Practice Set (Fourier Series)

Q1. A function $f(x)$ is given as

$$1-\frac{x^2}{4}, \quad 0<x<\pi$$

So what will be the value of n such that it will give positive a_n

(a) Odd numbers (b) Even numbers (c) $n=0$ (d) Both (a) & (b)

Q2. $f(x)=2a-|x|$, $0<x<2L$

$=0$, *otherwise*

where $|x|$ is the mod of x which always gives positive value, what will be $|b_2|^2$?

(a) $\frac{4L^2}{\pi^2}$ (b) $\frac{L^2}{\pi^2}$ (c) $\frac{2L^2}{\pi^2}$ (d) 0

Q3. A function in the range $0<x<\pi$ is $f(x)=\frac{1}{4}\pi x$

Choose the correct fourier series expansion from the given options:

(a) $\frac{\pi^2}{4}-\cos x+\frac{\pi}{2}\sin x-\frac{\pi}{4}\sin 2x-\frac{1}{9}\cos 3x+....$

(b) $\frac{\pi^2}{4}-\cos x-\frac{\pi}{2}\sin x+\frac{\pi}{4}\sin 2x+\frac{1}{9}\cos 3x-....$

(c) $\frac{\pi^2}{8}-\cos x+\frac{\pi}{2}\sin x-\frac{\pi}{4}\sin 2x-\frac{1}{9}\cos 3x+....$

(d) $\frac{\pi^2}{8}-\cos x-\frac{\pi}{2}\sin x+\frac{\pi}{4}\sin 2x+\frac{1}{9}\cos 3x-...$

Q4. If $f(x)$ is periodic function defined over a period $(0,2\pi)$, $f(x)=\frac{3x^2-6x\pi+2\pi^2}{12}$, what will be a_0?

(a) 2 (b) 1 (c) -1 d) 0

Q5. A function $f(x)=\cos^3 x+\sin^3 x$, which is periodic in the range $0<x<2\pi$.

If $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n\cos nx+b_n\sin nx\right]$

$$f(x)=\frac{a_0}{2}+a_1\cos x+b_1\sin x+a_2\cos 2x+b_2\sin 2x+a_3\cos 3x+b_3\sin 3x+...$$

What will be the value of a_1 and b_3 respectively?

(a) $\frac{3}{4},\frac{1}{4}$ (b) $\frac{3}{4},-\frac{1}{4}$ (c) $-\frac{1}{4},\frac{3}{4}$ (d) $-\frac{3}{4},\frac{1}{4}$

Q6. Find generalized form of a_n for the function $f(x)=x\sin x$, $0<x<2\pi$

(a) 0 (b) $-\dfrac{2}{1-n^2}$ (c) $\dfrac{-2}{n^2-1}$ (d) $\dfrac{2}{n^2-1}$

Q7. A function in the interval $-L<x<L$

$$f(x)=\frac{1}{4}(\pi+x)^2-\frac{\pi^2}{12}, \quad -L\le x\le 0$$
$$=\frac{1}{4}(\pi-x)^2-\frac{\pi^2}{12}, \quad 0\le x\le L$$

From the following options choose the correct answer matching with value of a_0. for $L=0$.

(a) $\dfrac{\pi^2}{6}$ (b) $-\dfrac{\pi^2}{6}$ (c) $\dfrac{\pi^2}{3}$ (d) $-\dfrac{\pi^2}{3}$

Q8. Given that $f(x)=x+x^2$ for $0<x<\pi$, find the fourier expression of $f(x)$

(a) $\dfrac{\pi}{2}+\dfrac{\pi^2}{3}-\left(\dfrac{4}{\pi}+4\right)\cos x-\left(2+2\pi-\dfrac{8}{\pi}\right)\sin x+...$

(b) $\dfrac{\pi}{2}+\dfrac{\pi^2}{3}+\left(\dfrac{4}{\pi}+4\right)\cos x-\left(2+2\pi-\dfrac{8}{\pi}\right)\sin x+...$

(c) $\dfrac{\pi}{2}+\dfrac{\pi^2}{3}-\left(\dfrac{4}{\pi}+4\right)\cos x+\left(2+2\pi-\dfrac{8}{\pi}\right)\sin x+...$

