Laplace Transformation

2. Inverse Laplace Transform:

Find the inverse Laplace transform of the following functions:

(a) $F(s)=\dfrac{5s+1}{s^2-25}$

Solution: $L^{-1}\{F(s)\}=5L^{-1}\left\{\dfrac{s}{s^2-25}\right\}+\dfrac{1}{5}L^{-1}\left(\dfrac{5}{s^2-25}\right)$

$$=5\cosh 5t+\frac{1}{5}\sinh 5t$$

(b) $F(s)=\dfrac{s}{L^2s^2+n^2\pi^2}=\dfrac{1}{L^2}\dfrac{s}{s^2+\left(\dfrac{n\pi}{L}\right)^2}$

Solution: $L^{-1}F(s)=\dfrac{1}{L^2}L^{-1}\left(\dfrac{s}{s^2+\left(\dfrac{n\pi}{L}\right)^2}\right)=\dfrac{1}{L^2}\cos\dfrac{n\pi t}{L}$

(c) $F(s)=\dfrac{1}{\left(s+\sqrt{2}\right)\left(s-\sqrt{3}\right)}$

Solution: $F(s)=\dfrac{1}{\left(s+\sqrt{2}\right)\left(s-\sqrt{3}\right)}=\dfrac{A}{\left(s+\sqrt{2}\right)}+\dfrac{B}{\left(s-\sqrt{3}\right)}$

$$A=\frac{-1}{\sqrt{2}+\sqrt{3}}\ \&\ B=\frac{1}{\sqrt{2}+\sqrt{3}}$$

$$F(s)=\frac{-1}{\sqrt{2}+\sqrt{3}}\frac{1}{\left(s+\sqrt{2}\right)}+\frac{1}{\sqrt{2}+\sqrt{3}}\frac{1}{\left(s-\sqrt{3}\right)}$$

$$L^{-1}F(s)=\frac{-1}{\sqrt{2}+\sqrt{3}}L^{-1}\frac{1}{\left(s+\sqrt{2}\right)}+\frac{-1}{\sqrt{2}+\sqrt{3}}L^{-1}\frac{1}{\left(s-\sqrt{3}\right)}$$

$$f(t)=\frac{-1}{\sqrt{2}+\sqrt{3}}e^{-\sqrt{2}t}+\frac{1}{\sqrt{2}+\sqrt{3}}e^{\sqrt{3}t}$$

(d) $F(s)=\dfrac{12}{s^4}-\dfrac{228}{s^6}$

Solution: $F(s)=\dfrac{2\times 3!}{s^{3+1}}-\dfrac{228\times 5!}{s^{5+1}}\times\dfrac{1}{5!}$

$$L^{-1}F(s)=2L^{-1}\left\{\frac{3!}{s^{3+1}}\right\}-\frac{228}{5!}L^{-1}\left\{\frac{5!}{s^{5+1}}\right\}$$

$$f(t)=2t^3-\frac{19}{10}t^5$$

(e) $F(s)=\dfrac{4s+32}{s^2-16}$

Solution: $F(s)=\dfrac{4s+32}{s^2-16}=\dfrac{4s}{s^2-4^2}+\dfrac{32}{s^2-4^2}$

$$L^{-1}F(s)=4L^{-1}\left\{\frac{s}{s^2-4^2}\right\}-8L^{-1}\left\{\frac{4}{s^2-4^2}\right\}$$

$$f(t)=4\cosh 4t+8\sinh 4t$$

(f) $F(s)=\dfrac{1}{(s+a)(s+b)}$

Solution: $F(s)=\dfrac{1}{(s+a)(s+b)}=\dfrac{A}{(s+a)}+\dfrac{B}{(s+b)}$

$$A = \frac{1}{b-a} \,\&\, B = \frac{1}{a-b}$$

$$F(s) = \frac{1}{(b-a)}\frac{1}{(s+a)} + \frac{1}{(a-b)}\frac{1}{(s+b)}$$

$$L^{-1}F(s) = \frac{1}{(b-a)}e^{-at} + \frac{1}{(a-b)}e^{-bt}$$