

CSIR NET-JRF, GATE, IIT-JAM, JEST, TIFR and GRE for Physics H.N. 28 A/1, Jia Sarai, Near IIT-Delhi, Hauz Khas, New Delhi-110016 Contact: +91-89207-59559, 8076563184 Website: <u>www.pravegaa.com</u> | Email: <u>pravegaaeducation@gmail.com</u>

Differential Equation

6. Coupled Differential Equation

Example: Predator-Prey Model

x(t) denotes the population of the prey, while y(t) denotes the population of the predator.

Their interaction can be modelled by,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x - \frac{1}{2}xy$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{3}{4}y + \frac{1}{4}xy$$

and we are interested only in the region $x \ge 0$ and $y \ge 0$.

First we find the equilibrium points, $x\left(1-\frac{1}{2}y\right) = 0$ and $y\left(-\frac{3}{4}+\frac{1}{4}x\right) = 0$. These are at (x, y) = (0, 0) and (x, y) = (3, 2). We examine each in turn. Close to (x, y) = (0, 0), we line arise the equations on the assumption that $|x| \ll 1$ and $|y| \ll 1$ so that



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$$\frac{\mathrm{d}x}{\mathrm{d}t} = x - \cdots$$
 and $\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{3}{4}y + \cdots$

This may be integrated immediately to give

$$x(t) = c_1 e^t$$
 and $y(t) = c_2 e^{-3t/4}$