Introduction To Statistical Mechanics

2. Statistical Ensemble

We proceed to a discussion of the system in terms of probability concepts. For this purpose we consider not an isolated instance of a single system, but instead imagine attention focused on *an ensemble consisting of a very large number of identical systems*, all prepared subject to whatever conditions are specified as known. The systems in this ensemble will, in general, be in different states and will, therefore, also be characterized by different macroscopic parameters (e.g., by different values of pressure or magnetic moment). But we can ask for the probability of occurrence of a particular value of such a parameter, i.e., we can determine the fraction of cases in the ensemble when the parameter assumes this particular value.

Example: Describe 3 spin $\frac{1}{2}$ particles?

Solution: Consider a system of three fixed particles, each having spin $\frac{1}{2}$ so that each spin can point either up or down (i.e., along or opposite some direction chosen as the z - axis).

The state of the particle i can be specified by its magnetic quantum number m_z, which can assume the two values $m_z = \pm\frac{1}{2}$. The state of the whole system is specified by giving the values of the three quantum numbers m_1, m_2, m_3. A particle has energy $-\mu H$ when its spin points up and energy μH when its spin points down.

We list in the table below all the possible states of the system. We also list some parameters, such as total magnetic moment and total energy, which characterise the system as a whole.

(For the sake of brevity $m = \frac{1}{2}$ is denoted simply by $+$, and $m = -\frac{1}{2}$ by $-$). States accessible to the system

State index r	Quantum Numbers m_1, m_2, m_3	Total magnetic moment	Total Energy
1	$+++$	3μ	$-3\mu H$
2	$++-$	μ	$-\mu H$
3	$+-+$	μ	$-\mu H$
4	$-++$	μ	$-\mu H$
5	$+--$	$-\mu$	μH
6	$-+-$	$-\mu$	μH
7	$--+$	$-\mu$	μH
8	$---$	-3μ	$3\mu H$

Example: Suppose that in the previous example of a system consisting of three spins the total energy of the system is known to be equal to $-\mu H$. If this is the only information available, then the system can be in which states?

Solution: The system can be in only one of the following three states:

$$(++-) \qquad (+-+) \qquad (-++)$$

Note: Of course, we do not know in which of these states the system may actually be, nor do we necessarily know the relative probability of finding the system in any one of these states.