{10,\pi/2}$$

$$=5\times10^{-6}\times8\times100\times1/2=2\,mJ$$

Q57. A pipe of $1m$ length is closed at one end. The air column in the pipe resonates at its fundamental frequency of $400\,Hz$. The number of nodes in the sound wave formed in the pie is ______. [Speed of sound $=320\,m/s$]

Topic: Waves and Optics

Sub Topic: Sound Wave

Ans. 5

Solution: $L=\frac{\lambda}{4}\Rightarrow\lambda=4L\Rightarrow\text{General expression }\lambda=\frac{4L}{n}=\frac{4}{n}\left[\text{since}, L=1\right]$

$$v=f\lambda$$

$$320=400\left(\frac{4}{n}\right)\Rightarrow n=5$$

Q58. The critical angle of a crystal is 30^0. Its Brewster angle is ______degrees (Round off to the nearest integer).

Topic: Waves and Optics

Sub Topic: Ray Optics

Ans. 27 to 27 or 63 to 63

Solution: For critical angle we can write

$$\sin(\theta_c)=\frac{n_1}{n_2}\Rightarrow\frac{n_1}{n_2}=Sin(30)=1/2$$

$$\tan(\theta_B)=\frac{n_2}{n_1}=2,\theta_B=\tan^{-1}(2)=63^0$$

Or

$$\sin(\theta_c)=\frac{n_2}{n_1}\Rightarrow\frac{n_2}{n_1}=\sin(30)=1/2$$

From the concept of Brewster angle

$$\tan(\theta_B)=\frac{n_2}{n_1}=1/2,\theta_B=\tan^{-1}(1/2)=27^0$$

Q59. In an LCR series circuit, a non-inductive resistor of 150Ω, a coil of $0.2H$ inductance and negligible resistance, and a $30\mu F$ capacitor are connected across an ac power source of $220V, 50Hz$. The power loss across the resistor is _____W. (Round off to 2 decimal places).

Topic: Electricity and Magnetism

Sub Topic: LCR Circuit

Ans. 297 to 299

Solution: $f=50Hz, L=1mH, R=150\,\Omega,\ C=50\,\mu F$

$$w=2\pi f=100\pi Hz, X_L=wL=100\pi Hz\times 1mH=0.314\Omega, X_L=\frac{1}{wC}=63.69\Omega$$

$$Z=\sqrt{R^2+(X_L-X_C)^2}=162.84\Omega,$$

$I = \frac{V}{Z} = \frac{220}{162.84} = 1.35\,A$

Power loss

$P = I^2 Z = 1.82 \times 162.84\,W = 297\,W$

Q60. A charge q is uniformly distributed over the volume of a dielectric sphere of radius a. If the dielectric constant $\in_r = 2$, then the ratio of the electrostatic energy stored inside the sphere to that stored outside is ______ (Round off to 1 decimal place).

Topic: Electricity and Magnetism

Sub Topic: Dielectrics

Ans. 0.1 to 0.1

Solution: $E_{inside} = \frac{qr}{4\pi\varepsilon R^3}$, $r < R$, $E_{outside} = \frac{q}{4\pi\varepsilon_0 R^2}$, $r > R$

Energy stored Inside

$$U_{inside} = \frac{\varepsilon}{2}\int_0^R r^2 E^2{}_{inside} \sin(\theta) dr d\theta d\phi = \frac{q^2}{80\pi\varepsilon_0 R}[\theta = 0 \to \pi, \phi = 0 \to 2\pi]$$

Energy stored outside

$$U_{outside} = \frac{\varepsilon_0}{2}\int E^2{}_{outside} r^2 \sin(\theta) dr d\theta d\phi = \frac{\varepsilon_0}{2}\int_R^\infty r^2 \left(\frac{q}{4\pi\varepsilon_0 r^2}\right)^2 \sin(\theta) dr d\theta d\phi = \frac{q^2}{8\pi\varepsilon_0 R}$$

$$\frac{U_{inside}}{U_{outside}} = 1/10 = 0.1$$