

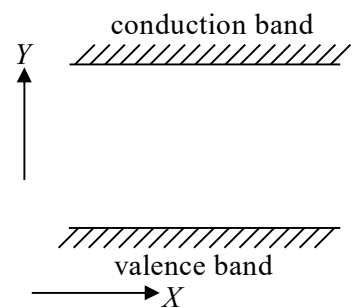
IIT-JAM 2021

SECTION – A

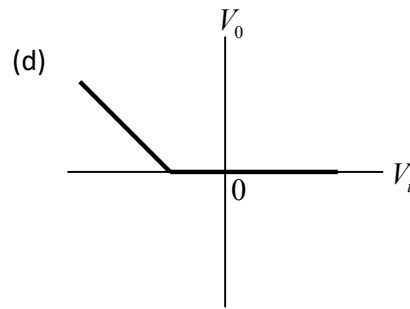
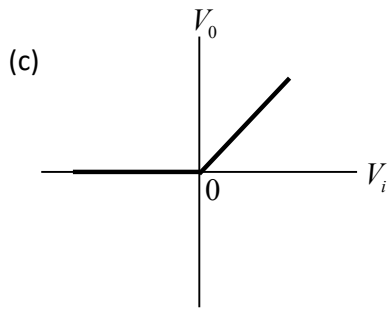
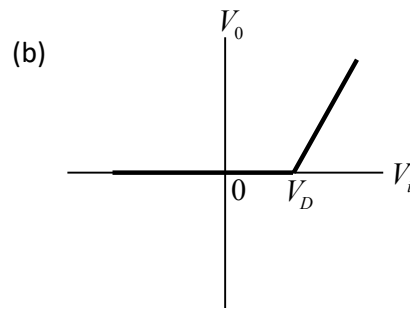
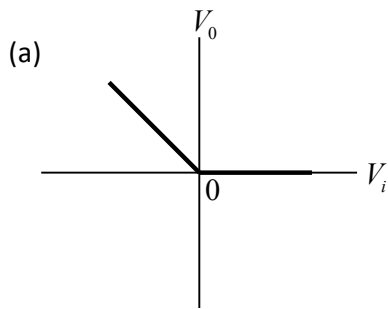
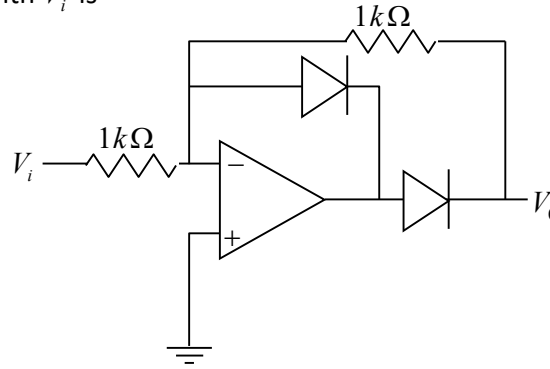
Multiple Choice Questions (MCQ)

Q1 – Q10 carry one mark each

- Q1. The function $e^{\cos x}$ is Taylor expanded about $x = 0$. The coefficient of x^2 is
 (a) $-\frac{1}{2}$ (b) $-\frac{e}{2}$ (c) $\frac{e}{2}$ (d) Zero
- Q2. Let M be a 2×2 matrix. Its trace is 6 and its determinant has value 8. Its eigenvalues are
 (a) 2 and 4 (b) 3 and 3 (c) 2 and 6 (d) -2 and -3
- Q3. A planet is in a highly eccentric orbit about a star. The distance of its closest approach is 300 times smaller than its farthest distance from the star. If the corresponding speeds are v_c and v_f , then $\frac{v_c}{v_f}$ is
 (a) $\frac{1}{300}$ (b) $\frac{1}{\sqrt{300}}$ (c) $\sqrt{300}$ (d) 300
- Q4. An object of density ρ is floating in a liquid with 75% of its volume submerged. The density of the liquid is
 (a) $\frac{4}{3}\rho$ (b) $\frac{3}{2}\rho$ (c) $\frac{8}{5}\rho$ (d) 2ρ
- Q5. An experiment with a Michelson interferometer is performed in vacuum using a laser of wavelength 610 nm . One of the beams of the interferometer passes through a small glass cavity 1.3 cm long. After the cavity is completely filled with a medium of refractive index n , 472 dark fringes are counted to move past a reference line. Given that the speed of light is $3 \times 10^8\text{ m/s}$, the value of n is
 (a) 1.01 (b) 1.04 (c) 1.06 (d) 1.10
- Q6. For a semiconductor material, the conventional flat band energy diagram is shown in the figure. The variables Y, X respectively, are
 (a) Energy, Momentum
 (b) Energy, Distance
 (c) Distance, Energy
 (d) Momentum, Energy



