Chapter 2

Newton's Laws of Motion

3. Application of Newton's Law of Motion

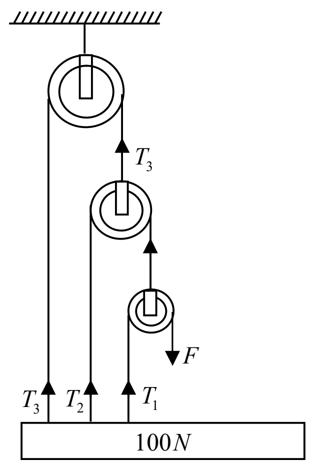
To apply Newton's law of motion one should follow following step:

Step 1- Draw free body diagram and identified external forces

Step 2 – Write down equation of constraint.

Step 3 – Write down Newton's law of motion.

Example: The force F in figure is just sufficient to hold the $100N$ block and weightless pulley in equilibrium. If There is no appreciable friction. Then find T_1, T_2 and T_3



Solution: For plank $T_1 + T_2 + T_3 = 100$

$F = T_1$

For pulley 1 $\quad T_2 = T_1 + F = T_2 = T_1 + T_1 \Rightarrow T_2 = 2T_1$

For pulley 2 $\quad T_3 = 2T_2 = 4T_1$

$T_1 + 2T_1 + 4T_1 = 100 \Rightarrow T_1 = \frac{100}{7} = F = \frac{100}{7}$, $T_2 = 2T_1 \Rightarrow T_2 = \frac{200}{7}$ and

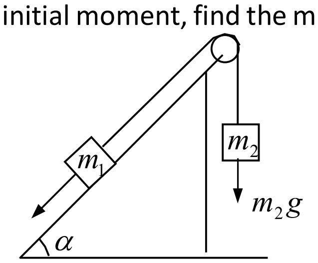
$T_3 = \frac{4T_1}{7} = \frac{400}{7}$

Example: The following parameters of the arrangement of are available: the angle α which the inclined plane forms with the horizontal, and the coefficient of friction k between the body m_1 and the inclined plane. The masses of the pulley and the threads, as well as the friction in the pulley, are negligible. Assuming both bodies to be motionless at the initial moment, find the mass ratio m_2 / m_1 at which the body m_2

(a) starts coming down;

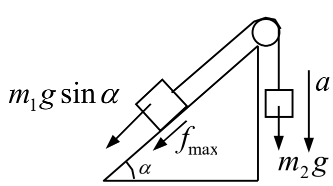
(b) starts going up;

(c) is at rest.



Solution: (a) for m_2 Starts coming down. For mass m_2 , $m_2 g > T$ and for mass m_1 moving up

$T > m_1 g \sin\alpha + f_{max}$



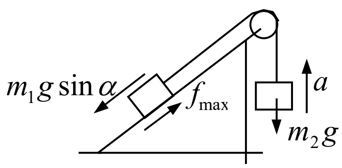
therefore $m_2 g > m_1 g \sin\alpha + f_{max}$

$m_2 g > m_1 g \sin\alpha + k_1 m_1 g \cos\alpha$

$\frac{m_2}{m_1} > \sin\alpha + k\cos\alpha$

(b) For m_2 starts coming up. For mass m_2 , $m_2 g < T$ and for mass m_1 moving up

$$T < m_1 g \sin\alpha - f_{\max}$$



$$m_1 g \sin\alpha > m_2 g + k m_1 g \cos\alpha$$

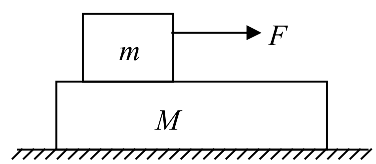
$$\sin\alpha > \frac{m_2}{m_1} + k\cos\alpha$$

$$\frac{m_2}{m_1} < \sin\alpha - k\cos\alpha$$

(c) At rest: friction will be static:

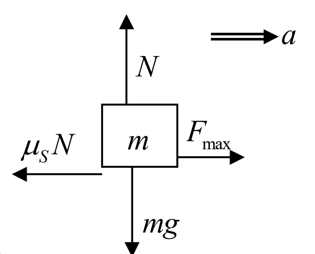
$$\sin\alpha - k\cos\alpha < \frac{m_2}{m_1} < \sin\alpha + k\cos\alpha$$

Example: A block of mass m is placed on another block of mass M lying on a smooth horizontal surface. The coefficient of static friction between m and M is μ_s. What is the maximum force that can be applied to m so that the blocks remains at rest relative to each other?



