Chapter 2

Time Independent Schrodinger Equation

Section 2.1: Stationary State

The Schrodinger wave equation is given by $i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V\psi(x)$

$V=V(x,t)$ but in this chapter $V=V(x)$

$\Psi(x,t)=\psi(x)\phi(t)$

Where $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V(x)\psi=E\psi$ this second-order differential equation is identified as the Schrödinger wave equation.

And $\langle Q(x,p)\rangle=\int\psi^*Q\left(x,\frac{\hbar}{i}\frac{d}{dx}\right)\psi\,dx$ $i\hbar\frac{1}{\phi}\frac{d\phi}{dt}=E\Rightarrow\phi(t)=\exp(iEt/\hbar)$

The property of $\Psi(x,t)=\psi(x)\phi(t)=\psi(x)\exp-\frac{iEt}{\hbar}$

1. They are in a stationary state which means $|\Psi(x,t)|^2=|\Psi(x,0)|^2=|\psi(x)|^2$

   If $Q(x,p)$ is any dynamic variable then $\langle Q(x,p)\rangle=\int\psi^*Q\left(x,\frac{\hbar}{i}\frac{d}{dx}\right)\psi\,dx$

2. They are states of definite total energy in classical mechanics the total energy is (kinetic plus potential) is called **Hamiltonian.**

$$H\psi=E\psi$$

Average value of $\langle H\rangle=\int\psi^*H\psi\,dx=E\int|\psi|^2dx=E,\langle H^2\rangle=\int\psi^*H^2\psi\,dx=E^2\int|\psi|^2dx=E^2$

The variance of $\sigma_H^2=\langle H^2\rangle-\langle H\rangle^2=0$

3. The general solution is a linear combination of separable combinations.

The solution is $(\psi_1(x),\psi_2(x),\psi_3(x)\psi_4(x)....)$ with the corresponding solution $(E_1,E_2,E_3...)$

$$\psi(x,t)=\sum_n c_n\psi_n(x)\exp\left(-\frac{iE_nt}{\hbar}\right)=\sum_n c_n\psi_n(x,t)$$