

Tensor Analysis

13. Permutation Symbols and Tensors

The symbol e_{pqr} is defined by the following relations:

$$e_{123} = e_{231} = e_{312} = +1, e_{213} = e_{132} = e_{321} = -1, \epsilon_{pqr} = 0$$

if two or more indices are equal.

The symbol e^{pqr} is defined in the same manner. The symbols e_{pqr} and e^{pqr} are called permutation symbols in three-dimensional space.

Further, let us define

$$\epsilon_{pqr} = \frac{1}{\sqrt{g}} e_{pqr}, \epsilon^{pqr} = \sqrt{g} e^{pqr}$$

It can be shown that ϵ_{pqr} and ϵ^{pqr} are respectively, covariant and contravariant tensors, called permutation tensors in three-dimensional space. Generalizations to higher dimensions are possible.