Solution: Imagine the situation when F is at its maximum value so that m is about to start slipping relative to M .

The mass m tries to drag M towards right due to friction.



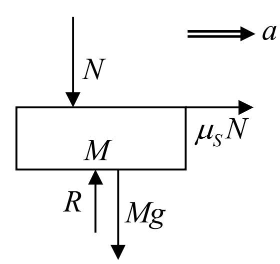
Equation of motion of mass m :

$$F - \mu_S N = ma$$

$$N = mg$$

Hence frictional force on M exerted by m will be towards right.

Let $a =$ magnitude of acceleration of blocks towards right.



Equation of motion of mass M :

$$\mu_S N = Ma \Rightarrow a = \frac{\mu_s mg}{M} \qquad R = N + Mg$$

Solving these equations, we get:

$$F = \mu_S N + ma \Rightarrow \mu_S mg + m.\frac{\mu_S mg}{M} = \frac{\mu_S mg(M+m)}{M}$$

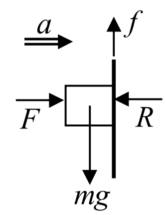
$$F_{\max} = \frac{\mu_S (m+M) mg}{M}$$

* If F is less than critical value, the blocks stick together without any relative motion.

* If F is greater than this critical value, the blocks slide relative to each other and their accelerations are different.

Example: The blocks of masses m and M are not attached to each other but are in contact. The coefficient of static friction between the blocks is μ but the surface beneath M is smooth. What is the minimum magnitude of the horizontal force F required to hold m against M ?

Solution: If m and M sticking together they will have same acceleration.

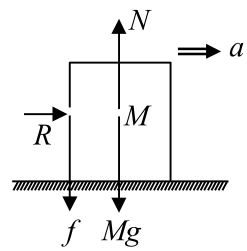


Let $a =$acceleration of blocks

Equation of motion for mass m

$F - R = ma$ and $f = mg$ f is frictional force

Equation of motion for mass M



$$R = Ma$$

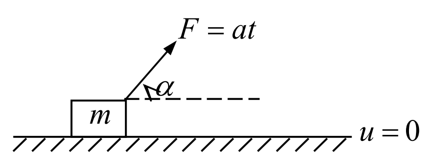
$$f + Mg = N$$

Solve to get:

$f = mg$ and $R = \dfrac{MF}{M+m}$

for no slipping, $f \le \mu_S R$ $\quad mg \le \dfrac{MF}{M+m} \quad \Rightarrow F \ge \dfrac{mg(M+m)}{\mu_S M}$

Example: At the moment $t = 0$ the force $F = at$ is applied to a small body of mass m resting on smooth horizontal plane (a is a constant). The permanent direction of this force forms an angle α with the horizontal. Find

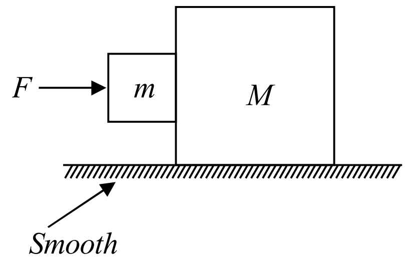


(a) The velocity of the body at the moment of its breaking off the plane;

(b) The distance traversed by the body up to this moment.

Solution: (a) If one will draw the free body diagram $F\sin\alpha + N = mg$ in y direction

$F\cos\alpha = ma_1$ in x-direction, where a_1 is acceleration of block.



At time of breaking off the plane vertical component of F must be equal to weight mg.

Then, $F\sin\alpha = mg = at\sin\alpha \Rightarrow t = \dfrac{mg}{a\sin\alpha}$

Equation of motion of block:

$$F\cos\alpha = ma_1,\ a_1 = \frac{at\cos\alpha}{m} = \frac{dv}{dt}$$

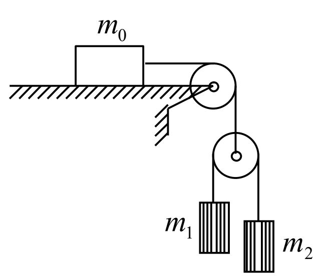
$$\Rightarrow \int_0^V \frac{mdv}{a\cos\alpha} = \int_0^{t_1} t\ dt \Rightarrow \frac{mv}{a\cos\alpha} = \frac{t_1^2}{2}$$

$$\frac{mv}{a\cos\alpha} = \frac{1}{2}\frac{m^2g^2}{a^2\sin^2\alpha} \Rightarrow v = \frac{mg^2\cos\alpha}{2a\sin^2\alpha}$$