(d) $\dfrac{\pi}{2}+\dfrac{\pi^2}{3}+\left(\dfrac{4}{\pi}+4\right)\cos x+\left(2+2\pi-\dfrac{8}{\pi}\right)\sin x+...$

Q9. Find the generalized form of b_n for a function $f(x)=4x(1-x)$ over a period $0\le x\le 1$

(a) $\dfrac{16}{n^3\pi^3}$ (b) $\dfrac{32}{n^3\pi^3}$, for odd n (c) $\dfrac{16}{n^2\pi^2}$ (d) $\dfrac{32}{n^2\pi^2}$, for even n

Q10. A function $f(x)=x^2\sin\pi x$ which exist in the range $0<x<2$. using fourier series what will be correct value of a_0?

(a) $-\dfrac{2}{\pi}$ (b) $-\dfrac{4}{\pi}$ (c) $\dfrac{4}{\pi}$ (d) $\dfrac{2}{\pi}$

Q11. An exponential function of $x, e^{-a|x|}$ is given over the period $0<x<2$.
What will be the corresponding Fourier series expansion for the given function?

(a) $\dfrac{1}{2a}(1-e^{-2a})+\dfrac{8ae^{-2a}}{4a^2+\pi^2}\cos x+\dfrac{4\pi e^{-2a}}{4a^2+\pi^2}\sin x$

(b) $\dfrac{1}{2a}(1-e^{-2a})+\dfrac{8ae^{-2a}}{4a^2+\pi^2}\cos x-\dfrac{4\pi e^{-2a}}{4a^2+\pi^2}\sin x$

(c) $\dfrac{1}{2a}(1+e^{-2a})+\dfrac{8ae^{-2a}}{4a^2+\pi^2}\cos x+\dfrac{4\pi e^{-2a}}{4a^2+\pi^2}\sin x$

(d) $\dfrac{1}{2a}(1+e^{-2a})+\dfrac{8ae^{-2a}}{4a^2+\pi^2}\cos x-\dfrac{4\pi e^{-2a}}{4a^2+\pi^2}\sin x$

Q12. For the function

$$f(x) = x - x^2, \qquad 0 < x < 4$$

The value of a_0 and b_2 in the fourier series expansion is respectively

(a) $\frac{20}{3}, \frac{6}{\pi}$ (b) $-\frac{20}{3}, \frac{6}{\pi}$ (c) $\frac{20}{3}, -\frac{6}{\pi}$ (d) $-\frac{20}{3}, -\frac{6}{\pi}$

Q13. A periodic function $f(x)$ of period 2π is given by $f(x) = x|x|$ for $-\pi < x < \pi$ then find the generalized from of b_n.

(a) $\frac{2}{\pi}\left[(-1)^n\left[\frac{1}{n^3} - \frac{\pi^2}{n}\right] - \frac{2}{n^3}\right]$ (b) $\frac{2}{\pi}\left[(-1)^n\left[\frac{\pi^2}{n} - \frac{1}{n^3}\right] - \frac{2}{n^3}\right]$

(c) $\frac{4}{\pi}\left[(-1)^n\left[\frac{1}{n^3} - \frac{\pi^2}{n}\right] - \frac{2}{n^3}\right]$ (c) $\frac{4}{\pi}\left[(-1)^n\left[\frac{\pi^2}{n} - \frac{1}{n^3}\right] - \frac{2}{n^3}\right]$

Q14. A periodic function in the range $-2 < x < 2$ is defined as

$$f(x) = \begin{cases} 0 & , \quad -2 < x < -1 \\ k & , \quad -1 < x < 1 \\ 0 & , \quad 1 < x < 2 \end{cases}$$

The value of a_1 will be

(a) $\frac{4k}{\pi}$ (b) $\frac{4\pi}{k}$ (c) $\frac{2k}{\pi}$ (d) $\frac{2\pi}{k}$

Q15. A sinusoidal voltage $E\sin 2\omega t$, where t is time and ω is frequency, is passed through a half wave rectifier that clips the negative portion of the wave , find the a_0 value.

$$u(t) = \begin{cases} 0, & -L < t < 0 \\ E\sin 2\omega t, & 0 < t < L \end{cases}$$

