Chapter 10

Identical Particles

1. Maxwell-Boltzmann Distribution

In statistical mechanics, the **Maxwell–Boltzmann distribution** describes particle speeds in gases, where the particles move freely without interacting with one another, except for very brief elastic collision in which they may exchange momentum and kinetic energy, but do not change their respective states of intermolecular excitation, as a function of the temperature of the system, the mass of the particle, and speed of the particle. Particle in this context refers to the gaseous atoms or molecules – no difference is made between the two in its development and result.

Maxwell –Boltzmann system constituent identical particles who are **distinguishable** in nature and there is **not any restriction** on no of particles which can occupy any energy level. The wave function of particle will not overlap to each other because mean separation of particles is more than the thermal wavelength , which is identified by λ . (where $\lambda = \frac{h}{\sqrt{2\pi m k_B T}}$ is defined as the thermal wavelength)

Suppose there are l states with energies, $E_1, E_2, E_3.......E_l$ and degeneracy $g_1, g_2, g_3.......g_l$. Respectively, in which the particles are distributed. If there is N numbers of distinguishable particles out of these $n_1, n_2, n_3.......n_l$ particles is adjusted in energy level $E_1, E_2, E_3.......E_l$ respectively. So $\sum_{i=1}^{i=l} n_i = N \quad \sum_{i=1}^{i=l} E_i n_i = U$.

The total nunmber of arrangements of the particles in the given distributions

$$W = \frac{\lfloor N}{\lfloor n_1 \lfloor n_2 \lfloor n_3 ....\lfloor n_l} g_1^{n_1} g_2^{n_2} ......g_l^{n_l} \ , \quad W = \lfloor N \prod_{i=1}^{i=l} \frac{g_i^{n_i}}{\lfloor n_i}$$

Example: Two distinguishable particles have to be adjusted in a state whose degeneracy is three

(a) How many ways the particles can be adjusted?

(b) Show all arrangement .

Solution: (a) $N = 2, n = 2, g = 3$ and no. of microstate is $W = \frac{\lfloor N g^n}{\lfloor n}$ 9 ways.

(b) Total no. of arrangement for 2 distinguishable in state whose degeneracy is 3.

First level	Second level	Third level
AB	0	0
0	AB	0
0	0	AB
A	B	0
B	A	0
A	0	B
B	0	A
0	A	B
0	B	A

Derivation of Maxwell-Boltzmann Distribution

The Maxwell-Boltzmann distribution law for the particles in the states is

$$n_i = g_i \exp\left(-\alpha - \beta E_i\right) \ , \ n_i = g_i(\exp - \alpha)(\exp - \beta E_i)$$

After using the values $e^{-\alpha} = \frac{N}{V}\left(\frac{h^2}{2\pi m k_B T}\right)^{3/2}$ where $\beta = \frac{1}{k_B T}$