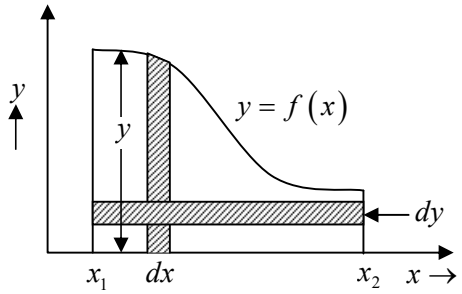
Multiple Variable Calculus

Outline

- Multiple variables in integration
- Jacobean transformation
- Properties of partial derivatives
- Taylor's Series in 1D and 2D
- Limit, Continuity, Derivatives, etc.

Double Integral



Single Integral $\int_{x_1}^{x_2} y\,dx$ **Double Integral** $\int_{y_1}^{y_2}\int_{x_1}^{x_2} dxdy$

$$I_1 = \int_{x_1}^{x_2}\int_{y_1}^{y_2} f(x,y)\,dydx$$

$y_1 = f_1(x)$ x_1 Constant

$y_2 = f_2(x)$ x_2 Constant

$$I_2 = \int_{y_1}^{y_2}\int_{x_1}^{x_2} f(x,y)\,dxdy$$

$x_1 \to f_1(y)$

$x_2 \to f_2(y)$

y_1 and y_2 constant

Note: If x_1, x_2, y_1, y_2 are constant either way will work

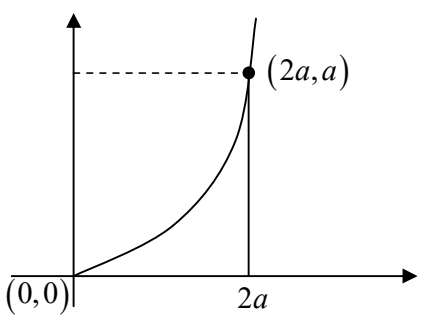
Example: $\int_0^5\int_0^{x^2} x\left(x^2+y^2\right)dydx$

Solution: $= \int_0^5 \left[x^3 y + \frac{xy^3}{3}\right]_0^{x^2} dx = \int_0^5 \left[x^5 + \frac{x^7}{3}\right]dx = \left[\frac{x^6}{6} + \frac{x^8}{24}\right]_0^5$

$$= \frac{5^6}{6} + \frac{5^8}{24} = 1880 \cdot 2$$

Example: Evaluate $\iint_A xydxdy$ where A is the domain bounded by x-axis, absicca $x = 2a$ and the curve $x^2 = 4ay$

Solution:



$$\iint_A xydxdy = \int_0^{2a}\int_0^{x^2/4a} xydydx = \int_0^{2a}\left[\frac{xy^2}{2}\right]_0^{x^2/4a} dx = \int_0^{2a}\frac{x^5}{32a^2}dx$$

$$= \frac{1}{6}\times\frac{1}{32a^2}\left[x^6\right]_0^{2a} = \frac{1}{192a^2}\cdot 2^6\times a^6 = \frac{a^4}{3}$$