Practice Set 2 (Mechanics and Classical Mechanics)

Langrangian Formulation

Q1. Geodesis on a surface is a curve along which the distance between any two points at the surface is a minimum, then geodesis on a right circular cylinder of radius 'a' is given by

(a) Circle of radius a (b) Circular helix (c) Straight line (d) Cycloid

Q2. If the Lagrangian of a dynamical system in two dimensions is $L=\frac{1}{2}m\dot{x}^2+m\dot{x}\dot{y}$, then which one is not correct test 3 2016

(a) $\dot{x}=\frac{p_y}{m}$ (b) $\dot{y}=\frac{p_x-p_y}{m}$

(c) both p_x and p_y are constants of motion (d) $\dot{y}=\frac{p_x+p_y}{m}$

Q3. If Lagrangian of system is given by $L=\frac{x\dot{x}^2}{2}-\frac{\dot{x}x^2}{2}$, then the equation of motion is:

(a) $x\ddot{x}+\frac{\dot{x}^2}{2}=0$ (b) $\ddot{x}+x\dot{x}+\dot{x}^2=0$ (c) $\ddot{x}+2x\dot{x}+\dot{x}^2=0$ (d) $\ddot{x}-2x\dot{x}+\frac{\dot{x}^2}{2}=0$

Q4. If Lagrangian of the system is given as

$L=-\frac{m}{2}q\ddot{q}-\frac{k}{2}q^2$ then equation of motion is given by.

(a) $m\ddot{q}+kq=0$ (b) $\frac{m}{2}\ddot{q}+kq=0$ (c) $m\ddot{q}-kq=0$ (d) $\frac{m}{2}\ddot{q}-kq=0$

Q5. Lagrangian of the particular physical system can be written as.

$$L=\frac{m}{2}(a\dot{x}^2+2b\dot{x}\dot{y}+c\dot{y}^2)-\frac{k}{2}(ax^2+2bxy+cy^2)$$

Then which one is a correct relation where $u=ax+by$ and $v=bx+cy$, $\dot{u}=\frac{du}{dt},\dot{v}=\frac{dv}{dt}$

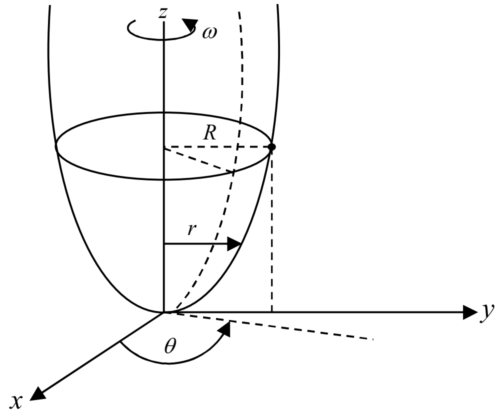
(a) $\frac{u}{v}=1$ (b) $\frac{u}{v}=\frac{\dot{u}}{\dot{v}}$ (c) $\frac{u}{v}=\frac{\ddot{u}}{\ddot{v}}$ (d) $\frac{u}{v}=\frac{\dot{v}}{\dot{u}}$

Q6. A particle of mass m and coordinate q has the Lagrangian given by $L=\frac{1}{2}m\dot{q}^2-\frac{\lambda}{2}q\dot{q}^2$ then equation of motion is given by

(a) $\ddot{q}(m+\lambda q)-\frac{\lambda}{2}\dot{q}^2$ (b) $\ddot{q}(m-\lambda q)+\frac{\lambda}{2}\dot{q}^2$

(c) $\ddot{q}(m-\lambda q)-\frac{\lambda}{2}\dot{q}^2$ (d) $\ddot{q}(m+\lambda q)+\frac{\lambda}{2}\dot{q}^2$

Q7. A bead slides along a smooth wire bent in the shape of a parabola $z = cr^2$ (Figure). when the wire is rotating about its vertical symmetry axis with angular velocity ω. Which of the following is correct lagrangian



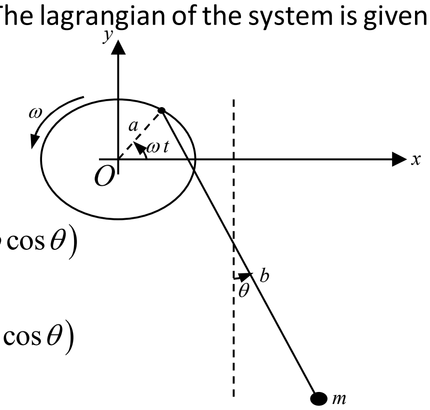
(a) $L = \frac{m}{2}\left(\dot{r}^2 + r^2\omega^2\right) - mgcr^2$

