

Tensor Analysis

5. Tensors of Rank Greater Than Two, Tensor Fields

Tensors of rank three or more are easily defined. Specifically, for example, A_{k2}^{an} are the components of a mixed tensor of rank 5, contravariant of order 3 and covariant of order 2, where they transform according to the relations

Scalars or Invariants

Suppose ϕ is a function of the coordinates x^k , and let $\bar{\phi}$ denote the functional value under a transformation to a new set of coordinates \bar{x}^k . Then ϕ is called a scalar or invariant with respect to the coordinate transformation if $\phi = \bar{\phi}$. A scalar or invariant is also called a tensor of rank zero.

Tensor Fields

If to each point of a region in N -dimensional space there corresponds a definite tensor, we say that a tensor field has been defined. This is a vector field or a scalar field according as the tensor is of rank one or zero. It should be noted that a tensor or tensor field is not just the set of its

components in one special coordinate system but all the possible sets under any transformation of coordinates.

Symmetric and Skew-Symmetric Tensors

If a tensor is symmetric with respect to any two contravariant and any two covariant indices, it is called Symmetric.

If a tensor is skew-symmetric with respect to any two contravariant and any two covariant indices, it is called skew-symmetric.