TIFR - 2022

Q1. The value of the first derivative of the function

$$f(x) = \frac{2}{\sqrt{3}} e^{-\sqrt{3}x^2|x|}$$

at $x = 0$ is $f'(0) =$

(a) undefined (b) 2 (c) 0 (d) $2/\sqrt{3}$

Q2. The binding energy ε_b of a nuclide $^{Z}_{A}X$ with atomic number Z and mass number A is given by the semi-empirical formula

$$\varepsilon_b = a_V A - a_5 A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} + a_A \frac{(A-2Z)^2}{A}$$

where the constant parameters and source of effect for each term are

Volume term	Surface term	Coulomb term	Asymmetry term
a_V	a_S	a_C	a_A
15.56MeV	17.8MeV	0.7MeV	23.29MeV

What is the mass difference between the two-mirror nuclei $^{13}_{6}C$ and $^{13}_{7}N$? It is known that both of them are spherical in shape and have a uniform charge distribution.

(a) 3.40MeV (b) 1.84MeV (c) 2.62MeV (d) 0.78MeV

Q3. Consider an electron with mass m_e, charge $-e$ and spin $1/2$, whose spin angular momentum operator is given by

$$\hat{\bar{S}} = \frac{\hbar}{2}\vec{\sigma}$$

This electron is placed in a magnetic field $\vec{\boldsymbol{B}} = B_x\hat{\mathbf{i}} + B_y\hat{j} + B_z\mathbf{k}$, where all three components (B_x, B_y, B_z) are nonvanishing.

At time $t = 0$, the electron is at rest in the $S_z = \hbar/2$ state. The earliest time when the state of the spin will be orthogonal to the initial state is

(a) Dependent on the direction of the magnetic field $\vec{B}$

(b) $\frac{2m_e}{ge|\vec{B}|}$

(c) infinity, i.e., it will never be orthogonal.

(d) $\frac{4m_e}{ge|\vec{B}|}$

Q4. An atom of mass M at rest emits or absorbs a photon of frequency v and recoils with a momentum p. The frequency of the internal transition of electronic levels is v_0 without accounting for recoil. Assuming the process is nonrelativistic, the fractional differences between the photon frequency for emission and absorption $(v - v_0)/v$, respectively, are given by

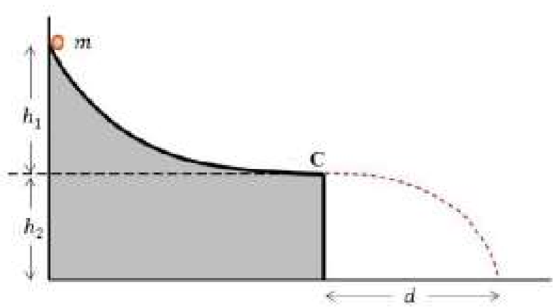
(a) $-\frac{hv}{2Mc^2}$ (emission), $+\frac{hv}{2Mc^2}$ (absorption)

(b) $-\frac{2hv_0}{Mc^2}$ (emission), $+\frac{2hv_0}{Mc^2}$ (absorption)

(c) $+\frac{2hv}{Mc^2}$ (emission), $-\frac{2hv}{Mc^2}$ (absorption)

(d) $+\frac{hv_0}{2Mc^2}$ (emission), $-\frac{hv_0}{2Mc^2}$ (absorption)

Q5. A small body of mass m is released from rest at the top of a frictionless curved surface as shown in the figure, and permitted to slide down the curve. At the endpoint C. the tangent to the curve is horizontal. The mass then falls on the ground at a distance d as shown in the figure below when the experiment is carried out on the surface of the Earth. The heights h_1 and h_2 are also shown in the figure.



Suppose the same experiment is repeated on the surface of the Moon, where the acceleration due to gravity is $g' = g/6$, where g is the value on Earth. The corresponding distance d' at which the mass will fall on the ground in the Moon is

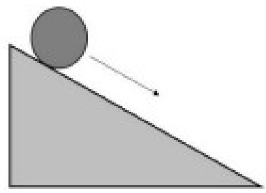
(a) $d\sqrt{h_1/h_2}$

(b) $6d$

(c) dependent on the shape of the curve

(d) d

Q6. A solid cylinder of uniform mass density rolls down a fixed inclined plane without slipping (see figure).