(b) $L = \frac{m}{2}\left(\dot{r}^2 + r^2\omega^2\right) + mgcr^2$

(c) $L = \frac{m}{2}\left(\dot{r}^2 + 4c^2r^2\dot{r}^2 + r^2\omega^2\right) - mgcr^2$

(d) $L = \frac{m}{2}\left(\dot{r}^2 + 4c^2r^2\dot{r}^2 + r^2\omega^2\right) + mgcr^2$

Q8. The point of support of a simple pendulum of length b moves on a massless rim of radius a rotating with constant angular velocity ω as shown is figure. The lagrangian of the system is given by



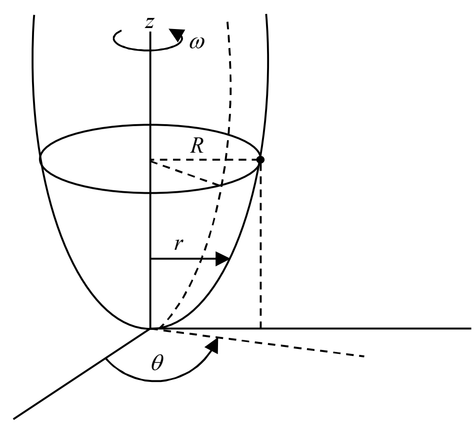
(a) $L = \frac{m}{2}\left[a^2\omega^2 + b^2\dot{\theta}^2\right] - mg\left(a\sin\omega t - b\cos\theta\right)$

(b) $L = \frac{m}{2}\left[a^2\omega^2 + b^2\dot{\theta}^2 + 2b\dot{\theta}a\omega\sin\left(\theta + \omega t\right)\right] - mg\left(a\sin\omega t + b\cos\theta\right)$

(c) $L = \frac{m}{2}\left[a^2\omega^2 + b^2\dot{\theta}^2 + 2b\dot{\theta}a\omega\sin\left(\theta + \omega t\right)\right] - mg\left(a\sin\omega t - b\cos\theta\right)$

(d) $L = \frac{m}{2}\left[a^2\omega^2 + b^2\dot{\theta}^2 + 2b\dot{\theta}a\omega\sin\left(\theta - \omega t\right)\right] - mg\left(a\sin\omega t - b\cos\theta\right)$

Q9. A bead slides along a smooth wire bent in the shape of a parabola $z = cr^2$ as shown in figure with lagrangian $L = \frac{m}{2}\left(\dot{r}^2 + 4c^2r^2\dot{r}^2 + r^2\omega^2\right) - mgcr^2$. The bead rotates in a circle of radius R when the wire is rotating about its vertical symmetry axis with angular velocity ω. Then the value of c is given by:



(a) $c = \frac{\omega^2}{g}$

(b) $c = \frac{\omega^2}{2g}$

(c) $c = \frac{2\omega^2}{g}$

(d) $c = \frac{4\omega^2}{g}$

Q10. An inverted pendulum consists of a particle of mass m supported by a rigid massless rod of length l. the pivot O has a vertical motion given by $z = A\sin\omega t$, the Lagrangian of the system is

(a) $\frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta - mlA\omega^2\sin\omega t\cos\theta$ (b) $\frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta - mlA\omega^2\sin\omega t\cos\theta$

(c) $\frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta + mlA\omega^2\sin\omega t\cos\theta$ (d) $\frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta + mlA\omega^2\sin\omega t\cos\theta$

Q11. The Lagrangian for a particle in cylindrical coordinates is given to be

$$L = \frac{1}{2}m\left(\dot{\rho}^2 + \rho^2\dot{\phi}^2 + \dot{z}^2/\left(1+(\rho/\lambda)^2\right)\right) - V(\rho)$$

where m is the mass of the particle and λ is a constant. Then which of the following is not correct?

(a) $m\dot{\rho}$ is not conserved (b) $m\rho^2\dot{\phi}$ is conserved

(c) $m\dot{z}/\left(1+(\rho/\lambda)^2\right)$ is a conserved quantity (d) $m\dot{z}$ is a conserved quantity .

