# Differential Equation

## 1. Ordinary Differential Equations (ODEs)

**Prerequisite:** Integral calculus

**Model:** If we want to solve a physics problem, we first must formulate the problem as a mathematical expression in terms of variables, functions, derivatives, and equations. Such an expression is known as a **mathematical model** of the given problem.

Now many physical concepts, such as velocity and acceleration, are derivatives. Hence a model is mostly an equation containing derivatives of an unknown function. Such a model is called a **differential equation**.

Physical System ⟹ Mathematical Model ⟹ Mathematical Solution ⟹ Physical Interpretation

Some physical examples that can be modelled to a ODE:

(a) Falling stone (b) Parachutist (c) Out flowing water

(a) Vibrating mass on a spring

(b) Beats of a vibrating system

(c) Current $I$ in an LCR circuit

(d) Deformation of a beam

(e) Pendulum

(f) Lotka–Volterra predator–prey model

**Ordinary differential equation (ODE): One independent variable**

An ODE is an equation that contains one or severalderivatives of an unknown function. The equation may also contain $y$ itself, knownfunction of *x* (or *t*), and constants. For example,

$$y' = \sin x$$

$$y'' + 10y = e^{-2x}$$

$$y' y''' - \frac{5}{2} y''^{2} = 0$$

**Partial Differential Equation (PDE): More than one independent variable**

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \text{ (Two independent variables x and y)}$$

**Explicit form**

$$F(x, y, y') = 0$$

**Example:** $x^{-3} y' - 4y^2 = 0$

**Implicit form**

$$y' = f(x, y)$$

$$y' = 4x^3 y^2$$

**What is a Solution for ODE?**

A function $y = f(x)$ is called a solution of a given ODE on some open interval $a < x < b$

(a) If $h(x)$ is defined and differentiable throughout the interval

(b) The equation becomes an identity if $y$ and $y'$ are replaced with $h$ and $h'$, respectively.

(c) The curve (the graph) of $h$ is called a solution curve.

Here, open interval $a < x < b$ means that the endpoints a and b are notregarded as points belonging to the interval. Also, $a < x < b$ includes infinite intervals $-\infty < x < b$, $a < x < \infty$, $-\infty < x < \infty$ (the real line) as special cases