(b) $\displaystyle\int_0^V \frac{mdv}{a\cos\alpha} = \int_0^t t\,dt,\ \frac{mv}{a\cos\alpha} = \frac{t^2}{2} \Rightarrow v = \frac{a\cos\alpha}{2m}t^2 \Rightarrow \frac{ds}{dt} = \frac{a\cos\alpha\, t^2}{2m}$

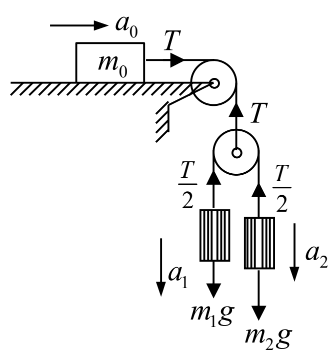
$$\int_0^s ds = \frac{a\cos\alpha}{2m}\int_0^t t^2 dt \quad \Rightarrow s = \frac{a\cos\alpha t^3}{6m} \Rightarrow s = \frac{m^2g^3\cos\alpha}{6\ \ a^2\sin^3\alpha}$$

Example: In the arrangement shown in figure the bodies have masses m_0, m_1, m_2, the friction is absent, the masses of the pulleys and the threads are negligible. Find the acceleration of the body m_1. Look into possible cases.



Solution: $T = 2T_1$ and from equation of constrain $a_0 = \dfrac{a_1 + a_2}{2}$

Equation of motion:



$$T = m_0\left(\frac{a_1 + a_2}{2}\right) \qquad \text{...(i)}$$

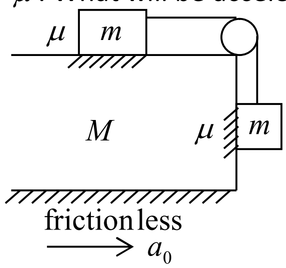
$$m_1 g - \frac{T}{2} = m_1 a_1 \qquad \text{...(ii)}$$

$$m_2 g - \frac{T}{2} = m_2 a_2 \qquad \text{...(iii)}$$

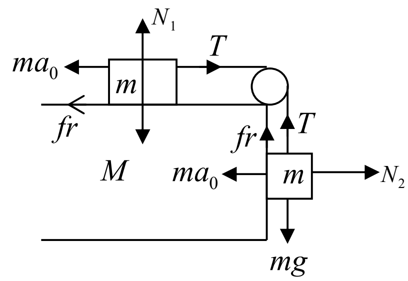
from (i), (ii) and (iii)

$$a_1 = \frac{4m_1m_2 g + m_0(m_1 - m_2)g}{4m_1m_2 + m_0(m_1 + m_2)}$$

Example: Coefficient of friction is μ. What will be acceleration of table such that system will be in equilibrium?



Solution: This whole wedge is moving with acceleration a_0.



Equation of motion for mass m which on the mass M

$N = mg$,

$ma_0 + fr = T \Rightarrow ma_0 + \mu N_1 = T$

$ma_0 + \mu mg = T \Rightarrow m(a_0 + \mu g) = T$ (1)

Equation of motion for mass m which is hanged vertical the mass M

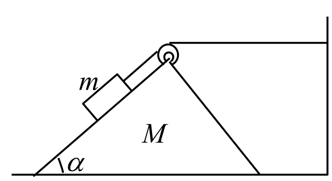
$N_2 - ma_0 = 0 \Rightarrow N_2 = ma_0$

$fr + T = mg \Rightarrow T = mg - fr, \Rightarrow T = mg - \mu ma_0 \Rightarrow T = m(g - \mu a_0)$ (2)

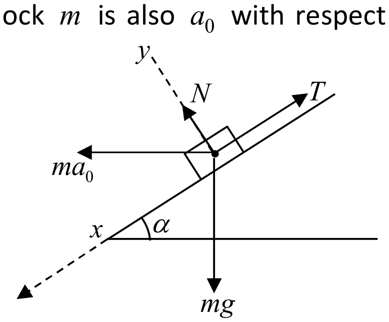
From (1) and (2) $ma_0 + m\mu g - mg + m\mu a_0 = 0$

$$-g(1-\mu) + a_0(\mu+1) = 0 \Rightarrow a_0 = g\frac{(1-\mu)}{(1+\mu)}$$

Example: In the arrangement shown in figure block of mass m slides on the surface of Wedge with inclination α. The masses of pulley and thread is negligible and friction on each surface is absent. If mass of Wedge is M then find the acceleration of wedge M.