Q7. For the given circuit, V_D is the threshold voltage of the diode. The graph that best depicts variation of V_0 with V_i is



Q8. Arrange the following telescopes, where D is the telescope diameter and λ is the wavelength, in order of decreasing resolving power:

- I. $D = 100\text{ m}$, $\lambda = 21\text{ cm}$
- II. $D = 2\text{ m}$, $\lambda = 500\text{ nm}$
- III. $D = 1\text{ m}$, $\lambda = 100\text{ nm}$
- IV. $D = 2\text{ m}$, $\lambda = 10\text{ mm}$

- (a) III, II, IV, I
- (b) II, III, I, IV
- (c) IV, III, II, I
- (d) III, II, I, IV

Q9. Metallic lithium has *bcc* crystal structure. Each unit cell is a cube of side a . The number of atoms per unit volume is

- (a) $\frac{1}{a^3}$ (b) $\frac{2}{\sqrt{2}a^3}$ (c) $\frac{2}{a^3}$ (d) $\frac{4}{a^3}$

Q10. The moment of inertia of a solid sphere (radius R and mass M) about the axis which is at a distance of $\frac{R}{2}$ from the center is

- (a) $\frac{3}{20}MR^2$ (b) $\frac{1}{2}MR^2$ (c) $\frac{13}{20}MR^2$ (d) $\frac{9}{10}MR^2$

Q11 – Q30. carry two marks each

Q11. Let (x, y) denote the coordinates in a rectangular Cartesian coordinate system C . Let (x', y') denote the coordinates in another coordinate system C' defined by

$$\begin{aligned} x' &= 2x + 3y \\ y' &= -3x + 4y \end{aligned}$$

The area element in C' , is

- (a) $\frac{1}{17}dx'dy'$ (b) $12dx'dy'$ (c) $dx'dy'$ (d) $x'dx'dy'$

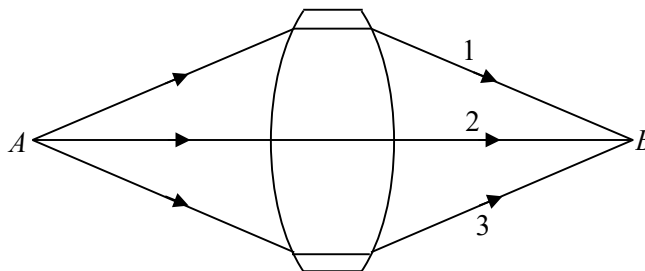
Q12. Three events, $E_1(ct = 0, x = 0)$, $E_2(ct = 0, x = L)$ and $E_3(ct = 0, x = -L)$ occur, as observed in an inertial frame S . Frame S' is moving with a speed v along the positive x -direction with respect to S . In S' , let t'_1, t'_2, t'_3 be the respective times at which E_1, E_2 and E_3 occurred. Then,

- (a) $t'_2 < t'_1 < t'_3$ (b) $t'_1 = t'_2 = t'_3$ (c) $t'_3 < t'_1 < t'_2$ (d) $t'_3 < t'_2 < t'_1$

Q13. The solution $y(x)$ of the differential equation $y \frac{dy}{dx} + 3x = 0, y(1) = 0$, is described by

- (a) an ellipse (b) a circle (c) a parabola (d) a straight line

Q14. In the figure below, point A is the object and point B is the image formed by the lens. Let l_1, l_2 and l_3 denote the optical path lengths of the three rays 1, 2 and 3, respectively. Identify the correct statement.



