Chapter 10
Identical Particles

2. Energy Distribution Function

Energy distribution function $f(E_i)$ is the average number of particles per level in the energy states E_i. Therefore $f(E_i) = \frac{n_i}{g_i} = \frac{N}{V}\left(\frac{h^2}{2\pi m k_B T}\right)^{3/2} e^{-E_i} / k_B T$

Energy distribution in different dimension

- $f(E) = \frac{N}{V}\left(\frac{h^2}{2\pi m k_B T}\right)^{3/2} e^{-E/k_B T}$ distribution function in three dimension where V is volume.
- $f(E) = \frac{N}{A}\left(\frac{h^2}{2\pi m k_B T}\right) e^{-E/k_B T}$ distribution function in two dimensions where A is area.
- $f(E) = \frac{N}{L}\left(\frac{h^2}{2\pi m k_B T}\right)^{\frac{1}{2}} e^{-E/k_B T}$ distribution function in one dimension. where L is length.

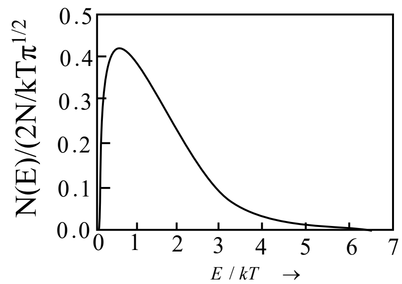
The number of particles $n(E)dE$ having energies in the range from E to $E+dE$ is $n(E)dE = f(E)g(E)dE$ where $f(E)$ is distribution function and $g(E)dE$ is number of level(quantum state) in the range of E to $E+dE$

- $g(E)dE$ in three dimension $g(E)dE = 2\pi V\left(\frac{2m}{h^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}}dE$ where V is volume of three dimensional space
- $g(E)dE$ in two dimension $g(E)dE = \pi A\left(\frac{2m}{h^2}\right)dE$ where A is area of the two dimensional space
- $g(E)dE$ in one dimension $g(E)dE = L\left(\frac{2m}{h^2}\right)^{\frac{1}{2}} E^{\frac{-1}{2}}dE$ where L is area of the one dimensional space

The number of particles $n(E)dE$ having energies in the range from E to $E+dE$ in three-dimensional space

$$n(E)dE = \frac{N}{V}\left(\frac{h^2}{2\pi mk_BT}\right)^{3/2} e^{-E/k_BT} 2\pi V\left(\frac{2m}{h^2}\right)^{\frac{3}{2}} E^{1/2}dE$$

$$= \frac{2\pi N}{(\pi k_BT)^{3/2}} E^{1/2} e^{-E/k_BT} dE$$



This is known as the Maxwell-Boltzmann energy distribution law for an-ideal gas.

Where $\lambda = \frac{h}{\sqrt{2\pi mk_BT}}$ is defined as the thermal wavelength.

Average Energy

For the Maxwell-Boltzmann energy distribution law, average energy $\langle E\rangle$ of the particles is

$$\langle E\rangle = \frac{\int_0^\infty E\, n(E)\, dE}{\int_0^\infty n(E)\, dE}$$

$$\langle E\rangle = \frac{1}{N}\frac{2\pi N}{(\pi k_BT)^{3/2}}\int_0^\infty E^{3/2} e^{-E/k_BT} dE$$

Let $E = k_BTx$ and therefore, $dE = k_BTdx$ Then we have

$$E = \frac{2\pi}{(\pi k_B T)^{3/2}} \int_0^\infty (k_B T x)^{3/2} e^{-x} k_B T dx \quad = \frac{2k_B T}{\sqrt{\pi}} \int_0^\infty x^{3/2} e^{-x} dx = \frac{3}{2} k_B T$$

Hence, the average of a particle is $\frac{1}{2}k_B T$ per degree of freedom, for three degree of freedom it is $\frac{3}{2}k_B T$

Example: If N number of distinguishable particle is kept into two dimensional box of area A what is average energy at temperature A.

Solution: for two dimensional system $g(E)dE = \pi A\left(\frac{2m}{h^2}\right)dE$ and distribution

Function is given by $f(E) = \frac{N}{A}\left(\frac{h^2}{2\pi m k_B T}\right)e^{-E/k_B T}$

$$\langle E \rangle = \frac{\int_0^\infty E\, n(E)\, dE}{\int_0^\infty n(E)\, dE} \quad \text{where } n(E)dE = f(E)g(E)dE = \frac{N}{A}\left(\frac{h^2}{2\pi m k_B T}\right)e^{-E/k_B T}\pi A\left(\frac{2m}{h^2}\right)dE$$

$$\langle E \rangle = k_B T$$