Solution: Let us assume the mass M is moving with acceleration a_0 towards the wall. Hence the length of thread is fixed then the acceleration of the block m is also a_0 with respect to surface of wedge in downward direction.



Now let's draw free body diagram for mass m.

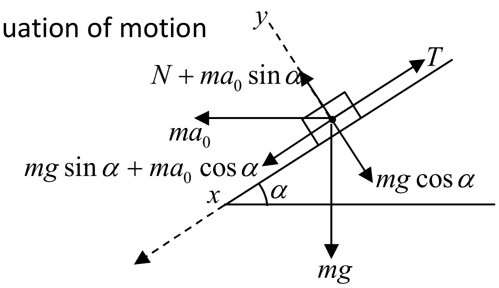
1. The weight mg in downward direction

2. The tension T along negative x axis

3. The normal force N along positive y axis.

4. If observer is attached to surface of wedge then he is on accelerating frame then he will measure Pseudo force ma_0 in horizontal direction.

Now resolve all forces along suitable axis and write equation of motion



The equation of motion along x - axis is

$$mg\sin\alpha + ma_0\cos\alpha - T = ma_0$$

$$T = mg\sin\alpha + ma_0\left(1-\cos\alpha\right) \qquad .....(1)$$

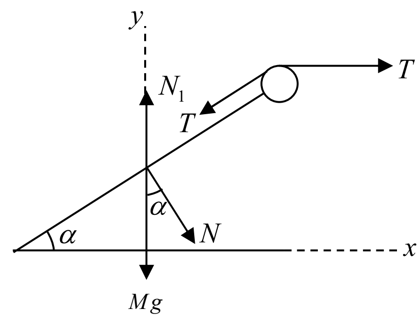
The equation of motion along y axis

$$N + ma_0\sin\alpha - mg\cos\alpha = 0$$

$$\Rightarrow N = mg\cos\alpha - ma_0\sin\alpha \qquad ....(2)$$

Now draw free body diagram for wedge M

1. The weight Mg in downward direction
2. Normal force N_1 in y direction
3. Reaction force N due to block m on wedge
4. Hence pulley is attached on wedge so tension on thread pull the wedge along x axis and parallel to surface of wedge



Now resolve the forces along suitable axis

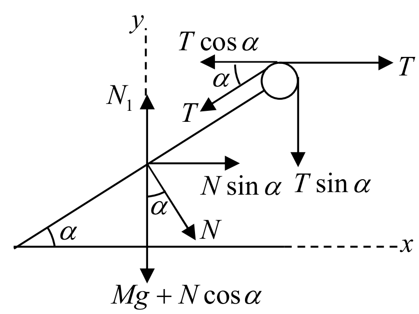
The equation of motion along x axis

$$T - T\cos\alpha + N\sin\alpha = Ma_0 \Rightarrow T = \frac{Ma_0 - N\sin\alpha}{1-\cos\alpha}$$

Put the value of N from equation (2)

$$\Rightarrow T = \frac{Ma_0 - \left(mg\cos\alpha - ma_0\sin\alpha\right)\sin\alpha}{1-\cos\alpha}$$

$$\Rightarrow T = \frac{Ma_0 - mg\sin\alpha\cos\alpha + ma_0\sin^2\alpha}{1-\cos\alpha} \qquad ....(3)$$



The equation of motion along y - axis is $N_1 - Mg + N\cos\alpha + T\sin\alpha = 0$

From equation (1) and equation (3)

$$\Rightarrow T = mg\sin\alpha + ma_0\left(1-\cos\alpha\right) = \frac{Ma_0 - mg\sin\alpha\cos\alpha + ma_0\sin^2\alpha}{1-\cos\alpha}$$

Now a_0 can be easily solve $a_0 = \dfrac{mg\sin\alpha}{M + 2m\left(1-\cos\alpha\right)}$