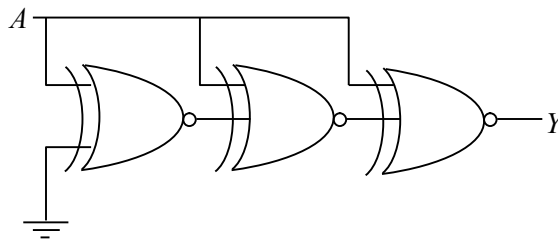
- (a) $l_1 = l_2 = l_3$
 (b) $l_1 > l_2 < l_3$
 (c) $l_1 = l_3 < l_2$
 (d) $l_1 = l_3 > l_2$

Q15. A particle initially at the origin in an inertial frame S , has a constant velocity $V\hat{i}$. Frame S' is rotating about the z -axis with angular velocity ω (anticlockwise). The coordinate axes of S' coincide with those of S at $t = 0$. The velocity of the particle (V'_x, V'_y) in the S' frame, at $t = \frac{\pi}{2\omega}$

is

- (a) $\left(-\frac{V\pi}{2}, -V\right)$ (b) $(-V, -V)$ (c) $\left(\frac{V\pi}{2}, -V\right)$ (d) $\left(\frac{3V\pi}{2}, -V\right)$

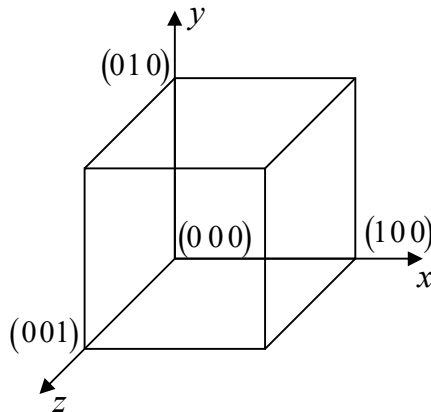
Q16. For the given circuit, the output Y is



- (a) 0 (b) 1 (c) A (d) \bar{A}

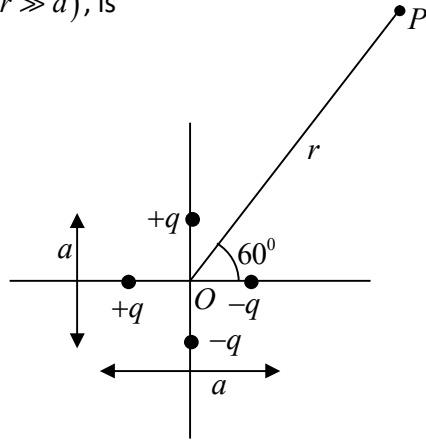
Q17. The total charge contained within the cube (see figure), in which the electric field is given by

$\vec{E} = K(4x^2\hat{i} + 3y\hat{j})$, where ϵ_0 is the permittivity of free space, is



- (a) $7K \epsilon_0$ (b) $5K \epsilon_0$ (c) $33K \epsilon_0$ (d) Zero

- Q18. Four charges are placed very close to each other, as shown. The separation between the two charges on the y -axis is a . The separation between the two charges on the x -axis is also a . The leading order (non-vanishing) form of the electrostatic potential, at point P , at a distance r from the origin ($r \gg a$), is



- (a) $\frac{1}{4\pi\epsilon_0} \frac{qa}{2r^2} (\sqrt{3} - 1)$ (b) $\frac{1}{4\pi\epsilon_0} \frac{2qa}{r^2}$
- (c) $\frac{1}{4\pi\epsilon_0} \frac{qa}{r^2} (\sqrt{5} - 1)$ (d) $\frac{1}{4\pi\epsilon_0} \frac{qa}{r^2} (1 - \sqrt{3})$
- Q19. At $t = 0$, N_0 number of a radioactive nuclei A start decaying into B with a decay constant λ_a . The daughter nuclei B decay into nuclei C with a decay constant λ_b . Then, the number of nuclei B at small time t (to the leading order) is
- (a) $\lambda_a N_0 t$ (b) $(\lambda_a - \lambda_b) N_0 t$ (c) $(\lambda_a + \lambda_b) N_0 t$ (d) $\lambda_b N_0 t$
- Q20. The electric field of an electromagnetic wave has the form $\vec{E} = E_0 \cos(\omega t - kz) \hat{i}$. At $t = 0$, a test particle of charge q is at $z = 0$, and has velocity $\vec{v} = 0.5c\hat{k}$, where c is the speed of light. The total instantaneous force on the particle is
- (a) $\frac{qE_0}{2} \hat{i}$ (b) $\frac{qE_0}{\sqrt{2}} (\hat{i} + \hat{j})$ (c) $\frac{qE_0}{2} (\hat{i} - \hat{k})$ (d) Zero
- Q21. The *rms* velocity of molecules of oxygen gas is given by v at some temperature T . The molecules of another gas have the same *rms* velocity at temperature $\frac{T}{16}$. The second gas is
- (a) Hydrogen (b) Helium (c) Nitrogen (d) Neon