The fraction of the total kinetic energy of the cylinder associated with its rotation about its centre of mass is

(a) $1/3$ (b) ¼ (c) $1/6$ (d) ½

Q7. A particle is executing simple harmonic motion in a straight line. When the distance of the particle from the equilibrium position is x_1 and x_2, the corresponding values of its velocity are v_1 and v_2 respectively. The time period of oscillation is given by

(a) $2\pi\sqrt{\frac{x_2^2-x_1^2}{v_2^2-v_1^2}}$ (b) $2\pi\frac{x_2-x_1}{v_2-v_1}$ (c) $2\pi\frac{x_2-x_1}{v_1-v_2}$ (d) $2\pi\sqrt{\frac{x_2^2-x_1^2}{v_1^2-v_2^2}}$

Q8. In a mercury vapour lamp an electric are passing through mercury vapour produces light. When the lamp is switched on, the arc is struck, and the liquid mercury starts evaporating as the temperature gradually inereases.

In a certain experiment, a Michelson interferometer is set up with a mercury vapour lamp as the light souree, and the lamp is switched on. Which of the following effects will be observed?

(a) Initially, fringes will appear with high contrast but low intensity, which will be reduced in contrast over the period of time as the light intensity builds up.

(b) High contrast fringes will appear as soon as the lamp is switched on and will remain thus so long as the lamp is on.

(c) Initially, no fringes will be observed, but then fringes will emerge with high contrast as the lamp heats up.

(d) No fringes will be observed as the source is incoherent and has many frequencies.

Q9. Consider a symmetric matrix

$$M = \begin{pmatrix} 1/3 & 0 & 2/3 \\ 0 & 1 & 0 \\ 2/3 & 0 & 1/3 \end{pmatrix}$$

An orthogonal matrix O which can diagonalize this matrix by an orthogonal transformation $O^T M O$ is given by $O =$

(a) $\begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{pmatrix}$

(b) $\begin{pmatrix} \sqrt{1/3} & 0 & \sqrt{2/3} \\ 0 & 1 & 0 \\ \sqrt{2/3} & 0 & -\sqrt{1/3} \end{pmatrix}$

(c) $\begin{pmatrix} 1/\sqrt{2} & 0 & i/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -i/\sqrt{2} \end{pmatrix}$

(d) $\begin{pmatrix} \sqrt{2/3} & 0 & \sqrt{1/3} \\ 0 & 1 & 0 \\ \sqrt{1/3} & 0 & -\sqrt{2/3} \end{pmatrix}$

Q10. An electromagnetic wave is described by the following expression

$$\vec{E}(z,t) = E_0 \sin kz \left\{ \hat{\boldsymbol{\imath}} \cos \omega t + \hat{\boldsymbol{\jmath}} \cos\left(\omega t - \frac{\pi}{2}\right) \right\}$$

Which of the following correctly describes this waveform?

(a) A right circular-polarised standing wave along the positive z-axis.

(b) A right circular-polarised travelling wave along the positive z-axis.

(c) A left circular-polarised travelling wave along the positive z-axis.

(d) A left circular-polarised standing wave along the positive z-axis.

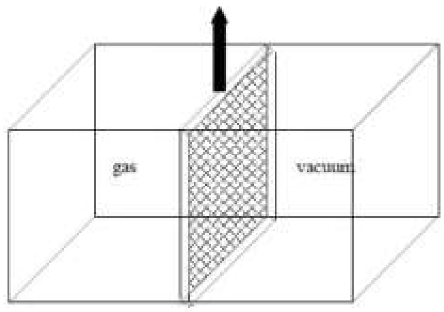
Q11. A faint star is known to emit light of a given frequency at an average rate of 10 photons per minute. An astronomer plans to measure this rate using a photon-counting detector. If she wants to measure the rate to an accuracy of 5%, approximately how long should be the exposure time?