Q12. If Lagrangian of system is given by

$$L = \frac{1}{2}ml^2(\dot{\theta}_1^2 + \dot{\theta}_2^2) + \frac{1}{2}mgl(\theta_1^2 + \theta_2^2) - \frac{1}{2}kl^2(\theta_1 - \theta_2)^2$$

Where ?1 and ?2 are generalized co-ordinate then the equation of motion is given by

(a) $ml^2\ddot{\theta}_1 - mgl\theta_1 - kl^2(\theta_2 - \theta_1) = 0$ (b) $ml^2\ddot{\theta}_1 - mgl\theta_1 + kl^2(\theta_2 - \theta_1) = 0$

(c) $ml^2\ddot{\theta}_2 - mgl\theta_2 - kl^2(\theta_2 - \theta_1) = 0$ (d) $ml^2\ddot{\theta}_2 - mgl\theta_2 - kl^2(\theta_2 - \theta_1) = 0$

Q13. If lagrangian of the system is given by

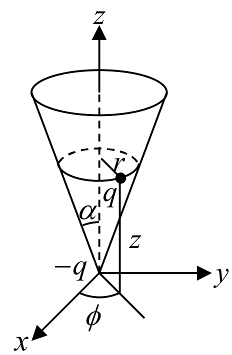
$$L = \frac{m}{2}\left[a^2\omega^2 + b^2\dot{\theta}^2 + 2b\dot{\theta}a\omega\sin(\theta - \omega t)\right] - mg\left(a\sin\omega t - b\cos\theta\right)$$

Where θ is generalized coordinate . Which one of the following is equation of motion

(a) $\ddot{\theta} = \frac{\omega^2 a}{b}\cos(\theta + \omega t) - \frac{g}{b}\sin\theta$ (b) $\ddot{\theta} = \frac{\omega^2 a}{b}\cos(\theta + \omega t) + \frac{g}{b}\sin\theta$

(c) $\ddot{\theta} = \frac{\omega^2 a}{b}\cos(\theta - \omega t) - \frac{g}{b}\sin\theta$ (d) $\ddot{\theta} = \frac{\omega^2 a}{b}\cos(\theta - \omega t) + \frac{g}{b}\sin\theta$

Q14. A small body of mass m and charge q is constrained to move without friction on the interior of a cone of opening angle 2α. A charge $-q$ is fixed at the apex of the cone as shown in figure below. There is no gravity. Find the frequency of small oscillations about equilibrium trajectories of the moving body in terms of $\dot{\phi}_0$, the equilibrium angular velocity of the body around the inside of the cone. Assume $v << c$ so that radiation is negligible.



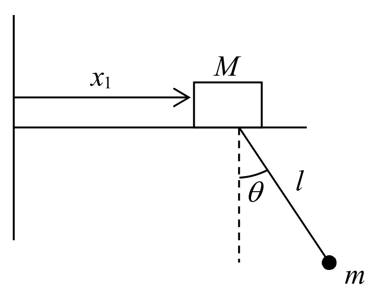
(a) $L = \frac{1}{2}m\left(\dot{r}^2 \text{cosec}^2\alpha + r^2\dot{\phi}^2\right) - \frac{q^2}{4\pi\varepsilon_0 r}$

(b) $L = \frac{1}{2}m\left(\dot{r}^2 \cot^2\alpha + r^2\dot{\phi}^2\right) - \frac{q^2 \sin\alpha}{4\pi\varepsilon_0 r}$

(c) $L = \frac{1}{2}m\left(\dot{r}^2 \text{cosec}^2\alpha + r^2\dot{\phi}^2\right) + \frac{q^2 \sin\alpha}{4\pi\varepsilon_0 r}$

(d) $L = \frac{1}{2}m\left(\dot{r}^2 \text{cosec}^2\alpha + r^2\dot{\phi}^2\right) + \frac{q^2}{4\pi\varepsilon_0 r}$

Q15. A pendulum of length l and mass m is attached to a block of mass M. The block slides on a horizontal frictional surface. Find kinetic energy of the system.



(a) $T = \frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}m\dot{x}_1^2$

(b) $T = \frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}m\left(\dot{x}_1^2 + 2l\dot{x}\dot{\theta}\cos\theta + l^2\dot{\theta}^2\right)$

(c) $T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left(\dot{x}_1^2 + l^2\dot{\theta}^2\right)$

(d) $T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left(\dot{x}_1^2 + l^2\dot{\theta}^2 - 2\dot{x}\dot{\theta}\cos\theta\right)$

Practice Set (Solution)

Langrangian Formulation

Ans. 1: (b)	Ans. 2: (d)	Ans. 3: (a)	Ans. 4: (a)	Ans. 5: (c)	Ans. 6: (c)	Ans. 7: (c)
Ans. 8: (d)	Ans. 9: (b)	Ans. 10: (c)	Ans. 11: (d)	Ans. 12: (a)	Ans. 13: (c)	Ans. 14: (c)
Ans. 15: (b)						