Q22. A system undergoes a thermodynamic transformation from state S_1 to state S_2 via two different paths 1 and 2. The heat absorbed and work done along path 1 are $50J$ and $30J$, respectively.

If the heat absorbed along path 2 is $30J$, the work done along path 2 is

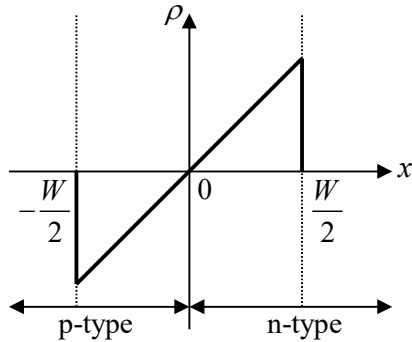
- (a) Zero (b) $10J$ (c) $20J$ (d) $30J$

Q23. The condition for maxima in the interference of two waves

$Ae^{i\left(\frac{k_0}{2}(\sqrt{3}x+y)-\omega t\right)}$ and $Ae^{i\left(\frac{k_0}{\sqrt{2}}(x+y)-\omega t\right)}$ is given in terms of the wavelength λ and m , an integer, by

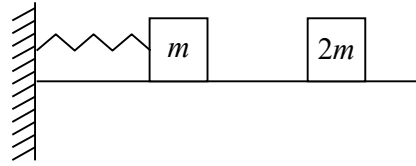
- (a) $(\sqrt{3}-\sqrt{2})x+(1-\sqrt{2})y=2m\lambda$ (b) $(\sqrt{3}+\sqrt{2})x+(1-\sqrt{2})y=2m\lambda$
 (c) $(\sqrt{3}-\sqrt{2})x-(1-\sqrt{2})y=m\lambda$ (d) $(\sqrt{3}-\sqrt{2})x+(1-\sqrt{2})y=(2m+1)\lambda$

Q24. A semiconductor pn junction at thermal equilibrium has the space charge density $\rho(x)$ profile as shown in the figure. The figure that best depicts the variation of the electric field E with x is (W denotes the width of the depletion layer)



- (a) (b)
- (c) (d)

- Q25. A mass m is connected to a massless spring of spring constant k , which is fixed to a wall. Another mass $2m$, having kinetic energy E , collides collinearly with the mass m completely inelastically (see figure). The entire set up is placed on a frictionless floor. The maximum compression of the spring is



- (a) $\sqrt{\frac{4E}{3k}}$ (b) $\sqrt{\frac{E}{3k}}$ (c) $\sqrt{\frac{E}{5k}}$ (d) $\sqrt{\frac{E}{7k}}$
- Q26. A linearly polarized light falls on a quarter wave plate and the emerging light is found to be elliptically polarized. The angle between the fast axis of the quarter wave plate and the plane of polarization of the incident light, can be
- (a) 30° (b) 45° (c) 90° (d) 180°
- Q27. The expression for the magnetic field that induces the electric field