(a) 20 minutes (b) 1 hour (c) 10 minutes (d) 40 minutes

Q12. The minimum energy required to dissociate a hydrogen molecule (H_2) into two atoms is 4.5eV. If the electron affinity of the hydrogen atom is 0.75eV, the minimum energy required to dissociate the hydrogen molecule into H^+ and H^- would be

(a) 14.35eV (b) 17.35eV (c) 5.25eV (d) 18.85eV

Q13. Consider a sealed but thermally conducting container of total volume V, which is in equilibrium with a thermal bath at temperature T. The container is divided into two equal chambers by a thin partition, which is thermally conducting but impermeable to particles. One of the chambers contains an ideal gas, while the other is a vacuum.



If the partition is removed suddenly and the ideal gas is allowed to expand and fill the entire container, then, once equilibrium has been reached, the entropy per molecule will increase by an amount

(a) $\frac{1}{2}k_B\ln 2$ (b) $+k_B\ln 2$ (c) $2k_B$ (d) $-k_B\ln 2$

Q14. A two-level system with zero ground state energy is in equilibrium at a nonzero finite temperature. If α is defined as the ratio

$$\alpha = \frac{\langle E^2\rangle}{\langle E\rangle^2}$$

where $\langle E\rangle$ denotes the mean energy and $\langle E^2\rangle$ denotes the mean squared energy, then

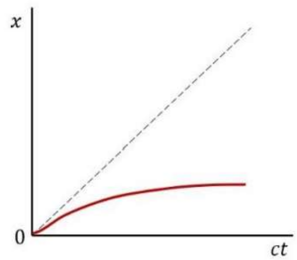
(a) $1 < \alpha \leq 2$ (b) $2 < \alpha < \infty$ (c) $\frac{1}{2} < \alpha < 1$ (d) $0 < \alpha < \frac{1}{2}$

Q15. Five identical bosons are distributed in energy levels E_1 and E_2 with degeneracy of 2 and 3, respectively. Find the number of microstates if there are three bosons in the energy level E_1 and two bosons in the energy level E_2.

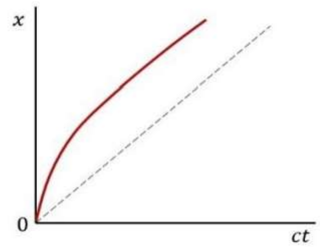
(a) 1024 (b) 120 (c) 24 (d) 6

Q16. A relativistic particle, moving in one dimension x, starts from rest at $x = 0$ and is subjected to a uniform and constant force field along the positive x-direction. If the dashed line corresponds to $x = ct$, which of the following curves (red line) would best represent the position $x(t)$ of the particle?

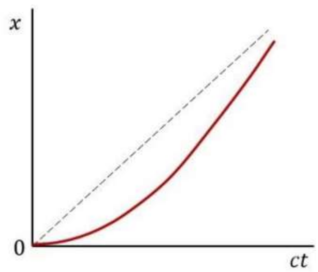
(a)



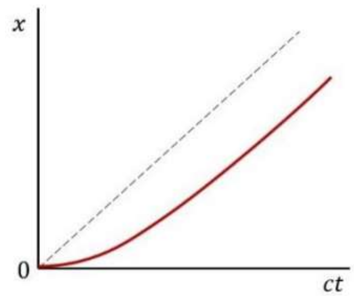
(b)



(c)



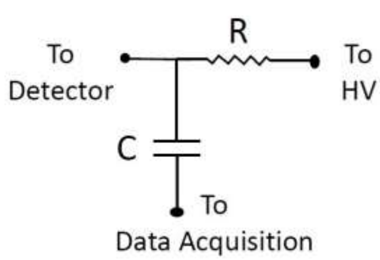
(d)



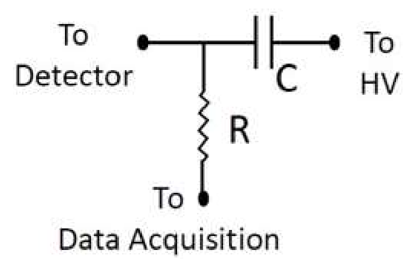
Q17. In an experimental setup, positively charged particles are detected by a detector which requires a negative DC high voltage of -2000 V. Every time a charged particle is detected by the detector, it gives a transient pulse of height 10mV.

