Differential Equation

6. Coupled Differential Equation

Example: Predator-Prey Model

$x(t)$ denotes the population of the prey, while $y(t)$ denotes the population of the predator. Their interaction can be modelled by,

$$\frac{dx}{dt} = x - \frac{1}{2}xy$$
$$\frac{dy}{dt} = -\frac{3}{4}y + \frac{1}{4}xy$$

and we are interested only in the region $x > 0$ and $y > 0$.

First we find the equilibrium points, $x\left(1 - \frac{1}{2}y\right) = 0$ and $y\left(-\frac{3}{4} + \frac{1}{4}x\right) = 0$. These are at $(x,y) = (0,0)$ and $(x,y) = (3,2)$. We examine each in turn.

Close to $(x,y) = (0,0)$, we line arise the equations on the assumption that $|x| \ll 1$ and $|y| \ll 1$ so that

$$\frac{dx}{dt} = x - \cdots \quad \text{and} \quad \frac{dy}{dt} = -\frac{3}{4}y + \cdots$$

This may be integrated immediately to give

$$x(t) = c_1 e^{t} \quad \text{and} \quad y(t) = c_2 e^{-3t/4}$$