$$\vec{E} = K(yz\hat{i} + 3z\hat{j} + y\hat{k})\cos(\omega t) \text{ is}$$

- (a) $-\frac{K}{\omega}(\hat{i} + \hat{j} - \hat{k})\sin(\omega t)$ (b) $-\frac{K}{\omega}(\hat{i} + \hat{j} + \hat{k})\sin(\omega t)$
- (c) $-\frac{K}{\omega}(\hat{i} - \hat{j} + \hat{k})\sin(\omega t)$ (d) $-\frac{K}{\omega}(\hat{i} + \hat{j} + \hat{k})\sin(\omega t)$
- Q28. In the Fourier series expansion of two functions $f_1(t) = 4t^2 + 3$ and $f_2(t) = 6t^3 + 7t$ in the interval $-\frac{T}{2}$ to $+\frac{T}{2}$, the Fourier coefficient a_n and b_n (a_n and b_n are coefficients of $\cos(n\omega t)$ and $\sin(n\omega t)$, respectively) satisfy
- (a) $a_n = 0$ and $b_n \neq 0$ for $f_1(t)$; $a_n \neq 0$ and $b_n = 0$ for $f_2(t)$
- (b) $a_n \neq 0$ and $b_n = 0$ for $f_1(t)$; $a_n = 0$ and $b_n \neq 0$ for $f_2(t)$
- (c) $a_n \neq 0$ and $b_n \neq 0$ for $f_1(t)$; $a_n = 0$ and $b_n \neq 0$ for $f_2(t)$
- (d) $a_n = 0$ and $b_n \neq 0$ for $f_1(t)$; $a_n \neq 0$ and $b_n \neq 0$ for $f_2(t)$

Q29. A thin circular disc lying in the xy -plane has a surface mass density σ , given by

$$\sigma(r) = \begin{cases} \sigma_0 \left(1 - \frac{r^2}{R^2}\right) & \text{if } r \leq R \\ 0 & \text{if } r > R \end{cases}$$

where r is the distance from its center. Its moment of inertia about the z -axis, passing through its center is

- (a) $\frac{\sigma_0 R^4}{4}$ (b) $\frac{\pi\sigma_0 R^4}{6}$ (c) $\sigma_0 R^4$ (d) $2\pi\sigma_0 R^4$

Q30. The radial component of acceleration in plane polar coordinates is given by

- (a) $\frac{d^2 r}{dt^2}$ (b) $\frac{d^2 r}{dt^2} - r \left(\frac{d\theta}{dt}\right)^2$
 (c) $\frac{d^2 r}{dt^2} + r \left(\frac{d\theta}{dt}\right)^2$ (d) $2 \frac{dr}{dt} \frac{d\theta}{dt} + r \frac{d^2 \theta}{dt^2}$

SECTION – B

MULTIPLE SELECT QUESTIONS (MSQ)

Q31 – Q.40 carry two marks each

Q31. A gaseous system, enclosed in an adiabatic container, is in equilibrium at pressure P_1 and volume V_1 . Work is done on the system in a quasi-static manner due to which the pressure and volume change to P_2 and V_2 , respectively, in the final equilibrium state. At every instant, the pressure and volume obey the condition $PV^\gamma = C$, where $\gamma = \frac{C_p}{C_v}$ and C is a constant. If the work done

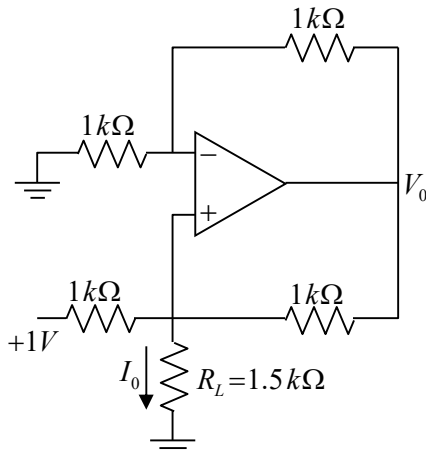
is zero, then identify the correct statement(s)

- (a) $P_2 V_2 = P_1 V_1$ (b) $P_2 V_2 = \gamma P_1 V_1$
 (c) $P_2 V_2 = (\gamma + 1) P_1 V_1$ (d) $P_2 V_2 = (\gamma - 1) P_1 V_1$

Q32. An isolated ideal gas is kept at pressure P_1 and volume V_1 . The gas undergoes free expansion and attains a pressure P_2 and volume V_2 . Identify the correct statements(s) $\left(\gamma = \frac{C_p}{C_v}\right)$