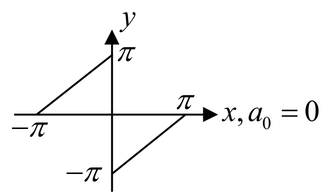
(a) $\frac{2E}{\omega L}[1 - \cos 2\omega L]$ (b) $\frac{E}{2\omega L}[1 - \cos 2\omega L]$

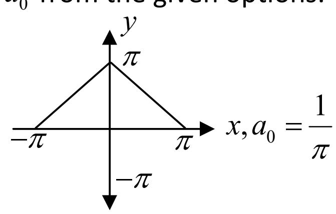
(c) $\frac{2E}{\omega L}[\cos 2\omega L - 1]$ (d) $\frac{E}{2\omega L}[\cos 2\omega L - 1]$

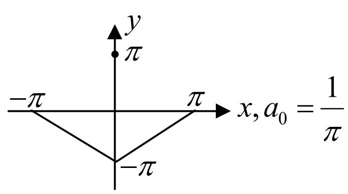
Q16. For the following function

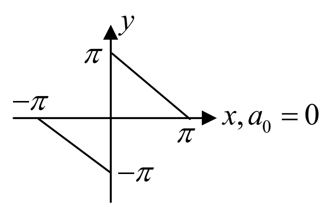
$$f(x, y) = \begin{cases} x + y = \pi, & \text{for} \quad 0 < x < \pi \\ x + y = -\pi, & \text{for} \quad -\pi < x < 0 \end{cases}$$

Choose the correct schematic curve as well as value of a_0 from the given options.

(a)


(b)


(c)


(d)


Q17. The function $f(x)=\sinh x$, which exist in the range $-\pi \le x \le \pi$. using fourier series the value of b_n for odd value of n is $A\times\sinh\,\pi$. What will be value of $\frac{A}{2}$.

(a) $\frac{2n}{1+n^2}$ (b) $\frac{-2n}{1+n^2}$ (c) $\frac{n}{1+n^2}$ (d) $\frac{-n}{1+n^2}$

Q18. A periodic function $f(x)$ of period π is defined in the interval $(-\pi/2 < x < \pi/2)$ as-

$$f(x)=\begin{cases}-1, & -\pi/2 < x < 0 \\ 1, & 0 < x < \frac{\pi}{2}\end{cases}$$

The appropriate fourier series expansion for $f(x)$ is-

(a) $\frac{4}{\pi}\sin x+\frac{4}{2\pi}\sin 2x+\frac{4}{3\pi}\sin 3x+\ldots$ (b) $\frac{4}{\pi}\sin x+\frac{4}{\pi}\sin 2x+\frac{4}{\pi}\sin 3x+\ldots$

(c) $\frac{4}{\pi}\sin x-\frac{4}{\pi}\sin 2x+\frac{4}{3\pi}\sin 3x+\ldots$ (d) $\frac{4}{\pi}\sin x+\frac{4}{\pi}\sin 2x+\frac{4}{3\pi}\sin 3x+\ldots$

Q19. The coefficient of e^{2ikx} in the fourier expansion of $u(x)=A\sin^2(\alpha x)$ for $K=-\alpha$ is

(a) $\frac{A}{2}$ (b) $\frac{A}{4}$ (c) $-\frac{A}{2}$ (d) $-\frac{A}{4}$

Q20. A function of θ is given as

$$f(\theta)=\begin{cases}\cos\theta, & -\pi \le \theta \le \pi \\ 0\,, & otherwise\end{cases}$$

If the fourier series of this function is written as $f(\theta)=\sum_{n=-\infty}^{\infty} c_n e^{in\theta}$, then value of $|C_1|^3$ will be

(a) $\frac{1}{8\pi^3}$ (b) $\frac{1}{64\pi^3}$ (c) $\frac{1}{512\pi^3}$ (d) 0

Answer

Ans. 1: (a)

Ans. 2: (b)

Ans. 3: (c)

Ans. 4: (d)

Ans. 5: (b)

Ans. 6: (b)

Ans. 7: (c)

Ans. 8: (c)

Ans. 9: (b)

Ans. 10: (b)

Ans. 11: (a)

Ans. 12: (c)

Ans. 13: (d)

Ans. 14: (c)
Ans. 15: (b)

Ans. 16: (b)

Ans. 17: (c)

Ans. 18: (d)

Ans. 19: (b)

Ans. 20: (d)