The data collection system used for the experiment needs to detect this pulse; however, it cannot operate at -2000 V. Which of the following circuits can be used to connect the data collection system to the detector to obtain these pulses?

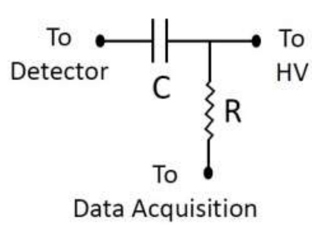
(a)



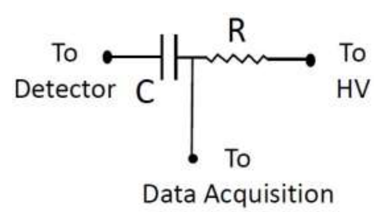
(b)



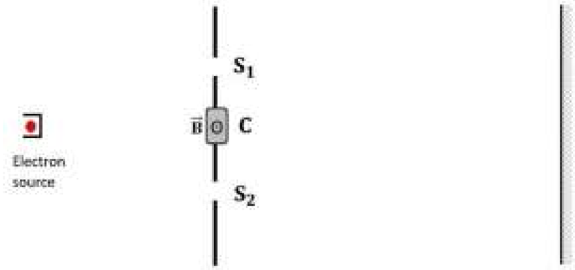
(c)



(d)



Q18. Consider an electron double slit experiment as shown in the figure below, with slits S_1 and S_2.

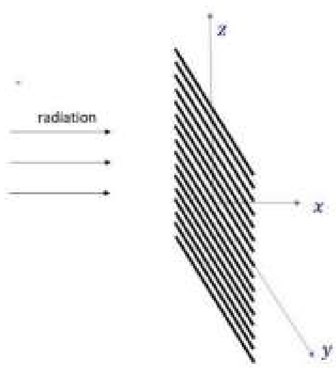


If now, within the shaded region marked C, a constant uniform magnetic field pointing outside the page is turned on, the fringe pattern

(a) will disappear. (b) will become dimmer.

(c) will remain unchanged. (d) will get shifted.

Q19. A beam of unpolarized microwave radiation is incident along the x-axis on a grid of metal wires in the yz-plane with wires running along the y-axis (see figure below).



If the width of each wire and the spacing between the adjacent wires is less than the wavelength of the microwave, the observation would be that

(a) No wave will pass through as the spacing is smaller than the wavelength.

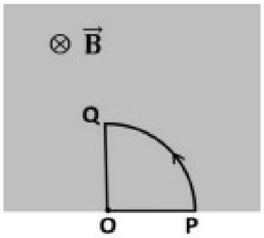
(b) The transmitted wave would be polarized along the z-axis.

(c) The transmitted wave would be unpolarized.

(d) The transmitted wave would be polarized along the y-axis.

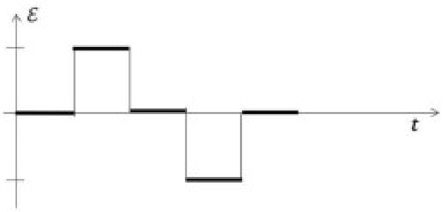
Q20. Consider the following situation. A uniform magnetic field $\vec{B}$ pointing into the plane of the paper is present everywhere inside the rectangular region shown shaded in the adjoining figure. Outside the rectangular region, there is no magnetic field.

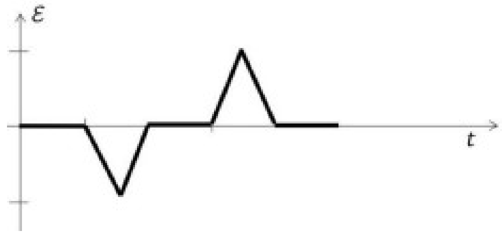
A closed loop of conducting wire is placed inside the rectangular region as shown in the figure at time $t = 0$. The loop is then rotated