- (a) This is an adiabatic process
 (b) $P_1 V_1 = P_2 V_2$
 (c) $P_1 V_1^\gamma = P_2 V_2^\gamma$
 (d) This is an isobaric process

Q35. For the given circuit, identify the correct statement(s)



- (a) $I_0 = 1\text{mA}$
- (b) $V_0 = 3\text{V}$
- (c) If R_L is doubled, I_0 will change to 0.5mA
- (d) If R_L is doubled, V_0 will change to 6V

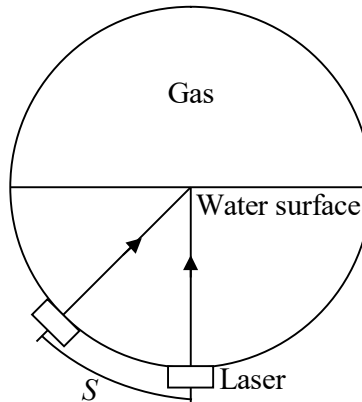
Q36. A Carnot engine operates between two temperatures, $T_L = 100\text{K}$ and $T_H = 150\text{K}$. Each cycle of the engine lasts for 0.5 seconds during which the power delivered is 500J/second . Let Q_H be the corresponding heat absorbed by the engine and Q_L be the heat lost. Identify the correct statement(s)

- (a) $Q_H = 750\text{J}$
- (b) $\frac{Q_H}{Q_L} \leq \frac{2}{3}$
- (c) The change in entropy of the engine and the hot bath in a cycle is 5J/K
- (d) The change in entropy of the engine in 0.5 seconds is zero.

Q37. A time independent conservative force \vec{F} has the form, $\vec{F} = 3y\hat{i} + f(x, y)\hat{j}$. Its magnitude at $x = y = 0$ is 8. The allowed form(s) of $f(x, y)$ is (are)

- (a) $3x+8$
- (b) $2x+8(y-1)^2$
- (c) $3x+8e^{-y^2}$
- (d) $2x+8\cos y$

- Q38. The figure shows the cross-section of hollow cylindrical tank, 2.2 m in diameter which is half filled with water (refractive index of 1.33). The space above the water is filled with a gas of unknown refractive index. A small laser moves along the bottom surface and aims a light beam towards the center (see figure). When the laser moves a distance of $S = 1.09\text{ m}$ or beyond from the lowest point in the water, no light enters the gas. Identify the correct statement(s) (speed of light is $3 \times 10^8\text{ m/s}$).



- (a) The refractive index of the gas is 1.05
- (b) The time taken for the light beam to travel from the laser to the rim of the tank when $S < 1.09\text{ m}$ is 8.9 ns
- (c) The time taken for the light beam to travel from the laser to the rim of the tank when $S > 1.09\text{ m}$ is 9.7 ns
- (d) The critical angle for the water-gas interface is 56.77°
- Q39. Identify the correct statement(s) regarding nuclei
- (a) The uncertainty in the momentum of a proton in a nucleus is roughly 10^5 times the uncertainty in the momentum of the electron in the ground state of Hydrogen atom
- (b) The volume of a nucleus grows linearly with the number of nucleons in it
- (c) The energy of γ rays due to de-excitation of nucleus can be of the order of MeV
- (d) ${}^{56}\text{Fe}$ is the most stable nucleus
- Q40. A particle of mass m is in an infinite square well potential of length L . It is in a superposed state of the first two energy eigenstates, as given by $\psi(x) = \frac{1}{\sqrt{3}}\psi_{n=1}(x) + \sqrt{\frac{2}{3}}\psi_{n=2}(x)$. Identify the correct statement(s). h is Planck's constant.

