Tensor Analysis

6. Fundamental Operations with Tensors

The following operations apply:

1 Addition: The sum of two or more tensors of the same rank and type (i.e. same number of contravariant indices and same number of covariant indices) is also a tensor of the same rank and type. Thus, if A_q^{mp} and B_q^{qw} are tensors, then $C_q^{N\psi} = A_q^{m} + B_q^{rv}$ is also a tensor. The addition of tensors is commutative and associative.

2 Subtraction: The difference of two tensors of the same rank and type is also a tensor of the same rank and type. Thus, if $A_q^{m\varphi}$ and B_q^{mp} are tensors, then $D_q^{Np} = A_q^{mp} - B_q^{mp}$ is also a tensor.

3 Outer Multiplicatio:. The prove of two tensors is a tensor whose rank is the sum of the ranks of the given tensors. The product, which involves ordinary multiplication of the components of the tensor, is called the outer product. For example, $A_q^{\infty r} B_x^{m} = C_{\sigma s}^{\infty m}$ is the

outer product of A_q^{pr} and B_s^{∞}, However, note that not every tensor can be written as a product of two tensors of lower rank. For this reason, division of tensors is not always possible.

4 Contraction: If one contravariant and one covariant index of a tensor are set equal, the result indicates that a summation over the equal indices is to be taken according to the summation convention. This resulting sum is a tensor of rank two less than that of the original tensor. The $A_{\bar{T}}^{\text{nur}} = B_q^{mp}$, a tensor of rank 3. Further, by setting $p = q$, we obtain $B_z^{\text{wp}} = C^{\text{m}}$, a tensor of rank 1.

5 Inner Multiplication: By the process of outer multiplication of two tensors followed by a contraction, we obtain a new tensor called an inner product of the given tensors. The process is called inner multiplication. For example, given the tensors A_q^{wp} and $B_{jr^+}^{r}$ the outer product is $A_{q^{+0}} B_{a'}^{r}$ Letting $q = r$, we obtain the inner product $A_r^{\sigma} B_{\text{sr}}$. Letting $q = r$ and $p = s$, another inner product $A_r^{\pi p} B_{\rho t}^{r}$ is obtained. Inner and outer multiplication of tensors is commutative and associative.

6 Quotient Law: Suppose it is not known whether a quantity X is a tensor or not. If an inner product of X with an arbitrary tensor is itself a tensor, then X is also a tensor. This is called the quotient law.