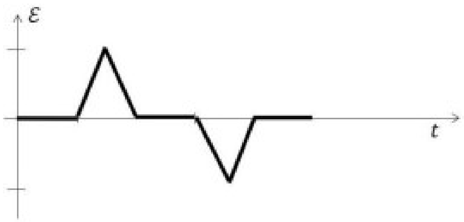


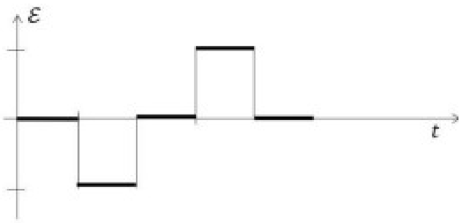
counter clockwise with a uniform angular velocity ω about an axis perpendicular to the paper passing through the point O.

If the direction along PQOP is taken to be positive, then a correct graph for the EMF ε generated in the loop is

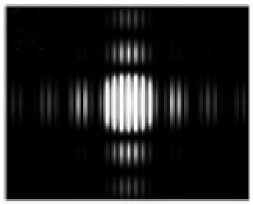
(a) 

(b) 

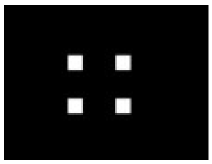
(c) 

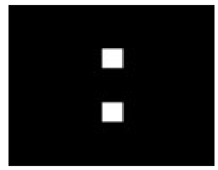
(d) 

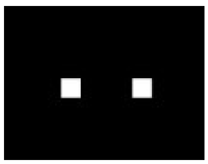
Q21. The following Fraunhofer diffraction pattern was observed in an experiment.

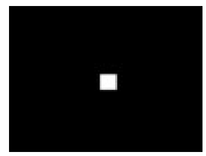


The aperture arrangement that would yield such a fringe pattern is

(a) 

(b) 

(c) 

(d) 

Q22. A surface is given by

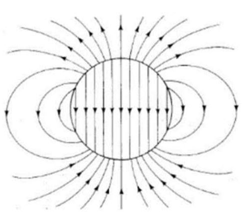
$$4x^2y - 2xy^2 + 3z^3 = 0$$

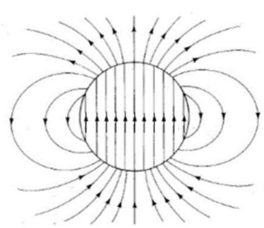
Which of the following is a vector normal to it at the point (2,3,1) ?

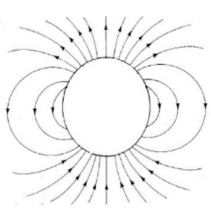
(a) $30\hat{\mathbf{i}} - 8\hat{\mathbf{j}} - 9\hat{\mathbf{k}}$ (b) $15\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 18\hat{\mathbf{k}}$ (c) $30\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$ (d) $30\hat{\mathbf{i}} + 8\hat{\mathbf{j}} - 9\hat{\mathbf{k}}$

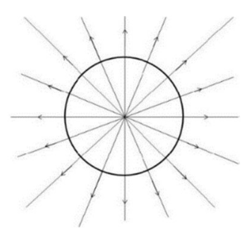
Q23. The electric field lines due to a uniformly polarized dielectric sphere (polarization along the positive Z-axis as shown in the figure)

Will look like

(a) 

(b) 

(c) 

(d) 

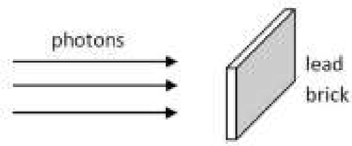
Q24. The pulse train at the output of an XNOR gate with the three inputs

$$A = 00011011$$
$$B = 10100011$$
$$C = 00101110$$

will be

(a) 10010110 (b) 01101001 (c) 10101000 (d) 01010111

Q25. A beam of photons of 1MeV energy each is shot at a 10 mm thick lead brick (see figure).



