Group Theory

4. The salient features of Cayley table

1. The identity occurs over in each row and in each column furthermore $a_n a_m = e$ as well. Thus each element a has unique inverse a^{-1}.

2. Indeed, each group element occurs precisely once in each row and in each column of Caley table is symmetric, the multiplication operation is commutative.

Example: Multiplication table for group.

Solution: $s = \{1, -1, i, -i\}$

	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

Such table is also called a Latin square, i.e. an $n \times n$ array of n different symbols, each of which appears exactly once in each row and column.

[redacted] Find missing element in the multiplication table of a group of order 3.

$$G = \{e, a, b\}$$

	e	a	b
e	e	a	b
a	a	p	q
b	b	r	s

Solution: p can not be e because that would require (q) to be b. Then two elements in last column would coincide

$\therefore p = b$, $q = e$, similarly we can show $r = e$ and $s = a$

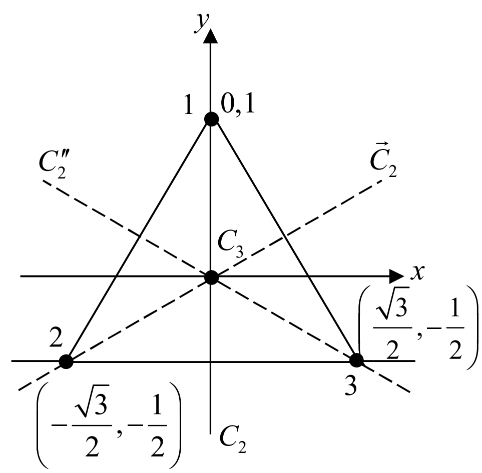
Construct multiplication table for

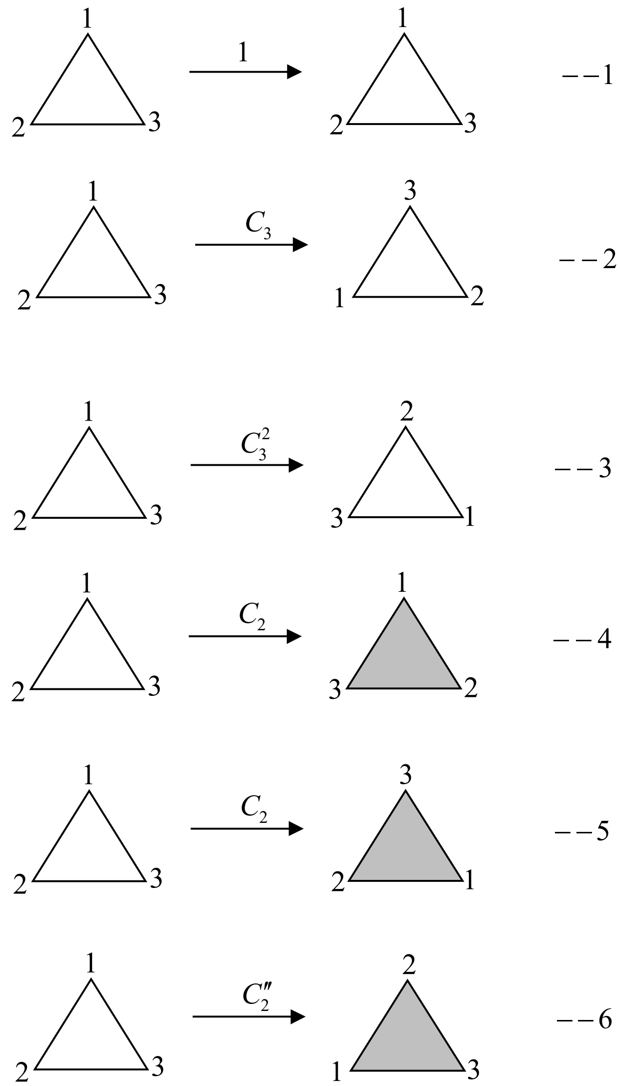
$z_5 = \{0,1,2,3,4\}$ addition modules 5 show that it is abelian

Solution: By inspecting the table we find that table is symmetric which indictie group is abelian.

D_3 symmetry of an equilateral triangles

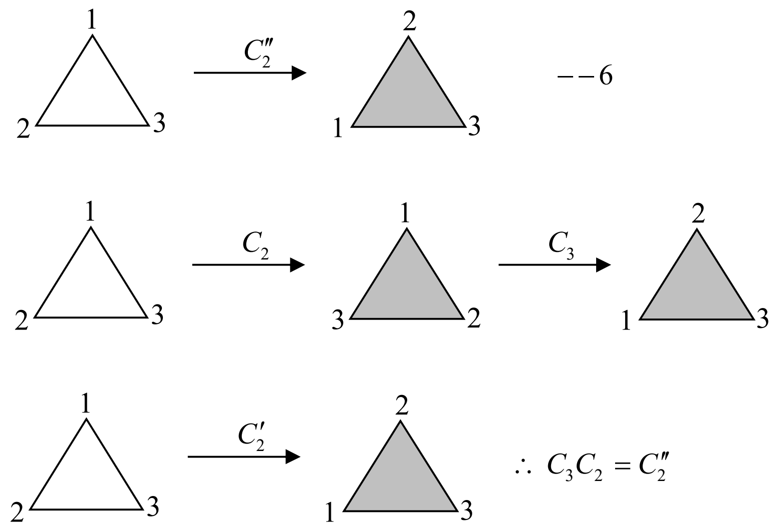
The symmetry operations of an equilateral triangle from a finite group with six elements.





Multiplication Table for Group D_3

$\odot$	I	C_3	C_3^2	C_2	C_2'	C_2''
I	I	C_3	C_3^2	C_2	C_2'	C_2''
C_3	C_3	C_3^2	I	C_2''	C_2	C_2'
C_3^2	C_3^2	I	C_3	C_2'	C_2''	C_2
C_2	C_2	C_2'	C_2''	I	C_3	C_3^2
C_2'	C_2'	C_2''	C_2	C_3^2	I	C_3
C_2''	C_2''	C_2	C_2'	C_3	C_3^2	I



$--6$

$\therefore\ C_3C_2 = C_2''$