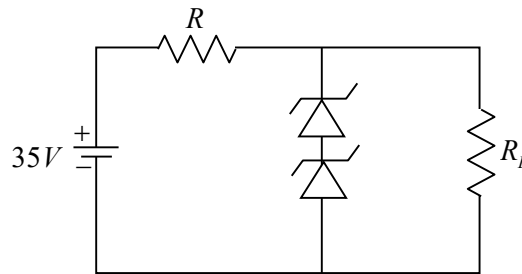
(a) $\langle p \rangle = 0$ (b) $\Delta p = \frac{\sqrt{3}h}{2L}$ (c) $\langle E \rangle = \frac{3h^2}{8mL^2}$ (d) $\Delta x = 0$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

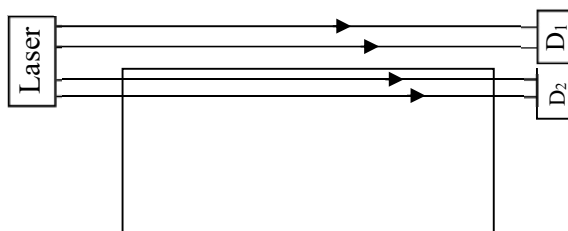
Q 41 – Q. 50 carry one mark each

- Q41. One of the roots of the equation, $z^6 - 3z^4 - 16 = 0$ is given by $z_1 = 2$. The value of the product of the other five roots is _____
- Q42. The following Zener diode voltage regulator circuit is used to obtain 20 V regulated output at load resistance R_L from a 35 V dc power supply. Zener diodes are rated at 5 W and 10 V . The value of the resistance R is _____ Ω .



- Q43. A small conducting square loop of side l is placed inside a concentric large conducting square loop of side L ($L \gg l$). The value of mutual inductance of the system is expressed as $\frac{n\mu_0 l^2}{\pi L}$. The value of n is _____ (Round off to two decimal places)
- Q44. Consider N_1 number of ideal gas particles enclosed in a volume V_1 . If the volume is changed to V_2 and the number of particles is reduced by half, the mean free path becomes four times of its initial value. The ratio $\frac{V_1}{V_2}$ is _____ (Round off to one decimal place).
- Q45. A particle is moving with a velocity $0.8c\hat{i}$ (c is the speed of light) in an inertial frame S_1 . Frame S_2 is moving with a velocity $0.8c\hat{i}$ with respect to S_1 . Let E_1 and E_2 be the respective energies of the particle in the two frames. Then, $\frac{E_2}{E_1}$ is _____ (Round off to two decimal places).
- Q46. At some temperature T , two metals A and B , have Fermi energies ϵ_A and ϵ_B , respectively. The free electron density of A is 64 times that of B . The ratio $\frac{\epsilon_A}{\epsilon_B}$ is _____.

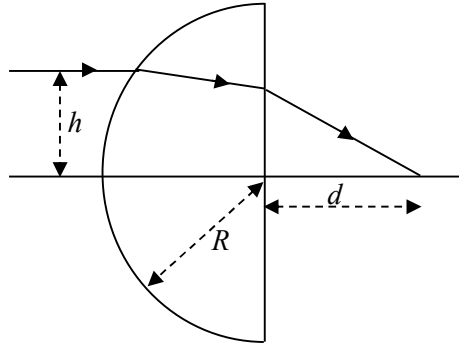
- Q47. A crystal has monoclinic structure, with lattice parameters, $a = 5.14 \text{ \AA}$, $b = 5.20 \text{ \AA}$, $c = 5.30 \text{ \AA}$ and angle $\beta = 99^\circ$. It undergoes a phase transition to tetragonal structure with lattice parameters, $a = 5.09 \text{ \AA}$ and $c = 5.27 \text{ \AA}$. The fractional change in the volume $\left| \frac{\Delta V}{V} \right|$ of the crystal due to this transition is _____ (Round off to two decimal places).
- Q48. A laser beam shines along a block of transparent material of length 2.5 m . Part of the beam goes to the detector D_1 while the other part travels through the block and then hits the detector D_2 . The time delay between the arrivals of the two light beams is inferred to be 6.25 ns . The speed of light $c = 3 \times 10^8 \text{ m/s}$. The refractive index of the block is _____ (Round off to two decimal places).



- Q49. An ideal blackbody at temperature T , emits radiation of energy density u . The corresponding value for a material at temperature $\frac{T}{2}$ is $\frac{u}{256}$. Its emissivity is _____ (Round off to three decimal places).
- Q50. A particle with positive charge 10^{-3} C and mass 0.2 kg is thrown upwards from the ground at an angle 45° with the horizontal with a speed of 5 m/s . The projectile moves through a horizontal electric field of 10 V/m , which is in the same direction as the horizontal component of the initial velocity of the particle. The acceleration due to gravity is 10 m/s^2 . The range is _____ m . (Round off to three decimal places).