Given that the density of lead is 11.29 $g^{-cm^{-3}}$, its atomic mass is 207.2a. m. u, and also that the interaction cross-section for these photons with a lead atom is 10^{-2} cm^2, the fraction of the incident photons that will cross the brick without losing any energy is

(a) 33% (b) 670 (c) 28% (d) 72%

Q1. Consider the Hamiltonian for a one-dimensional classical oscillator

$$H = \frac{1}{2m}(p^2 + m^2\omega^2 q^2)$$

A canonical transformation to variables (P, Q) via the generating function

$$F = \frac{m\omega q^2}{2}\cot Q$$

leads to which of the following Hamiltonians in the new coordinates?

(a) $H = \omega P$ (b) $H = P^2 + \omega^2 Q^2$ (c) $H = 2\omega Q$ (d) $H = 2\omega P$

Q2. The binding energy ε_b of a nuclide ${}^{Z}_{A}X$ with atomic number Z and mass number A is given by the semi-empirical formula

$$\varepsilon_b = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} + a_A \frac{(A-2Z)^2}{A}$$

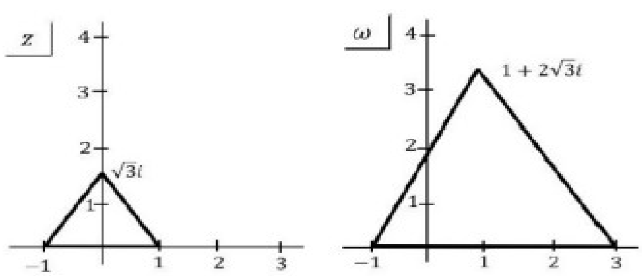
where the constant parameters and source of effect for each term are

Volume term	Surface term	Coulomb term	Asymmetry term
a_V	a_S	a_C	a_A
15.56MeV	17.8MeV	0.7MeV	23.29MeV

For a spherical neutron star consisting of only neutrons and having uniform nuclear density throughout its volume, the Coulomb term is replaced by gravitational energy. What would be the smallest radius of this neutron star?

(a) 2.165 km (b) 4.345 m (c) 4.345 km (d) 10.435 km

Q3. A complex analytic function $\omega = f(z)$ transforms an equilateral triangle in the complex z-plane to another equilateral triangle in the complex ω-plane as shown in the figure.



Which of the options below CANNOT be $f(z)$?

(a) $f(z) = 2e^{2\pi i/3}z + 2 + i\sqrt{3}$ (b) $f(z) = e^{5\pi i/6}z + 2i\sqrt{3}$

(c) $f(z) = 2ie^{5\pi i/6}z + i\sqrt{3}$ (d) $f(z) = 2z + 1$

Q4. For an electromagnetic wave propagating through a rectangular waveguide, the electric and magnetic fields

(a) are never perpendicular to each other

(b) are perpendicular to each other only in the TM mode

(c) are always perpendicular to each other

(d) are perpendicular to each other only in the TE mode

Q5. Consider a particle of mass m in a quartic potential

$$H = \frac{p^2}{2m} + ax^4$$

If we take a variational wavefunction

$$\psi(x, \lambda) = e^{-\lambda x^2}$$

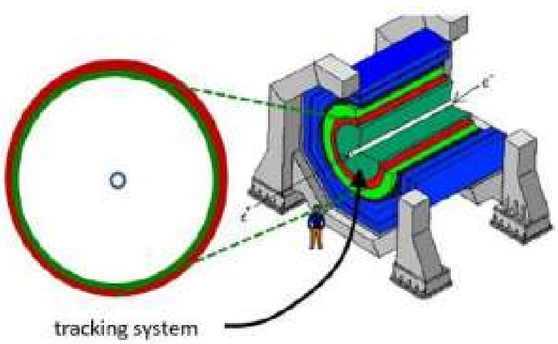
with $\lambda > 0$ and try to estimate the ground state energy, the value of λ should be chosen as [You may use the integral

$$\int_{-\infty}^{+\infty} dx(A + Bx^2 + Cx^4)e^{-\lambda x^2} = A\sqrt{\frac{\pi}{\lambda}} + \frac{B}{2}\sqrt{\frac{\pi}{\lambda^3}} + \frac{3C}{4}\sqrt{\frac{\pi}{\lambda^5}}$$

where A, B, C and $\lambda > 0$ are all constants.]

(a) $\left(\frac{15ma}{8n^2}\right)^{1/3}$ (b) $\left(\frac{5ma}{3\pi^2 h^2}\right)^{1/3}$ (c) $\left(\frac{ma}{2\pi\hbar^2}\right)^{1/3}$ (d) $\left(\frac{3m}{4n^2}\right)^{1/3}$

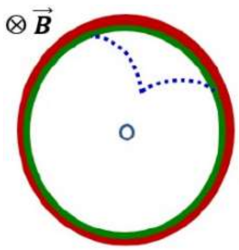
Q6. The figure below shows on the right a sketch of an electron-positron collider experiment where the innermost detector (shaded dark green) is a tracking system which records the tracks of charged particles which pass through it. On the left of the figure, a cross-sectional view of the same tracking system is shown. The narrow (white) pipe in the centre is where electrons and positrons are injected as shown and collide in the centre. (On the left it appears as a small central circle.) Inside the tracking system there is a strong uniform magnetic field collinear with the e^+ direction.

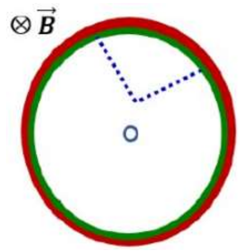


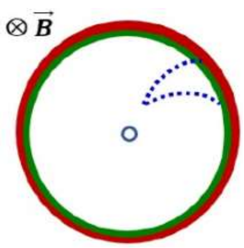
In one of the e^+e^-collisions, a high-energy K_S^0 meson is produced that subsequently decays as follows

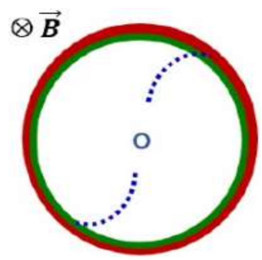
$$K_S^0 \rightarrow \pi^+ + \pi^-$$

A possible representation of the tracks (dotted lines) of the pions $\pi\pm$ in the tracking system would be

(a) 

(b) 

(c) 

(d) 

Q7. In an experiment that measures the angular distribution of the emission of particles. the angular distribution function is defined as

$$f(\theta) = \frac{n(\theta)}{n(\pi/2)}$$

where $n(\theta)$ is the number of particles detected at an angle θ.
If, for a certain sample, we count

$$n(\pi/4) = 16265 \; n(\pi/2) = 8192$$

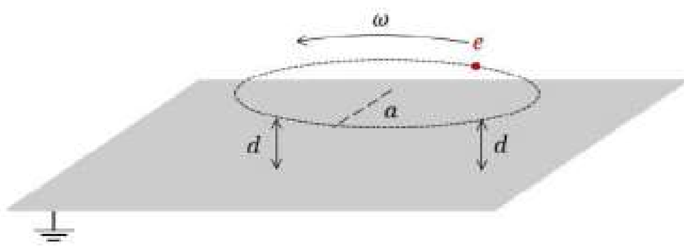
then the uncertainty

$$\left.\frac{\Delta f}{f}\right|_{\theta=\pi/4}$$

in this measurement at $\theta = \pi/4$ would be approximately

(a) 0.707% (b) 0.013% (c) 1.350% (d) 0.018%

Q8. A charge e is moving with an angular frequency ω along a circle of radius a always keeping a small distance $d(d \ll a)$ from a grounded infinite conducting plane.



The leading dependence of the radiated power $P(\omega)$ at a distance $r(r \gg a)$ will be

(a) $P(\omega) \propto \omega^4$ (b) $P(\omega) \propto \omega^6$ (c) $P(\omega) \propto \omega^3$ (d) $P(\omega) \propto \omega^2$

Q9. Consider a fermionic system with a Hamiltonian

$$\hat{H} = \begin{bmatrix} 0 & E_0 & 0 \\ E_0 & 0 & 2E_0 \\ 0 & 2E_0 & 0 \end{bmatrix}$$

Consider the grand canonical ensemble of this system at temperature T and zero chemical potential, where k_B is the Boltzmann constant. The grand canonical partition function of the system is given by

(a) $\operatorname{sech}\left(\sqrt{5}\frac{E_0}{k_B T}\right)$ (b) $4 + \cosh\left(\sqrt{5}\frac{E_0}{k_B T}\right)$(c) $\frac{1}{4+\cos\left(\sqrt{5}\frac{E_0}{k_B T}\right)}$ (d) $\cosh\left(\sqrt{5}\frac{E_0}{k_B T}\right)$

Q10. The angular position of a star is found to change by an amount of 0.2 arc seconds (relative to the very distant background stars) when measured by a telescope on the Earth on two different nights separated by exactly six months. Note that the distance between the Earth and Sun is known to be approximately 1.5×10^{13} cm.