Q51 – Q60 carry two marks each.

Q51. Consider a hemispherical glass lens (refractive index is 1.5) having radius of curvature $R = 12\text{ cm}$ for the curved surface. An incoming ray, parallel to the optical axis, is incident on the curved surface at a height $h = 1\text{ cm}$ above the optical axis, as shown in the figure. The distance d (from the flat surface of the lens) at which the ray crosses the optical axis is _____ cm (Round off to two decimal places).

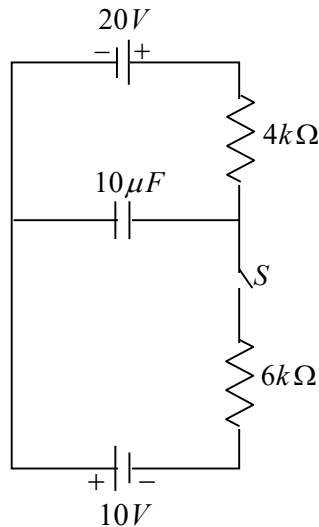


Q52. Twenty non-interacting spin $\frac{1}{2}$ particles are trapped in a three-dimensional simple harmonic oscillator potential of frequency ω . The ground state energy of the system, in units of $\hbar\omega$, is _____.

Q53. A thin film of alcohol is spread over a surface. When light from a tunable source is incident normally, the intensity of reflected light at the detector is maximum for $\lambda = 640\text{ nm}$ and minimum for $\lambda = 512\text{ nm}$. Taking the refractive index of alcohol to be 1.36 for both the given wavelengths, the minimum thickness of the film would be _____ nm (Round off to two decimal places).

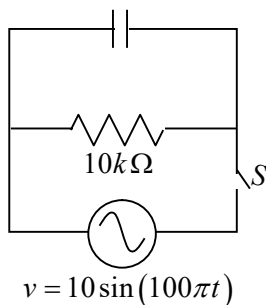
Q54. For the Boolean expression $Y = ABC + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$, the number of combinations for which the output $Y = 1$ is _____.

- Q55. An RC circuit is connected to two dc power supplies, as shown in the figure. With switch S open, the capacitor is fully charged. S is then closed at time $t = 0$. The voltage across the capacitor at $t = 2.4$ milliseconds is _____ V (Round off to one decimal place).



- Q56. A current I is uniformly distributed across a long straight nonmagnetic wire ($\mu_r = 1$) of circular cross-section with radius a . Two points P and Q are at distances $\frac{a}{3}$ and $9a$, respectively, from the axis of the wire. The ratio of the magnetic fields at points P and Q is _____.
- Q57. A particle A of mass m is moving with a velocity $v_i \hat{i}$, and collides elastically with a particle B , of mass $2m$, B is initially at rest. After collision, A moves with a velocity $v_A \hat{j}$. If v_B is the final speed of B , then $v_A^2 = kv_B^2$. The value of k is _____.
- Q58. In an X -ray diffraction experiment with Cu crystals having lattice parameter 3.61 \AA , X -rays of wavelength of 0.090 nm are incident on the family of planes $\{110\}$. The highest order present in the diffraction pattern is _____.

- Q59. A parallel plate capacitor having plate area of 50 cm^2 and separation of 0.1 mm is completely filled with a dielectric (dielectric constant $K = 10$). The capacitor is connected to a $10\text{ k}\Omega$ resistance and an alternating voltage $v = 10 \sin(100\pi t)$, as shown in the figure. The switch S is initially open and then closed at $t = 0$. The ratio of the displacement current in the capacitor, to the current in the resistance, at time $t = \frac{2}{\pi}$ seconds is _____ (Round off to three decimal places).



- Q60. The wavelength of characteristic K_{α} X-ray photons from Mo (atomic number 42) is _____ Å (Round off to one decimal place).
 (speed of light is $3 \times 10^8\text{ m/s}$; Rydberg constant $R = 1.09 \times 10^7 / \text{m}$)