If the energy flux received from the star is $F = 10^{-7}\text{ergs}^{-1}\text{ cm}^{-2}$, what is the approximate value of its luminosity?

(a) 10^{33} erg s $^{-1}$ (b) 10^{31}erg s^{-1} (c) 10^{29}ergs^{-1} (d) 10^{35} erg s $^{-1}$

Q11. A simple pendulum is oscillating freely in the vertical plane. If the string is shortened very slowly to half its length, the angular amplitude $\theta_{\max}$ will change by a factor

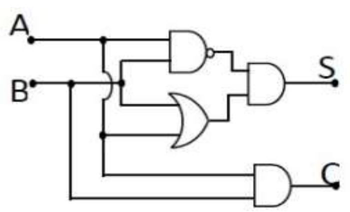
(a) 2 (b) Does not change. (c) $2^{3/4}$ (d) $\sqrt{2}$

Q12. A half-adder circuit is defined as a two-input, two-output logic circuit where the output S gives the sum of inputs up to a single bit, and the output C gives carryover in a single bit.
The expected truth table of the half-adder is given as

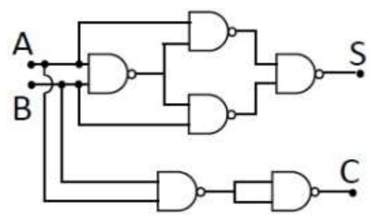
Input		Output	
A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Which of the following circuits does NOT behave like a half adder?

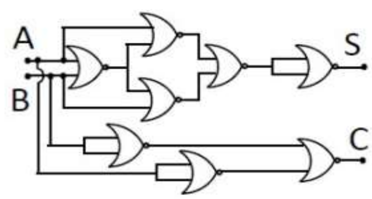
(a)



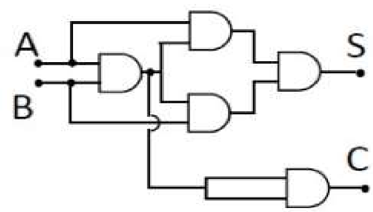
(b)



(c)



(d)



Q13. A lattice in the three-dimensional space has N sites, each occupied by an atom whose magnetic moment is μ. The lattice is in contact with a heat reservoir at a fixed temperature T. The atoms interact with an applied magnetic field

$$\vec{H} = H(\vec{x})\hat{z}$$

Ignoring the interactions between the atoms, the average magnetic susceptibility per lattice site is given by

(a) $\frac{\mu H}{3k_B T}$ (b) $\frac{\mu^2}{9k_B T}$ (c) $\frac{\mu^2}{3k_B T}$ (d) $\frac{\mu}{3k_B T}$

Q14. The energy gap between the $n = 1$ and the $n = 2$ energy levels of a hydrogen atom is denoted E_0 -Now, consider a muonic carbon ion C^{5+}, i.e., a carbon nucleus $({}^{12}_{6}C)$ orbited by a muon $\mu\left(q = -e, M_\mu = 210m_e\right)$. The energy of the photon emitted in the transition of the muon from the $n = 3$ level to the $n = 2$ level of this ion will be approximately

(a) $235E_0$ (b) $7560E_0$ (c) $1400E_0$ (d) $1050E_0$

Q15. Calculate the integral

$$\int_0^\infty \frac{dx}{\sqrt{x}(x^2 + 1)}$$

(a) 2π (b) $\frac{\pi}{\sqrt{2}}$ (c) $\pi\sqrt{2}$ (d) $\frac{\pi}{2}$